A MATHEMATICAL MODEL ON MAGNETOHYDRODYNAMIC SLIP FLOW AND HEAT TRANSFER OVER A NON-LINEAR STRETCHING SHEET

by

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Some analyses have been carried out to study the influence of suction/blowing, thermal radiation and temperature dependent fluid properties on the hydromagnetic incompressible electrically conducting fluid flow and heat transfer over a permeable stretching surface with partial slip boundary conditions. It is assumed that the fluid viscosity and the thermal conductivity vary as an inverse function and linear function of temperature. Using the similarity transformation, the governing system of non-linear partial differential equations are transformed into non-linear ordinary differential equations and are solved numerically using symbolic software MATHEMATICA 7.0. The effects of various physical parameters on the flow and heat transfer characteristics as well as the skin friction coefficient and Nusselt number are illustrated graphically. The physical aspects of the problem are highlighted and discussed.

Key words: *boundary layer flow, stretching sheet, variable viscosity, variable thermal conductivity, thermal radiation*

Introduction

A significant amount of research on fluid flow and heat transfer caused by continuously stretched or moving surfaces [1-8] under different conditions and in the presence of various physical effects has been reported. The study of flow and heat transfer of an electrically conducting fluid past a porous plate under the influence of a magnetic field has attracted the interest of numerous researchers in view of its applications in many engineering problems, such as magnetohydrodynamic (MHD) generators, nuclear reactors, geothermal energy extractions and the boundary layer control in the field of aerodynamics. Keeping in mind some specific industrial applications such as polymer processing technology, numerous attempts have been made to analyze the effect of transverse magnetic field on boundary layer flow characteristics. Eldabe and Ouaf [9] considered the problem of heat and mass transfer in a MHD flow of a micropolar fluid past a stretching surface with ohmic heating and viscous dissipation effects using the Chebyshev finite difference method. However, the effect of thermal radiation [10] on the flow and heat transfer have not been provided in the most investigations. The effect of radiation on MHD flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for design of reliable equipment, nuclear plants, gas turbines

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and various propulsion devices or aircraft, missiles, satellites and space vehicles. Based on these applications, Cogley *et al.* [11] showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Abdus Satter and Hamid Kalim [12] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. Makind [13] examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate. Hossain and Takhar [14] have considered the radiation effect on mixed convection boundary layer flow of an optically dense viscous incompressible fluid along vertical plate with uniform surface temperature. Ibrahim *et al*. [15] discussed the case of mixed convection flow of a micropolar fluid past a semi-infinite, steady moving porous plate with varying suction velocity normal to the plate in presence of thermal radiation and viscous dissipation. Raptis [16] investigate the steady flow of a viscous fluid through a porous medium bounded by a porous plate subjected to a constant suction velocity in presence of thermal radiation.

In all the above mentioned papers the thermophysical properties of the ambient fluid were assumed to be constant. However, it is well known [17-23] that these physical properties may change with temperature, especially for fluid viscosity and thermal conductivity. For lubricating fluids, heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid and the properties of the fluid are no longer assumed to be constant. The increase in temperature leads to increase in the transport phenomena by reducing the physical properties across the thermal boundary layer and so the heat transfer at the wall is also affected. Therefore to predict the flow and heat transfer rates, it is necessary to take into account the variable fluid properties. Recently Prasad *et al*. [24] studied the effect of variable viscosity and electric conductivity on the hydro-magnetic flow and heat transfer over a stretching plate.

Analyses has been restricted to flow and heat transfer with no-slip boundary condition. But no-slip assumption is not consistent with all physical characteristics *i. e.*, in some practical flow situations it is essential to replace the no-slip boundary condition by the partial slip boundary condition. When fluid flows in micro electromechanical system (MEMS), the no slip condition at the solid-fluid interface is no longer applicable. A slip flow model more accurately describes the non-equilibrium region near the interface. A partial slip may occur on a stationary and moving boundary when the fluid is particulate such as emulsions, suspensions, foams, and polymer solutions. Beavers and Joseph [25] investigated the fluid flow over a permeable wall using slip boundary condition. In biomedical engineering, when blood flows through an artery slip flow is evident from experimental observations [26, 27]. The slip flows under different flow configurations have been studied in recent years [28-36].

In this paper, we extended the work of Prasad *et al.* [24] to investigate the slip effect and thermal radiation on the hydro-magnetic flow and heat transfer over a permeable stretching surface with variable fluid properties. The resulting governing equations are transformed into a system of non-linear ordinary differential equations by applying a suitable similarity transformation. These equations are solved numerically using MATHEMATICA 7.0 and discussed the results from the physical point of view.

Mathematical formulation of the problem

Let us consider a steady two dimensional laminar convective flow of an electrically conducting incompressible fluid over a permeable stretching surface under the influence of a transverse magnetic field B in the presence of thermal radiation. The magnetic Reynolds number of the flow is taken to be small enough so that induced magnetic field is assumed to be negligible in comparison with applied magnetic field so that $B = [0, B(x)]$, where $B(x)$ is the applied magnetic field acting normal to the plate and varies in strength as a function of x. The flow is assumed to be in the x-direction which is taken along the plate and y-axis is normal to it. There is a constant suction/blowing velocity v_w normal to the plate. It is to be mentioned that the hole size of porous plate is taken to be constant. The viscosity and thermal conductivity of the fluid are assumed to be functions of temperature. The temperature of the plate surface are always greater than their free stream values. The flow configuration and the co-ordinate system are shown in fig. 1.

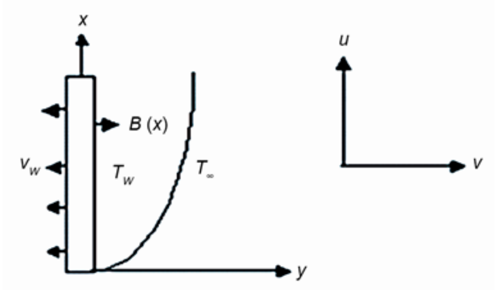

Figure 1. Physical model and coordinate system of the problem

Under boundary layer and Boussinesq approximation, the governing equations that describe the physical situation are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_\infty \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y} \right) - \sigma B^2(x)u \tag{2}$$

$$\rho_\infty c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left(\kappa \frac{\partial T}{\partial y} \right) - \frac{\partial q_r}{\partial y} \tag{3}$$

where u and v are velocity components along x- and y-axis, respectively, σ – the electrical conductivity of the fluid, T – the temperature of the fluid within the boundary layer, κ – the thermal conductivity of the fluid, c_p – the specific heat at constant pressure p, and ρ_∞ – the constant fluid density. Here μ is the dynamic viscosity and is considered to vary as an inverse function of temperature as:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \delta(T - T_\infty)] \tag{4}$$

where δ is the thermal property of fluid, T_∞ – the temperature of the fluid outside the boundary layer, and μ_∞ is the dynamic viscosity at ambient temperature.

Equation (4) can be written as:

$$\frac{1}{\mu} = A(T - T_\infty) \tag{5}$$

where $A = \delta/\mu_\infty$ and $T_r = T_\infty - 1/\delta$.

In general $A > 0$ corresponds to liquids and $A < 0$ to gases when the temperature at the plate is larger than that of the temperature at far away from the plate. The relations between the viscosity and the temperature for air and water are [24]:

– for air

$$\frac{1}{\mu} = -123.2(T - 742.6) \quad \text{based on} \quad T_\infty = 293 \text{ K } (20\,^\circ\text{C})$$

– for water

$$\frac{1}{\mu} = -29.83(T - 258.6) \quad \text{based on} \quad T_\infty = 288 \text{ K } (15\,^\circ\text{C})$$

In the eq. (2), the applied magnetic field $B(x)$ is assumed of the form [24]:

$$B(x) = B_0 x^{(m-1)/2} \tag{6}$$

to obtain a similarity solution, where B_0 is a constant.

Using eqs.(4) and (6) in eq.(2) we obtain:

$$\rho_\infty \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y}\left(\frac{\mu_\infty}{1+\delta(T-T_\infty)}\frac{\partial u}{\partial y} \right) - \sigma B_0{}^2 x^{(m-1)/2} u \tag{7}$$

The radiative heat flux term by using the Rosseland approximation for radiation [10] is given by:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{8}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that T_4 can be expressed as a linear combination of the temperature, we expand T_4 in Taylor's series about T_∞ and neglecting higher order terms, we get:

$$T^4 = 4T_\infty{}^3 T - 3T_\infty{}^4 \tag{9}$$

Thus we have:

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty{}^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{10}$$

Using eq. (10) in eq. (3) we obtain:

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left[\left(\kappa + \frac{16T_\infty{}^3\sigma^*}{3k^*} \right)\frac{\partial T}{\partial y} \right] + q''' \tag{11}$$

The appropriate boundary conditions for the present problem are:

$$\left.\begin{aligned} & u = u_w = bx^m + L\frac{\partial u}{\partial y}, \quad \text{v} = \text{v}_m, \quad T = T_w \ \text{ for } y = 0 \\ & u \to 0, \quad \text{T} \to T_\infty \ \text{ as } \ y \to \infty \end{aligned}\right\} \tag{12}$$

where b is the stretching rate and the surface is accelerated or decelerated from the extruded slit according as m is positive or negative, respectively, and L is the slip length.

Now we transform the system of eqs. (7) and (11) into a dimensionless form. To this end, let the dimensionless similarity variable be:

$$\eta = \frac{y}{x}\sqrt{\frac{(m+1)\mathrm{Re}_x}{2}} \tag{13}$$

where $\mathrm{Re}_x = u_w x/\upsilon_\infty$, and the dimensionless stream function $f(\eta)$ and dimensionless temperature $\theta(\eta)$ are:

$$f(\eta) = \frac{\psi}{u_w x\sqrt{\mathrm{Re}_x}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{14}$$

where the dimensionless stream function $\psi(x, y)$ identically satisfies the continuity eq. (1) with:

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{15}$$

The dimensionless temperature θ can also be written as:

$$\theta = \frac{T - T_r}{T_w - T_\infty} + \theta_r \tag{16}$$

where $\theta_r = T_r - T_\infty/(T_w - T_\infty) = -2/\delta(m+1)(T_w - T_\infty)$, and its value is determined by viscosity/temperature characteristics of the fluid under consideration and the operating temperature difference. If θ_r is large *i. e.,* if $T_w - T_\infty$ is small, the effects of variable viscosity on the flow can be neglected. On other hand, for smaller values of θ_r, either the fluid viscosity changes markedly with temperature or the operating temperature difference is high. It is important to note that θ_r is negative for liquids and positive for gases.

Using eq. (16), eq. (4) becomes:

$$\mu = \mu_\infty \frac{\theta_r}{\theta_r - \theta} \tag{17}$$

This viscosity model is very much appropriate for the present study.

We consider the temperature dependent thermal conductivity relationship in the form [29]:

$$\kappa = \kappa_\infty \left(1 + \varepsilon \frac{T - T_\infty}{\Delta T}\right) \tag{18}$$

where ε is the thermal conductivity parameter and $\Delta T = T_w - T_\infty$. This relation can be written as:

$$\kappa = \kappa_\infty (1 + \varepsilon\theta) \tag{19}$$

Now introducing eqs. (14) into eqs. (7) and (11), we obtain:

$$\frac{\theta_r}{\theta_r - \theta} f''' + ff'' + \frac{\theta_r}{(\theta_r - \theta)^2} f''\theta' - \beta f'^2 - Mf' = 0 \tag{20}$$

$$(1 + \varepsilon\theta + Nr)\theta'' + \varepsilon\theta'^2 + \Pr f\theta' = 0 \tag{21}$$

where prime denotes differentiation with respect to η and $\beta = 2m/(m + 1)$ is the stretching parameter, $M = (2\sigma B_0^2)/[\rho_\infty b(m + 1)]$ is the magnetic field parameter, $\mathrm{Pr}_\infty = \mu_\infty c_p/\kappa_\infty$ is the Prandtl number and $Nr = 16T_\infty^3\sigma^*/3k^*\kappa_\infty$ is the thermal radiation.

The corresponding boundary conditions (12) become:

$$\left.\begin{array}{l} f = S, \quad f' = 1 + \zeta f'', \quad \theta = 1 \ \text{ for } \eta = 0 \\ f' = 0, \quad \theta = 0 \ \text{ as } \ \eta \to \infty \end{array}\right\} \tag{22}$$

where

$$S = -\frac{v_w}{u_w\sqrt{\mathrm{Re}_x}}$$

represents suction/blowing velocity at the plate for $v_w < 0$, and $v_w > 0$, respectively, and

$$\zeta = L\sqrt{\frac{2(m+1)}{\mathrm{Re}_x}}$$

is the slip parameter.

The quantities of main physical interest are the skin friction coefficient (rate of shear stress) and the Nusselt number (rate of heat transfer). The equation defining the wall shear stress is:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{23}$$

The local skin friction coefficient is defined as:

$$C_f = \sqrt{\frac{2(m+1)}{\mathrm{Re}_x}}\ \frac{\theta_r}{\theta_r - 1} f''(0) \tag{24}$$

The quantity of heat transfer through the unit area of the surface is given by:

$$q_w = -\kappa_\infty\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{25}$$

The rate of heat transfer in terms of the dimensionless Nusselt number is defined as:

$$\mathrm{Nu} = \sqrt{\frac{(m+1)\mathrm{Re}_x}{2}}\theta'(0) \tag{26}$$

Method of solution

The non-linear differential eqs. (20) and (21) with boundary conditions (22) have been solved in the symbolic computation software MATHEMATICA 7.0 using finite difference code that implements the 3-stage Lobatto IIIa formula for partitioned Runge-Kutta method. This is a collocation formula and the collocation polynomial provides a C1-continuous solution that is fourth order accurate. Mesh selection and error control are based on the residual of the continuous solution. The system cannot be solved on an infinite interval, and it would be impractical to solve it for even a very large finite interval. Effort has been made to solve a sequence of problems posed on increasingly larger intervals to verify the solution's consistent behavior as the boundary approaches to ∞. The plot of each successive solution has been superimposed over those of previous solutions so that they can easily be compared for consistency. For numerical computation infinity condition has been taken at a large but finite value of η where no considerable variation in velocity, temperature *etc*., occur.

Code verification

In the absence of thermal radiation, suction/blowing velocity of the plate and no-slip at the boundary, the present investigation coincides with that of Prasad *et al*. [24]. To check the validity of the present code, the values of $\theta'(0)$ have been calculated for different values of thermal conductivity parameter ε when Nr = 0, S = 0, and ζ = 0 using symbolic software MATHEMATICA 7.0 in tab. 1. From tab. 1, it has been observed that the data produced by present code and those of Prasad *et al*. [24] show excellent agreement and so justifies the use of the present numerical code.

Table 1. Comparison of $\theta'(0)$ via ε with Nr = 0, S = 0, and ζ = 0

ε	Present results		Prasad *et al.* [24]	
	$\beta = 0$	$\beta = 1$	$\beta = 0$	$\beta = 1$
0.0	–0.510583	–0.485798	–0.51059	–0.48582
0.1	–0.472928	–0.449700	–0.47293	–0.44971
0.2	–0.440993	–0.419091	–0.44099	–0.41908
0.3	–0.413489	–0.392752	–0.41350	–0.39275

Numerical results and discussions

Numerical results for the velocity and temperature functions of the transverse coordinate η are calculated for different values of suction/blowing parameter S, thermal radiation parameter Nr, variable viscosity parameter θ_r, thermal conductivity parameter ε, stretching parameter β, and slip parameter ζ are depicted in figs. 2 to 17 and in tabs. 1 and 2.

Effect of suction/blowing parameter S

The effect of suction/blowing parameter S on the velocity and temperature distribution for a permeable plate in presence of thermal radiation are presented in figs. 2-5. It is observed from figs. 2 and 3 that velocity distribution across the boundary layer decrease with increase of suction parameter S but effect is opposite for blowing parameter S. Figure 4 shows the effect of the suction parameter S on temperature distribution. It is noticed that for the increment of suction, temperature at fixed η decreases whereas the effects of blowing is entirely opposite as shown in fig. 5. Table 2 depict the effects of the suction/blowing parameter S on the skin friction coefficient $f''(0)$ and Nusselt number $\theta'(0)$. It is observed from this table that as S increases skin friction coefficient increase in magnitude and the Nusselt number is fluctuating in nature.

Effect of thermal radiation parameter Nr

The effect of thermal radiation parameter Nr on the velocity and temperature profiles are presented in figs. 6 and 7. It can be easily seen from fig. 6, that the velocity decreases as η increases for a fixed value of Nr. For a non-zero fixed value of η, velocity distribution across the boundary layer decreases with the increasing values of Nr but the effect is not significant.

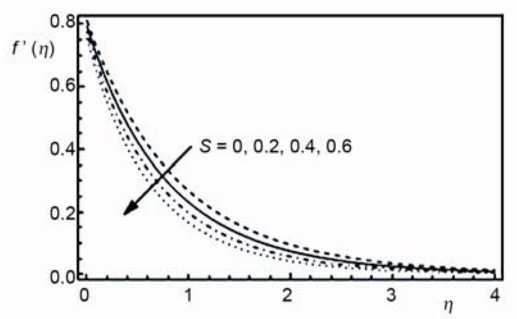

Figure 2. Velocity profiles for various values of *S* (suction)

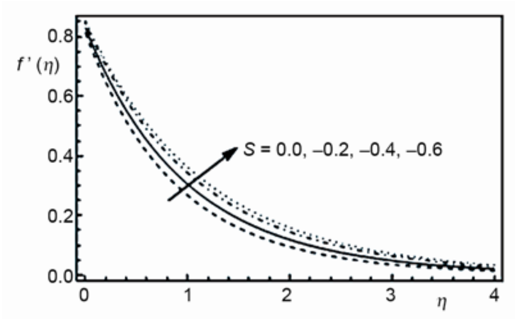

Figure 3. Velocity profiles for various values of *S* (blowing)

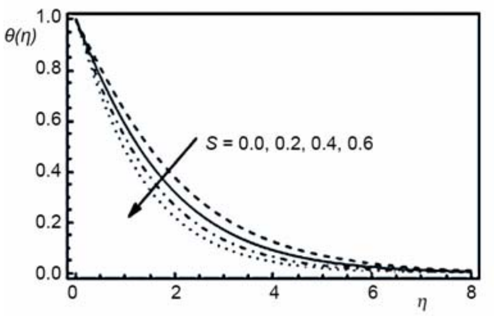

Figure 4. Temperature profiles for various values of *S* (suction)

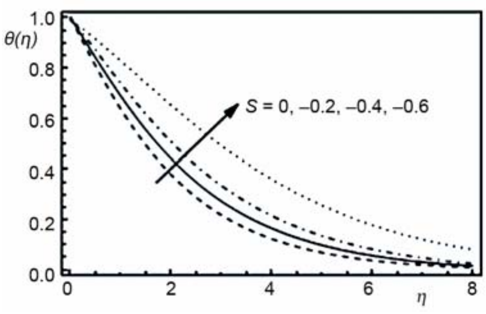

Figure 5. Temperature profiles for various values of *S* (blowing)

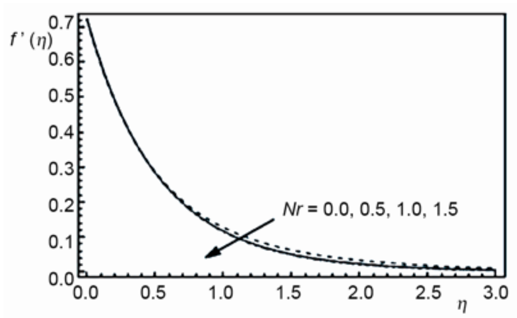

Figure 6. Velocity profiles for various values of *Nr*

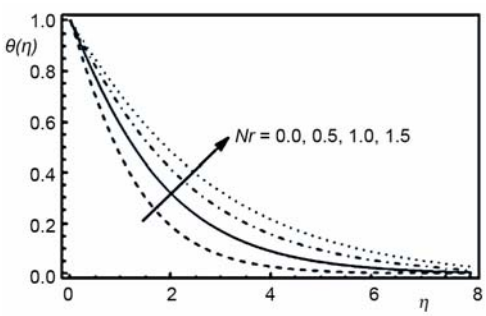

Figure 7. Temperature profiles for various values of *Nr*

Figure 7 illustrates the effect of thermal radiation *Nr* on temperature distribution. From the figure we see that the temperature distribution increases uniformly with increasing thermal radiation parameter and hence increase the thickness of thermal boundary layer. Table 2 shows the effects of thermal radiation parameter *Nr* on the skin friction coefficient $f''(0)$ and Nusselt number $\theta'(0)$. It is observed from this table that as *Nr* increases both of skin friction coefficient and Nusselt number decrease in magnitude but the values are large in absence of thermal radiation parameter.

Table 2. Effects of various parameters on $f''(0)$ and $\theta'(0)$

S	Nr	ε	θ_r	β	ζ	$f''(0)$	$\theta'(0)$
0.0	0.0	0.05	–2.5	1.0	02	–1.26391	–0.392356
0.2	0.0	0.05	–2.5	1.0	02	–1.41477	–0.473951
0.2	0.5	0.05	–2.5	1.0	02	–1.40922	–0.38062
0.4	0.5	0.05	–2.5	1.0	02	–1.57413	–0.449102
0.6	0.5	0.05	–2.5	1.0	02	–1.75311	–0.522234
–0.2	0.5	0.05	–2.5	1.0	02	–1.12769	–0.315783
–0.4	0.5	0.05	–2.5	1.0	02	–1.00762	–0.249191
–0.6	0.5	0.05	–2.5	1.0	02	–0.59526	–0.139946
0.2	0.5	0.05	–2.5	1.0	02	–1.40922	–0.330620
0.2	1.0	0.05	–2.5	1.0	02	–1.40580	–0.318332
0.2	1.5	0.05	–2.5	1.0	02	–1.40353	–0.27655
0.2	0.5	0.05	–2.5	1.0	02	–1.40922	–0.38062
0.2	0.5	0.10	–2.5	1.0	02	–1.41290	–0.461140
0.2	0.5	0.15	–2.5	1.0	02	–1.41238	–0.449109
02	0.5	0.05	–0.1	1.0	02	–4.73997	–0.279891
02	0.5	0.05	–0.5	1.0	02	–2.19185	–0.398127
02	0.5	0.05	–1.0	1.0	02	–1.73849	–0.439821
02	0.5	0.05	–5.0	1.0	02	–1.23943	–0.487467
02	0.5	0.05	–2.5	0.0	02	–1.01802	–0.503773
02	0.5	0.05	–2.5	0.5	02	–1.23007	–0.487370
02	0.5	0.05	–2.5	1.0	02	–1.41345	–0.473889
02	0.5	0.05	–2.5	1.5	02	–1.57674	–0.462468
02	0.5	0.05	–2.5	–0.5	02	–0.76128	–0.524660
02	0.5	0.05	–2.5	–1.0	02	–0.42379	–0.553352
02	0.5	0.05	–2.5	–1.5	02	0.10732	–0.599350
02	0.5	0.05	–2.5	1.0	0.0	0.97859	–0.365631
02	0.5	0.05	–2.5	1.0	0.4	0.49836	–0.297022
02	0.5	0.05	–2.5	1.0	0J6	0.33579	–0.316458

Effect of thermal conductivity parameter ε

For different values of thermal conductivity parameter, the velocity and temperature profiles are plotted in figs. 8 and 9. It is obvious from fig. 8 that velocity distribution across the boundary layer decreases with the increasing values of ε parameters but the effect is negligible. It is seen from fig. 9 that the temperature decreases rapidly as η increases and there is significant increase in the rate of increase of temperature with the increase of ε. From tab. 2 we see that the effect of ε on skin friction coefficient $f''(0)$ is not impressive but Nusselt number $\theta'(0)$ decreases with increasing ε in presence of suction parameter and thermal radiation parameter. It is seen that the results are found to be in good agreement with the previous work Prasad *et al*. [24].

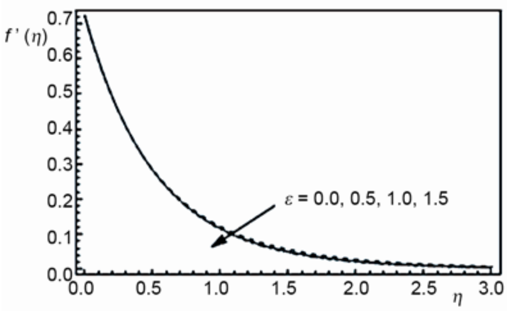

Figure 8. Velocity profiles for various values of ε

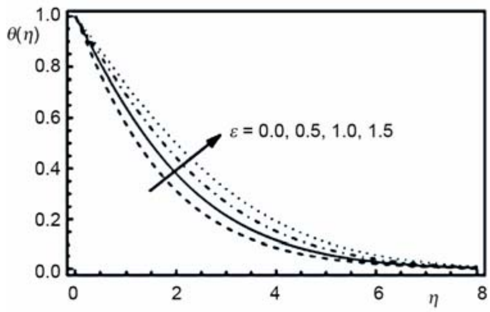

Figure 9. Temperature profiles for various values of ε

Effect of variable viscosity parameter θ_r

Figures 10 and 11 depict the effects of variable viscosity parameter θ_r on the velocity and temperature profiles for accelerating surface ($\beta > 0$). The variations of the velocity profiles against transverse co-ordinate η are shown in fig. 10 for various values of viscosity parameter θ_r. The results show that as an increasing the parameter θ_r the velocity profiles decreases and so decrease the momentum boundary layer thickness. This is due to the fact that for a given fluid when δ is fixed, smaller θ_r implies higher temperature difference between the wall and the ambient fluid. The influence of viscosity parameter θ_r on temperature distribution are enlightened in fig. 11. It is seen that as θ_r decreases, the thickness of the boundary layer decreases with a consequent reduction of the temperature in the boundary layer. The effect of variable viscosity parameter θ_r on skin friction coefficient $f''(0)$ and Nusselt number $\theta'(0)$ are presented in tab. 2. It is observed from this table that as θ_r increases, skin friction coefficient decreases and whereas Nusselt number increases. These observations show good agreement with the results of Prasad *et al*. [24].

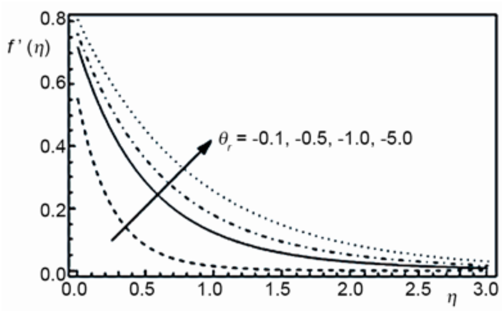

Figure 10. Velocity profiles for various values of θ_r

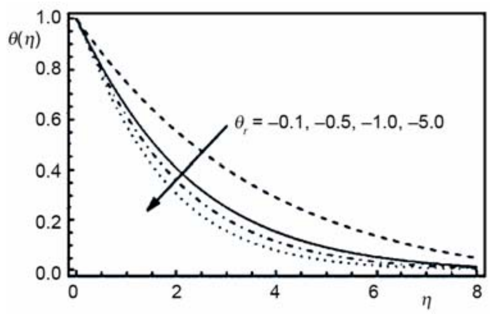

Figure 11. Temperature profiles for various values of θ_r

Effect of stretching parameter β

The boundary layer velocity profiles for several values of the stretching parameter β are plotted in figs. 12 and 13. It is noticed from those figures that the value of the fluid velocity decreases with increasing β for an accelerating surface *i. e.,* when $\beta > 0$ and the behavior is totally same for a decelerating surface *i. e.*, when $\beta < 0$. In this case velocity is very high close

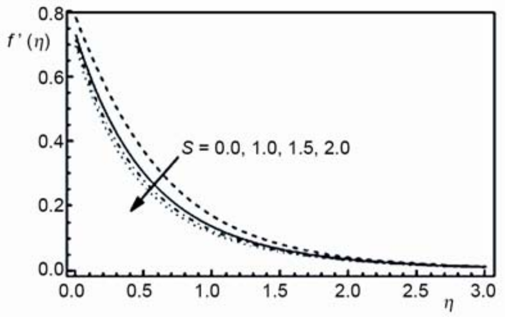

Figure 12. Velocity profiles for various values of $\beta > 0$

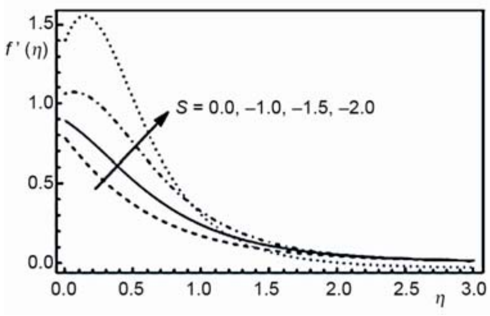

Figure 13. Velocity profiles for various values of $\beta < 0$

to the wall for smaller values of β. The graphs for temperature profiles are plotted in figs. 14 and 15 for various values of β. It is observed from the figures that the temperature distribution increases uniformly with increasing β for an accelerating surface *i. e.,* when $\beta > 0$ and the effects is exactly same for a decelerating surface *i. e.,* when $\beta < 0$ and hence reduce the thermal boundary layer thickness. The impact of the stretching parameter β on skin friction coefficient and Nusselt number may be analyzed from tab. 2. From the table it can be seen that the effect of β is to enhance the skin friction and to decrease the Nusselt number in magnitude.

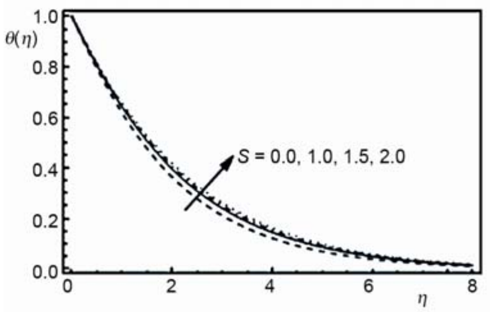

Figure 14. Temperature profiles for various values of $\beta > 0$

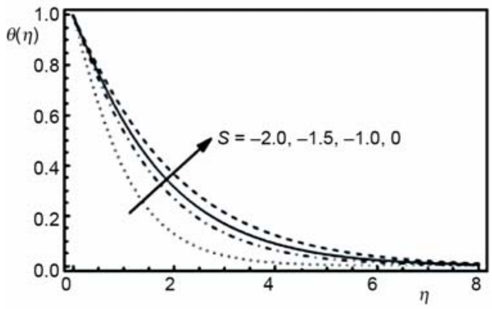

Figure 15. Temperature profiles for various values of $\beta < 0$

Effect of slip parameter ζ

The profiles of fluid velocity and temperature *vs*. boundary layer coordinate η have been plotted in figs. 16 and 17 for various values of the slip parameter ζ. It is noticed from fig. 16 that the fluid velocity within the boundary layer decreases with the increase of ζ and, as a result, thickness of momentum boundary layer decreases. Figure 17 shows that the fluid temperature is the maximum near the boundary layer region and it decreases on increasing boundary layer co-ordinate η to approach free stream value. Also fluid temperature increases on increasing ζ in the boundary layer region and, as a consequence, thickness of the thermal boundary layer increases. Table 2 presents the values of $f''(0)$ and $\theta'(0)$ which are proportional, respectively, to skin friction and rate of heat transfer (Nusselt number) from the surface of the plate for various values of ζ. It is evident from the table that $f''(0)$ decreases on increasing ζ. On the other hand the effect of the slip parameter ζ on $\theta'(0)$ is opposite, that is, ζ has a tendency to enhance the magnitude of $\theta'(0)$. These results are in good agreement with the results obtained by Wang [34].

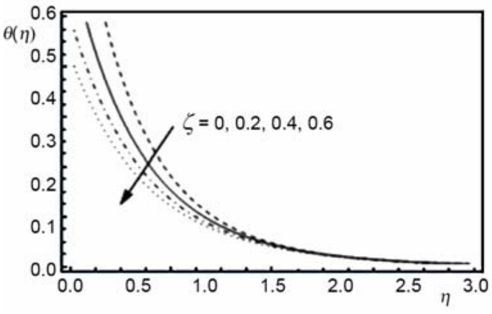

Figure 16. Velocity profiles for various values of ζ

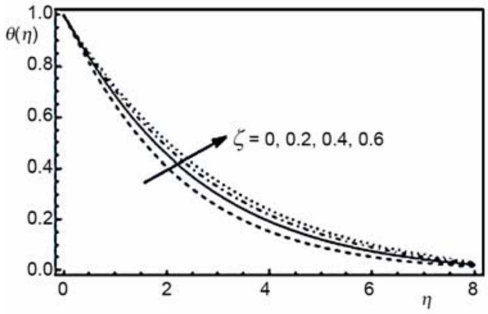

Figure 17. Temperature profiles for various values of ζ

Conclusions

In this paper, we have theoretically studied the effects of slip conditions, suction/blowing, and thermal radiation on steady boundary layer MHD flow and heat transfer of an incompressible electrically conducting fluid over a permeable stretching surface with temperature dependent fluid viscosity and thermal conductivity. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on material parameters. We can conclude the following results from our investigation.

- The velocity distribution are decreasing for increasing values of suction/blowing parameter S, thermal radiation parameter Nr, thermal conductivity parameter ε, variable viscosity parameter θ_r, stretching parameter β, and slip parameter ζ.
- The temperature profile increases with increasing of thermal radiation parameter Nr, thermal conductivity parameter ε, variable viscosity parameter θ_r, stretching parameter β, and slip parameter ζ, while the opposite effect is observed for suction/blowing parameter S.
- The skin friction coefficient decreases with increase of thermal radiation parameter Nr, thermal conductivity parameter ε and variable viscosity parameter θ_r but effect is reverse for suction/blowing parameter S, stretching parameter β, and slip parameter ζ.
- Nusselt number decreases in magnitude for increasing of thermal radiation parameter Nr, thermal conductivity parameter ε, and stretching parameter β while it increases for increasing variable viscosity parameter θ_r and slip parameter ζ. The effect is fluctuating in nature for suction/blowing parameter S.
- The results obtained in this work are more generalized form of Prasad *et al.* [24] and can be taken as a limiting case by taking $S \to 0$ and $Nr \to 0$.

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