SERIES SOLUTIONS FOR THE FLOW IN THE VICINITY OF THE EQUATOR OF AN MAGNETOHYDRODYNAMIC BOUNDARY-LAYER OVER A POROUS ROTATING SPHERE WITH HEAT TRANSFER

by

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Original scientific paper
DOI: 10.2298/TSCI120301155R

DTM-Pade analytical method is employed to solve the flow and heat transfer near the equator of an magnetohydrodynamic boundary-layer over a porous rotating sphere. This method is used to give solutions of non-linear ordinary differential equations with boundary conditions at infinity. The velocity components in all directions (meridional, rotational, and radial) and temperature fields are derived. The obtained results are verified with the results of numerical solution. A very good agreement can be observed between them. The effect of involved parameters such as magnetic strength parameter, rotation number, suction/blowing parameter, and Prandtl number on the all-different types of velocity components, temperature field and surface shear stresses in meridional and rotational directions, infinite radial velocity, and rate of heat transfer is checked and discussed.

Key words: *magnetohydrodynamic flow, rotating sphere, heat transfer, DTM-Pade, boundary-layer, shear stresses*

Introduction

Flows over rotating bodies such as disks, surfaces, spheres, *etc.* have increasing applications in several various industrial devices like nuclear reactors, turbines, *etc.* In this paper, we focused on the magnetohydrodynamic (MHD) boundary-layer flows over the porous rotating spheres. In the case of flows over rotating sphere, several works have been done in recent years. Singh [1] studied the laminar boundary-layer flow over a rotating sphere. Ranger [2] investigated the rotation of a conducting sphere in the presence of a uniform magnetic field. Ingham [3] solved boundary-layer equations in the form of the flow near the equator of a rotating sphere in a rotating fluid. Takhar *et al.* [4] discussed the unsteady laminar MHD flow with heat transfer in the stagnation zone of an impulsively rotating and translating sphere in the attendance of the buoyancy forces. Chamkha *et al.* [5] analyzed the unsteady MHD rotating flow over a rotating sphere near the equator numerically using an implicit finite-difference scheme. Kumari and Nath [6] carried out the transient rotating flow of a laminar incompressible viscous electrically conducting fluid over a rotating sphere near the equator. As the results show, the body and fluid angular velocities and magnetic field have considerable effects on the flow fields. On the other hand, the surface shear stresses change significantly, as the involved parameters vary. Dinarvand *et al.* [7] studied the unsteady laminar MHD flow of an impulsively rotating and translating sphere in the presence of buoyancy forces ana-

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lytically *via* the homotropy analysis method (HAM). Beg *et al.* [8] investigated the convective heat transfer on a rotating sphere in the attendance of strong magnetic field, impulsive motion and buoyancy forces numerically by using the Blottner's finite-difference method. They also considered the effect of involved parameters such as magnetic field, buoyancy parameter, Prandtl number, and thermal conductivity parameter on the translational velocities and temperature fields. Sweet *et al.* [9] demonstrated the 3-D MHD rotating flow of a viscous fluid over a rotating sphere in the vicinity of the equator analytically *via* HAM.

The ability to induce currents in a movable conductive fluid is the basic concept of MHD. For the examples of MHD fluids one can be cited to plasmas, liquid metals and electrolytes. The presence of magnetic fields can induce the forces in which act on the fluid [10]. Magnetic fields are used to pump, heat and stir liquid metals in industry [11].

Non-linear equations are used to describe some of physical systems in the form of mathematical modeling. Concurrent with the development of computers, rising use of analytical methods can be observed in comparison with the numerical methods. Despite all the applications, there are many cons for the numerical methods such as disability to apply infinite boundary condition, *etc.* There are a lot of analytical methods, such as HPM [12, 13], HAM [14], differential transform method (DTM) [15, 16] which are applied to solve non-linear equations. In the current paper, DTM is applied to solve the non-linear differential equations. The main use of the proposed method is that it can be used straightly to non-linear differential equations without requiring linearization and discretization [17] also does not require perturbation parameter. Ayaz employed DTM to solve differential-algebraic equations [15] and system of differential equations [16]. Arikoglu and Ozkol [18] applied DTM to solve the boundary value problems for differential-difference equations and integro-differential equations [19]. The combination of DTM and Pade approximation was used to solve high non-linear equations, which is called DTM-Pade. This method is used to maximize the convergence of DTM. The DTM-Pade was used to solve different kinds of non-linear problems with the high order of non-linearity [20-22].

In this article, the system of non-linear equations for the flow and heat transfer near the equator of an MHD boundary-layer over a porous rotating sphere is derived. This problem was firstly studied by Turkyilmazoglu [23]. The similarity solutions are applied to convert the systems of non-linear equations into set of ordinary differential equations. In order to solve the set of ordinary differential equations (ODE), the combination of DTM-Pade is employed. In the continued of this paper, the effect of involved parameters such as magnetic parameter, Prandtl number, rotation number and suction/blowing parameter on the different kinds of velocity components, shear stresses, temperature fields and rate of heat transfer has been investigated.

Problem statement and mathematical formulation

Consider steady laminar viscous incompressible electrically conducting rotating infinite fluid near the equator of a rotating sphere. The body as well as the fluid swirl in the same or opposite directions with angular velocities Ω_b and Ω_f, respectively. A spherical polar co-ordinate system, which is fixed in space with origin at the center of the sphere, $\theta = 0$ is the axis of rotation. The radius of the sphere is a, the distance r^* is gauged radially from the center of the sphere, ϕ is the azimuth, and θ is the latitude measured from the axis of rotation. The movement is assumed to be axisymmetric, which is, independent of the azimuthal angle. The surface of the sphere is supposed to be electrically insulated. The magnetic field is assumed to be applied in the radial r-direction. The magnetic parameter $m = (\sigma B^2)/(\rho \Omega_\mathrm{f})$ depends on the strength of the magnetic field, the electrical conductivity of the fluid and the fluid density. Due to the assumptions, the governing equations of the conducting fluid and heat flow are then expressed in the form (for more details see [23]):

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v\sin\theta)=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial\theta}-\frac{v^2+w^2}{r}=-\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left[\nabla^2 u-\frac{2u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial\theta}-\frac{2v\cot\theta}{r^2}\right] \tag{2}$$

$$u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial\theta}+\frac{uv-w^2\cot\theta}{r}=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\frac{1}{\mathrm{Re}}\left[\nabla^2 v+\frac{2}{r^2}\frac{\partial u}{\partial\theta}-\frac{v}{r^2\sin^2\theta}\right]-mv \tag{3}$$

$$u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial\theta}+\frac{uw+vw\cot\theta}{r}=\frac{1}{\mathrm{Re}}\left[\nabla^2 w-\frac{w}{r^2\sin^2\theta}\right]-m(w-r\sin\theta) \tag{4}$$

$$u\frac{\partial T}{\partial r}+\frac{v}{r}\frac{\partial T}{\partial\theta}=\frac{1}{\mathrm{PrRe}}\left[\nabla^2 T\right]+Ek\Phi+mEk\,\mathrm{Re}\,JL \tag{5}$$

where

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) \tag{6}$$

$$\Phi=2\left[\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{u}{r}\right)^2+\left(\frac{u}{r}+\frac{v\cot\theta}{r}\right)^2\right]+$$

$$+\left[r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)+\frac{1}{r}\frac{\partial u}{\partial\theta}\right]^2+\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{w}{\sin\theta}\right)\right]^2+\left[r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right]^2 \tag{7}$$

where Φ and $JL = v^2 + w^2$ are viscous dissipation and Joule heating terms, respectively. u, v, and w show the dimensionless velocity components in the radial r, meridional θ, and rotational ϕ directions, respectively, that obtained by dividing the dimensional velocity components by $a\Omega_\mathrm{f}$. In addition, $Ek = (\Omega_\mathrm{f}^2 a^2)/(c_p\mathrm{Re})$, and $r = r^*/a$ is the dimensionless radial distance. The associated boundary conditions of the above equations become:

$$\begin{gathered} u(1,\ \theta)=u_s,\quad v(1,\ \theta)=0,\quad w(1,\ \theta)=\lambda r\sin\theta,\quad T(1,\ \theta)=T_w, \\ v(\infty,\ \theta)=0,\quad w(\infty,\ \theta)=r\sin\theta,\quad T(\infty,\ \theta)=T_\infty \end{gathered} \tag{8}$$

where u_s indicates surface injection ($u_s > 0$), and suction ($u_s < 0$), $\lambda = \Omega_\mathrm{b}/\Omega_\mathrm{f}$ is the ratio of the angular velocity of the sphere to the angular velocity of the distant fluid, which is called the rotation number. The ambient temperature and wall temperature are assumed to be constant.

By using the Von Karman [24] similarity solution over a rotating disk, one can obtain the subsequent similarity variables:

$$\eta=\sqrt{\mathrm{Re}}(r-1),\quad u=\frac{U}{\sqrt{\mathrm{Re}}},\quad v=rV,\quad w=rW,\quad T=T_\infty+(T_w-T_\infty)\Theta \tag{9}$$

and by applying these similarity variables into the governing eqs. (1)-(5) and boundary conditions (8), a set of simplified partial differential equations can be obtained:

$$U_\eta + V_\theta + V\cot\theta = 0 \tag{10}$$

$$UV_\eta + VV_\theta - W^2\cot\theta = -\frac{\sin 2\theta}{2} + V_{\eta\eta} - mV \tag{11}$$

$$UW_\eta + VW_\theta + VW\cot\theta = W_{\eta\eta} - m(W - \sin\theta) \tag{12}$$

$$U\Theta_\eta + v\Theta_\theta = \frac{1}{\mathrm{Pr}}\Theta_{\eta\eta} + \mathrm{Ec}\left[V_\eta^2 + W_\eta^2\right] + \mathrm{Ec}m\left[V^2 + W^2\right] \tag{13}$$

$$U(0,\ \theta) = s,\quad V(0,\ \theta) = 0,\quad W(0,\ \theta) = \lambda\sin\theta,\quad \Theta(0,\ \theta) = 1,$$

$$V(\infty,\ \theta) = 0,\quad W(\infty,\ \theta) = \sin\theta,\quad \Theta(\infty,\ \theta) = 0 \tag{14}$$

where $s = u_s\,\mathrm{Re}^{-1/2}$ is the scaled suction/blowing parameter, which is physically corresponding to mass transfer. Near the pole, the above problem decreases to the steady equivalent of the well-known Karman swirling flow. Near the equator $\theta \approx \pi/2$ and hereupon it can be stated that [5]:

$$U(\eta,\ \theta) = H(\eta),\quad V(\eta,\ \theta) = -F(\eta)\cos\theta,\quad W(\eta,\ \theta) = G(\eta),\quad \Theta(\eta,\ \theta) = \Theta(\eta) \tag{15}$$

which eq. (15) simplifies the mean flow equations near the equator to the set of ordinary differential equations:

$$F'' - HF' - F^2 - G^2 + 1 - mF = 0 \tag{16}$$

$$G'' - HG' - m(G-1) = 0 \tag{17}$$

$$F + H' = 0 \tag{18}$$

$$\frac{1}{\mathrm{Pr}}\Theta'' - H\Theta' + \mathrm{Ec}\left[G'^2 + mG^2\right] = 0 \tag{19}$$

with the boundary conditions:

$$F(0) = 0,\quad G(0) = \lambda,\quad H(0) = s,\quad \Theta(0) = 1,$$

$$F(\infty) = 0,\quad G(\infty) = 1,\quad \Theta(\infty) = 0 \tag{20}$$

It should also be noted that the viscous dissipation term and Joule heating term JL decrease to $\mathrm{Ec}G'^2$ and $m\mathrm{Ec}G^2$, respectively, as given in eq. (19). After solving the mean flow quantities from the system of eqs. (16)-(19) and (20), one can evaluate the skin friction coefficients, the torque and the rate of heat transfer, which are of principal physical interest. The act of the viscosity in the fluid neighbor to the sphere sets up a rotational shear stress, which opposes the rotation of the sphere. As a result, it is necessary to prepare a torque at the shaft to preserve a steady rotation. In order to obtain the meridional shear stress τ_θ and rotational shear stress τ_ϕ the Newtonian formula is applied:

$$\tau_\theta = \mu \left[r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{1}{r}\frac{\partial u}{\partial \theta} \right]\Bigg|_{r=1} = RF'(0) \tag{21}$$

$$\tau_\phi = \mu \left[\frac{1}{r\sin\theta}\frac{\partial u}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{w}{r}\right) \right]\Bigg|_{r=1} = RG'(0) \tag{22}$$

The rate of heat transfer from the fluid to the sphere surface is calculated by the application of Fourier's law and the normalized Nusselt number can be derived from it. Hence, we calculate $F'(0)$, $G'(0)$, $H(\infty)$, and $\theta'(0)$ due to realize the underlying physics of the problem:

$$q = -\frac{\partial T}{\partial r}\Bigg|_{r=1} = -R\theta'(0) \tag{23}$$

Analytical approximations by means of DTM-Pade

Taking differential transform of eqs. (16)-(19), one can obtain, for more details, see [20-22]:

$$(k+1)(k+2)F(k+2) - \sum_{r=0}^{k}\left[\begin{array}{l}(k+1-r)H(r)F(k+1-r)+\\ +F(r)F(k-r)+G(r)G(k-r)\end{array}\right] - mF(k)+1=0 \tag{24}$$

$$(k+1)(k+2)G(k+2) - \sum_{r=0}^{k}\left[(k+1-r)H(r)G(k+1-r)\right] - m\left[G(k)-1\right]=0 \tag{25}$$

$$(k+1)H(k+1)+F(k)=0 \tag{26}$$

$$\begin{array}{c}(k+1)(k+2)\Theta(k+2)-\\ -\mathrm{Pr}\sum_{r=0}^{k}\left\{\begin{array}{c}(k+1-r)H(r)\Theta(k+1-r)\\ -\mathrm{Ec}\left[(k-r+1)(r+1)G(r+1)G(k-r+1)+mG(r)G(k-r)\right]\end{array}\right\}=0\end{array} \tag{27}$$

where $F(k)$, $G(k)$, $H(k)$, and $\Theta(k)$ are the differential transforms of $F(\eta)$, $G(\eta)$, $H(\eta)$, and $\theta(\eta)$ displayed by:

$$F(\eta) = \sum_{k=0}^{\infty} F(k)\eta^k \tag{28}$$

$$G(\eta) = \sum_{k=0}^{\infty} G(k)\eta^k \tag{29}$$

$$H(\eta) = \sum_{k=0}^{\infty} H(k)\eta^k \tag{30}$$

$$\theta(\eta) = \sum_{k=0}^{\infty} \Theta(k)\eta^k \tag{31}$$

$$H(0)=s, \quad F(0)=0, \quad F(1)=\alpha, \quad G(0)=\lambda, \quad G(1)=\beta, \quad \theta(0)=1, \quad \theta(1)=\gamma \tag{32}$$

when (32) are the transformed boundary conditions and *α, β,* and *γ* are constants. Substituting eq. (32) into eqs. (24)-(27) and with a recursive method and by the use of eqs. (28)-(31), we obtain the values of $F(\eta)$, $G(\eta)$, $H(\eta)$, and $\theta(\eta)$:

$$F(\eta)=\alpha\eta+\frac{1}{2}\eta^2(-1+s\alpha+\lambda^2)+\frac{1}{6}\eta^3\left[m\alpha+2\beta\lambda+s(-1+s\alpha+\lambda^2)\right]+\frac{1}{12}\eta^4\cdot$$

$$\cdot\left\{\frac{\alpha^2}{2}+\beta^2+\lambda\left(-m+s\beta+m\lambda\right)+\frac{1}{2}m(-1+s\alpha+\lambda^2)+\frac{1}{2}s\left[m\alpha+2\beta\lambda+s(-1+s\alpha+\lambda^2)\right]\right\}+\cdots \quad (33)$$

$$G(\eta)=\beta\eta+\lambda+\frac{1}{2}\eta^2(-m+s\beta+m\lambda)+\frac{1}{6}\eta^3\left[m\beta+s(-m+s\beta+m\lambda)\right]+$$

$$+\frac{1}{12}\eta^4\left\{-\frac{\alpha\beta}{2}+\frac{1}{2}m(-m+s\beta+m\lambda)+\frac{1}{2}s\left[m\beta+s(-m+s\beta+m\lambda)\right]\right\}+\cdots \quad (34)$$

$$H(\eta)=s-\frac{\alpha\eta^2}{2}+\frac{1}{6}\eta^3(1-s\alpha-\lambda^2)+\frac{1}{24}\eta^4\left[-m\alpha-2\beta\lambda-s(-1+s\alpha+\lambda^2)\right]+\cdots \quad (35)$$

$$\theta(\eta)=1+\gamma\eta+\frac{1}{2}\Pr\eta^2\left[s\gamma-\mathrm{Ec}(\beta^2+m\lambda^2)\right]+$$

$$+\frac{1}{6}\Pr\eta^3\left\{-2\mathrm{Ec}\left[m\beta\lambda+\beta(-m+s\beta+m\lambda)\right]+\Pr s\left[s\gamma-\mathrm{Ec}(\beta^2+m\lambda^2)\right]\right\}+$$

$$+\frac{1}{12}\Pr\eta^4\begin{pmatrix}-\dfrac{\alpha\gamma}{2}-\mathrm{Ec}\left[m\beta^2+(-m+s\beta+m\lambda)^2\right]\\ -2\mathrm{Ec}\left\{\dfrac{1}{2}m\lambda(-m+s\beta+m\lambda)+\dfrac{1}{2}\beta\left[m\beta+s(-m+s\beta+m\lambda)\right]\right\}\\ +\dfrac{1}{2}\Pr s\left\{-2\mathrm{Ec}\left[m\beta\lambda+\beta(-m+s\beta+m\lambda)\right]+\Pr s\left[s\gamma-\mathrm{Ec}(\beta^2+m\lambda^2)\right]\right\}\end{pmatrix}+\cdots \quad (36)$$

Using the infinity boundary conditions of eq. (20), one can obtain *α, β,* and *γ*. The number of required terms is determined by the convergence of the numerical values to one's desired accuracy.

Convergence of the DTM

DTM is one of the analytical methods, which provides the solution in the form of a rapidly convergent series. The Pade approximant is used to increase the convergence radius of the truncated series solution. We obtained the approximants using *MATHEMATICA* software. As it is illustrated in figs. 1(a-d), without using the Pade approximant, the different orders of DTM solution, cannot satisfy boundary conditions at infinity. Therefore, it is necessary to use DTM-Pade to provide an effective way to handle infinite boundary value problems. The Pade approximation is applied to eqs. (33)-(36).

Results and discussions

A comparison has been done to verify the accuracy of the results. Figures 1(a-d) show a very good agreement between the results of DTM-Pade of order [20, 20], and the result of numerical solutions, which are obtained with the shooting method.

The effect of magnetic strength parameter on the velocity components in the meridional, rotational and radial directions and temperature field is illustrated in figs. 2(a-d). The results show that the velocity components in meridional and magnitude of radial orientation reduce, as the magnetic strength parameter increases. The behavior of the rotational velocity component per magnetic strength parameter variations is in contrast with the other velocity components. It must be noted that the temperature profile increases, as the magnetic strength parameter increases. Figures 3(a-c) show the effect of magnetic strength parameter on the surface shear stresses in the meridional and rotational directions and on the radial velocity component in which comes from infinity. The results represent that the surface shear stresses in the meridional orientations reduce as the magnetic strength parameter increases. In addition, surface shear stresses in the meridional direc-

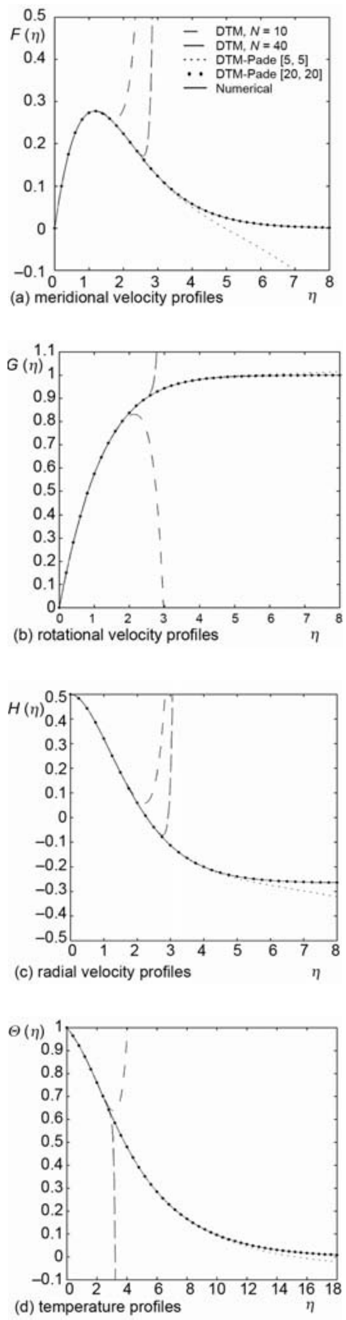

Figure 1. The obtained results by the DTM for different values of *N* and different orders of DTM-Pade in comparison with the numerical solution when λ = Ec = = 0, *m* = Pr = 1, and *s* = 0.5

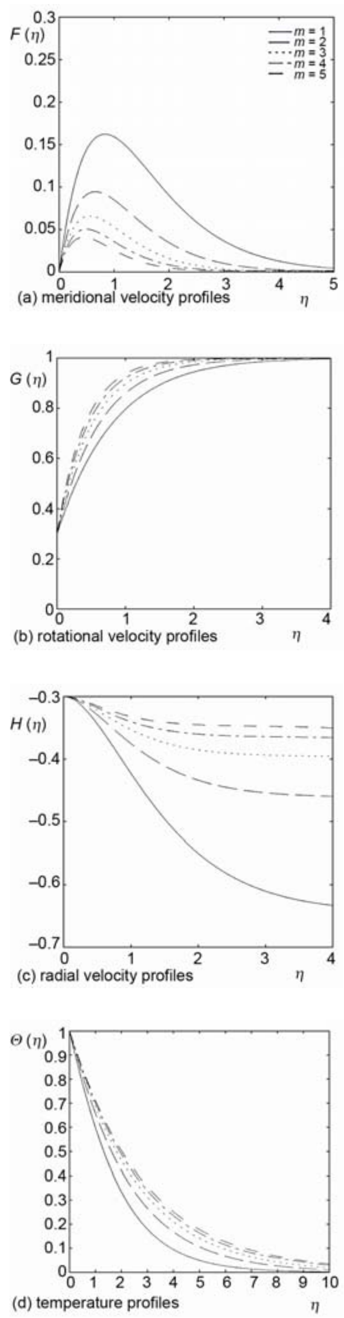

Figure 2. The effect of magnetic strength parameter on the flow quantities for the selected parameters Ec = 0, $\lambda = -s = 0.3$, and Pr = 1

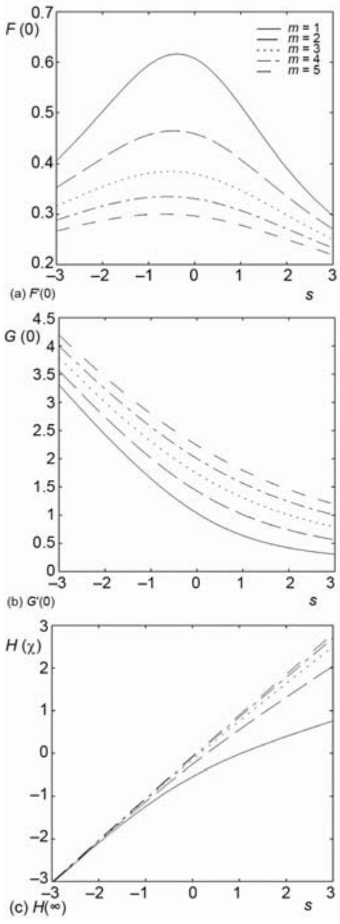

Figure 3. The effect of magnetic strength parameter for various parameters –3 ≤ *s* ≤ 3, and *λ* = 0

tions become maximize when no mass transfer occurs for all of magnetic strength parameter. The surface shear stresses in the rotational directions increase as the *m* increases. The radial incoming velocity from infinity increases, as the magnetic parameter increases. It should also be remarked that for the case of wall suction the effect of magnetic strength parameter on the $H(\infty)$will be minimal in comparison with the wall injections.

Figures 4(a-d) displays the effect of ratio of the body and fluid angular velocities on the different components of the velocity and temperature field without mass transfer in the presence of the conducting case. As the results show, the velocity profiles in all directions change more when the fluid and body rotate in the opposite side than when they swirl in the same direction. In other words, as the rotation number increases, the velocity profile in all directions reduces considerably. It can be also concluded that the temperature field increases further as the fluid and body rotate in the same direction than they rotate in opposite orientation.

The effect of rotation number on the surface shear stresses in the meridional and rotational directions and on the radial incoming velocity from infinity is presented in figs. 5(a-c). When the fluid and body rotate in the same direction, the shear stresses in the rotational directions change fewer than when they rotate in the opposite directions. A contrasting behavior can be seen for the infinite radial velocity components in comparison with the shear stresses in the rotational direction variations.

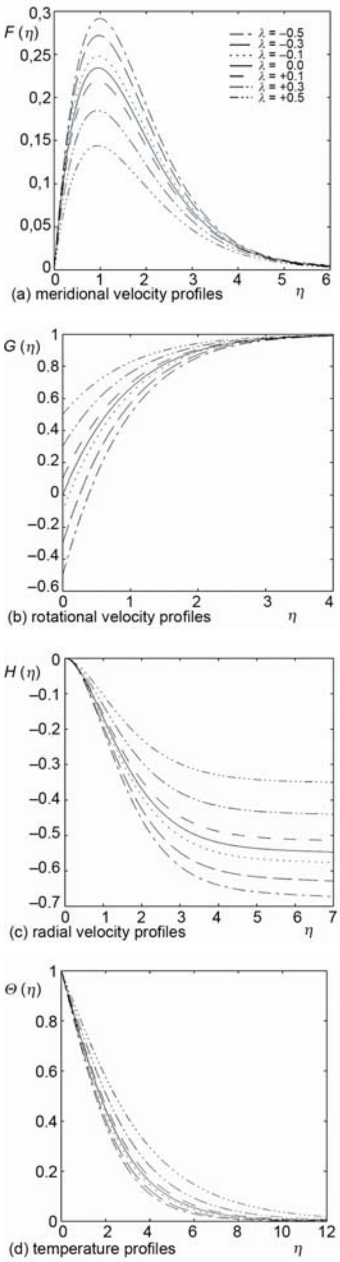

Figure 4. The effect of rotation number on the flow quantities for the selected parameters *s* = Ec = 0, and *m* = Pr = 1

The effect of mass transfer rate in the form of suction/blowing parameter on the all velocity components and temperature field is discussed in figs. 6(a-d). The results present that

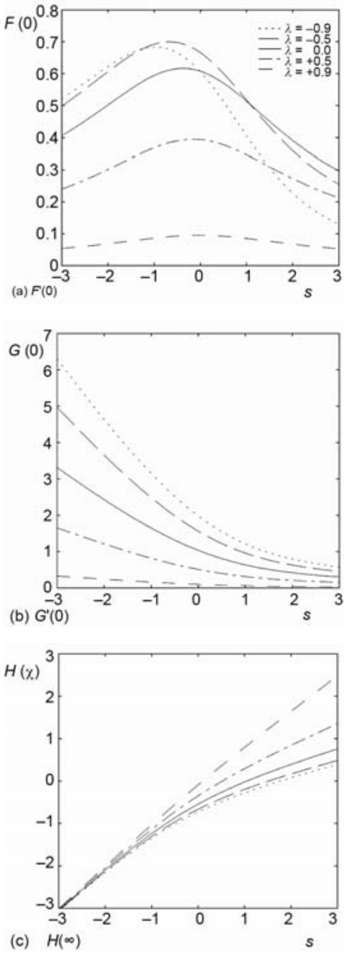

Figure 5. The effect of rotation number for various parameters –3 ≤ *s* ≤ 3, and *m* = 1

when the mass transfer appears in the case of wall injection ($s > 0$), the velocity components in the meridional and radial direction increase further than the wall suction case ($s < 0$). As the value of wall suction rate becomes greater, the rotational velocity component increases and the temperature field decreases.

The effect of Prandtl number variation on the temperature profile is displayed in fig. 7(a). The result demonstrates that as the Prandtl number increases, the temperature field reduces considerably. It means that the flow with large Prandtl number hampers extension of heat in the fluid. Figure 7(b) illustrates the effect of Prandtl number on the rate of heat transfer. As the results show the Prandtl number variations change the rate of heat transfer particularly, for the case of wall suction. In the case of wall suction, the rate of heat transfer increases, as the Prandtl number increases.

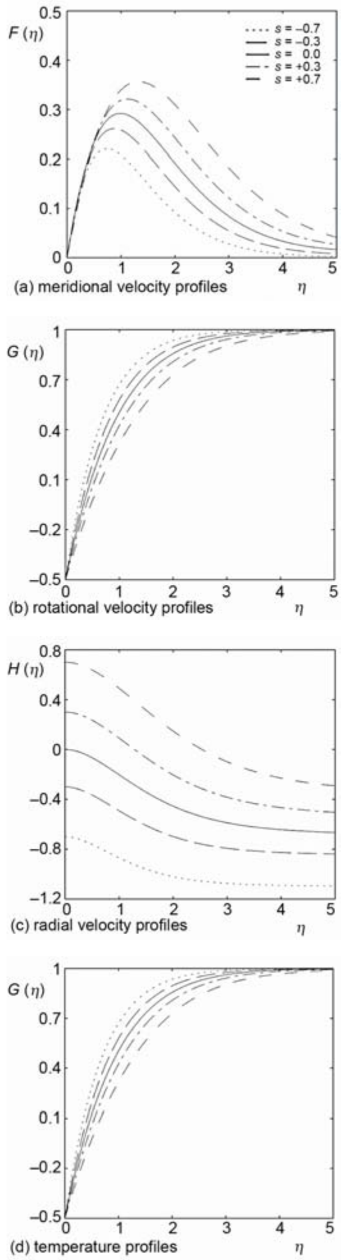

Figure 6. The effect of suction/ /blowing parameter on the flow quantities for the selected parameters *s* = Ec = 0, *m* = Pr = = 1, and *λ* = –0.5

Conclusions

A mathematical formulation has been derived for the flow and heat transfer near the equator of an MHD boundary-layer over a porous rotating sphere. An analytical method, which is named DTM-Pade, is applied to solve the governing equations. The effect of involved parameters on the all velocity components, temperature fields, the surface shear stresses in all directions and the rate of heat transfer is presented. A summary of the most important results is given below. As the magnetic strength parameter increases, the velocity components in meridional and magnitude of radial orientation reduce and rotational velocity component and temperature field increase. It should be noted that the surface shear stresses in the

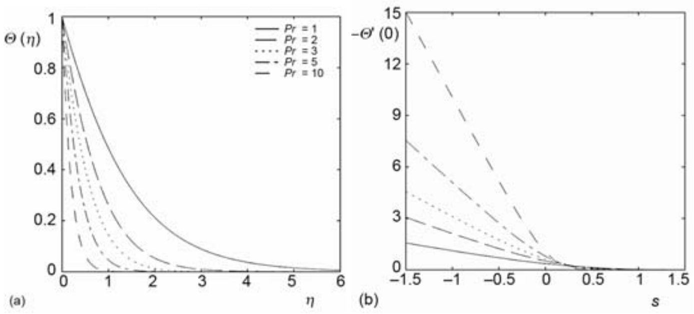

Figure 7. Effect of Prandtl number on (a) temperature profiles for Ec = 0, λ = –0.1, s = –0.5, and m = 1, and (b) $-\theta'(0)$ for – 1.5 ≤ s ≤ 1.5, Ec = 0, and $m = -\lambda = 1$

meridional orientations reduce and the surface shear stresses in the rotational directions and the radial incoming velocity from infinity increase, as the magnetic strength parameter increases. The velocity profiles in all directions and the shear stresses in the rotational direction change more when the fluid and body rotate in the opposite side than when they rotate in the same direction. It can be also noted that the temperature field increases further as the fluid and body rotate in the same direction than they rotate in opposite orientation, similar to the infinite radial velocity component. When mass transfer appears in the form of wall injection, the velocity components in the meridional and radial directions increase more than the wall suction case. The rotational velocity component increases and the temperature field decreases, as the value of wall suction rate increases. As the Prandtl number increases, the temperature field reduces considerably. In addition, the Prandtl number variations change the rate of heat transfer particularly, for the case of wall suction.

Nomenclature

B – magnetic field, [kgs^{-2}A^{-1}]
c_p – specific heat at constant pressure, [Jkg^{-1}K^{-1}]
Ec – Eckert number, (= EkRe/($T_w - T_\infty$), [–]
F – self-similar meridional velocity
G – self-similar rotational velocity
H – self-similar radial velocity
k – thermal conductivity, [Wm^{-1}K^{-1}]
m – magnetic parameter, (= $\sigma B^2/\rho\Omega_f$), [–]
Pr – Prandtl number, (= $\upsilon/c_p k$), [–]
p – dimensionless mean pressure
q – heat flux, [Wm^{-1}]
Re – Reynolds number, (=$a^2\Omega_f/\upsilon$), [–]
r – radial direction in spherical polar co-ordinates, [m]
s – suction/blowing parameter, [ms^{-1}]
T – temperature, [K]
u_s – surface velocity, [ms^{-1}]

Greek symbols

θ – spherical direction in spherical polar coordinates
ϕ – azimuthal direction in spherical polar coordinates
υ – kinematic viscosity, [m^{2}s^{-1}]
ρ – fluid density, [kgm^{-3}]
σ – fluids' electrical conductivity, [Sm^{-1}]
Ω – angular velocity, [s^{-1}]

Subscripts

b – body
f – fluid
w – wall
∞ – ambient

Acknowledgment

We express our gratitude to the anonymous referees for their constructive reviews of the manuscript and for helpful comments.

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Paper submitted: March 1, 2012
Paper revised: August 27, 2012
Paper accepted: August 27, 2012