

## STUDY OF ENTROPY GENERATION IN A SLAB WITH NON-UNIFORM INTERNAL HEAT GENERATION

by

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*Analysis of entropy generation in a rectangular slab with a non-uniform internal heat generation is presented. Dimensionless local and total entropy generation during steady-state heat conduction through the slab are obtained. Two different boundary conditions have been considered in the analysis, the first with asymmetric convection and the second with constant slab surface temperature. Temperature distribution within the slab is obtained analytically. The study investigates the effect of some relevant dimensionless heat transfer parameters on entropy generation. The results show that there exists a minimum local entropy generation but there does not exist a minimum total entropy generation for certain combinations of the heat transfer parameters. The results of calculations are presented graphically.*

Key words: *slab, internal heat generation, entropy generation, convection*

### Introduction

In all types of heat transfer processes thermodynamic irreversibility is associated with entropy generation, which results in the destruction of available work. Thermal irreversibility due to the temperature differences in thermal contacts and internal heat generation is the most common source of irreversibility in conductive systems. Heat conduction systems with internal heat generation have been widely used in many engineering applications, such as thermal insulation, metal casting and space vehicles. Entropy generation and its minimization were investigated extensively by Bejan [1-3], who introduced the concept and optimization method of entropy generation minimization. Basic analysis of steady-state 1-D heat conduction in a slab with constant and uniform internal heat generation can be found in many heat transfer textbooks [4-6].

Entropy generation of 1-D steady-state heat conduction in a semi-infinite wall with constant surface temperature has been investigated with an exact solution method as well as an integral method [7]. A study of entropy generation of a liquid metal laminar natural convection in a differentially heated cubic cavity and in the presence of an external magnetic field orthogonal to the isothermal walls has been carried out [8]. Entropy generation in Poiseuille-Benard channel flow in different angles has been presented [9]. Analysis of entropy generation has been applied to determine the effect of a heater location on entropy generation in a cavity [10], to estimate the entropy generation rate in a human body [11], to study the concept of a variable friction factor of fluid-driven deformable powder beds undergoing fluidization [12], and to analyze the heat-balance integral method of Goodman with two simple 1-D heat conduction problems with prescribed temperature and flux boundary conditions [13].

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A steady-state 1-D heat conduction problem in a rectangular slab with temperature-dependent internal heat generation was investigated to determine the local and total entropy generation rates [14] using asymmetric convective cooling. It was proved in that work [14], that the local and entropy generation rates can be minimized for certain combinations of heat transfer parameters. Analysis of minimum entropy generation for 1-D, 2-D, and 3-D steady-state heat conduction in Cartesian co-ordinates was presented [15]. The results of the authors [15] revealed that minimization of entropy generation in heat conduction processes is always possible by introducing additional heat sources. Another work using asymmetric convective cooling to minimize the entropy generation rate in a steady heat conduction problem in a slab with uniform internal heat generation was presented [16]. It was shown that [16] when Biot numbers (Bi) for each surface are equal the entropy generation rate shows a monotonic increase, however, when the Biot numbers are different there exists a minimum entropy generation rate for specific cooling conditions. Transient heat conduction with uniform internal heat generation in a slab was investigated [17]. The slab [17] is insulated on one surface and subjected to convection and radiation on the other surface. Transient entropy generation rate due to an instantaneous internal heat generation in a solid slab was presented [18]. The authors [18] introduced a non-dimensional heat generation parameter in order to evaluate entropy generation rate. The results [18] revealed that for certain values of non-dimensional heat generation parameter the non-dimensional temperature and entropy generation rate variables present a very sensible dependence between both parameters, indicating a direct relationship between the basic heat transfer mechanisms: heat conduction, heat convection, and internal heat generation. More works related to entropy generation can be found in [19-22].

The aim of this work is to find the effects of various parameters (heat transfer parameters) on the local and total entropy generation rates in 1-D heat conduction in a slab with non-linear internal heat generation.

### Energy analysis

Consider a 1-D slab of thickness  $L$ , made of a material with uniform and constant thermal conductivity  $k$ , subjected to a laser irradiation. Due to this laser irradiation, the slab is experiencing absorption of energy which is the equivalent of internal heat generation. This internal heat generation is function of location  $x$ , that is:

$$\dot{q} = ae^{-mx} \quad (1)$$

where  $a$  and  $m$  are constants.

#### Case 1: Asymmetric convection

The left face of the slab is exposed to a coolant at temperature  $T_1$  with a convective heat transfer coefficient  $h_1$ . The right face of the slab is exposed to a coolant at temperature  $T_2$  with a convective heat transfer coefficient  $h_2$ . The schematic diagram of the slab with its boundary conditions is shown in fig. 1.

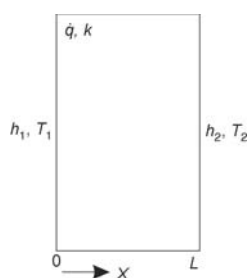


Figure 1. Schematic diagram of slab

The 1-D steady state heat conduction equation of the slab is:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad (2)$$

and the corresponding boundary conditions are:

$$k \frac{dT}{dx}(x=0) = h_1 [T(x=0) - T_1], \quad k \frac{dT}{dx}(x=L) = -h_2 [T(x=L) - T_2] \quad (3, 4)$$

Introducing the following dimensionless parameters:

$$\theta = \frac{T - T_2}{T_1 - T_2}, \quad X = \frac{x}{L}, \quad M = mL, \quad Q = \frac{aL^2}{k(T_1 - T_2)}, \quad \text{Bi}_1 = \frac{h_1 L}{k}, \quad \text{and} \quad \text{Bi}_2 = \frac{h_2 L}{k} \quad (5)$$

Therefore the non-dimensional form of energy equation, eq. (2), can be written as:

$$\frac{d^2\theta}{dX^2} + Qe^{-MX} = 0 \quad (6)$$

with the boundary conditions:

$$\frac{d\theta}{dX}(X=0) = \text{Bi}_1[\theta(X=0) - 1], \quad \frac{d\theta}{dX}(X=1) = -\text{Bi}_2\theta(X=1) \quad (7, 8)$$

The solution of energy equation, eq. (6), subjected to the boundary conditions eqs. (7) and (8), is obtained as:

$$\theta(X) = -\frac{Q}{M^2}e^{-MX} + CX + D \quad (9)$$

where  $C$  and  $D$  are given as:

$$C = \frac{\text{Bi}_1 \text{Bi}_2 (Qe^{-M} - Q - M^2)}{M^2 (\text{Bi}_1 + \text{Bi}_2 + \text{Bi}_1 \text{Bi}_2)} - \frac{Q(\text{Bi}_1 e^{-M} + \text{Bi}_2)}{M(\text{Bi}_1 + \text{Bi}_2 + \text{Bi}_1 \text{Bi}_2)} \quad (10)$$

$$D = \frac{\text{Bi}_1 \text{Bi}_2 (Q + M^2)}{M^2 (\text{Bi}_1 + \text{Bi}_2 + \text{Bi}_1 \text{Bi}_2)} + \frac{Q(\text{Bi}_2 e^{-M} - Me^{-M} + \text{Bi}_2 M + \text{Bi}_1 + M) + M^2 \text{Bi}_1}{M^2 (\text{Bi}_1 + \text{Bi}_2 + \text{Bi}_1 \text{Bi}_2)} \quad (11)$$

Case 2: Constant surface temperature ( $\text{Bi}_1 \rightarrow \infty$  and  $\text{Bi}_2 \rightarrow \infty$ )

In this case we consider that the left and the right surfaces of the slab are kept at constant temperatures. In other words, as  $\text{Bi}_1 \rightarrow \infty$  and  $\text{Bi}_2 \rightarrow \infty$ , the left surface of the slab approaches  $T_1$  and the right surface of the slab approaches  $T_2$ . Hence the boundary conditions can be written as:

$$T(x=0) = T_1, \quad T(x=L) = T_2 \quad (12, 13)$$

Considering the dimensionless parameters given in eq. (5), the non-dimensional form of the energy equation, eq. (2), can be written as:

$$\frac{d^2\theta}{dX^2} + Qe^{-MX} = 0 \quad (14)$$

with the boundary conditions:

$$\theta(X=0) = 1 \quad \theta(X=1) = 0 \quad (15, 16)$$

The solution of eq. (14) subjected to the boundary conditions eqs. (15) and (16), is obtained as:

$$\theta(X) = -\frac{Q}{M^2}e^{-MX} + EX + F \quad (17)$$

where  $E$  and  $F$  are given as:

$$E = \frac{Q}{M^2}(e^{-M} - 1) - 1, \quad F = 1 + \frac{Q}{M^2} \quad (18, 19)$$

For  $M=0$ , the problem becomes identical to the case with constant internal heat generation and eq. (14) then reduces to  $d^2\theta/dX^2 + Q = 0$ ; its solution with respect to the boundary conditions in eqs. (15) and (16) becomes:

$$\theta(X) = -\frac{Q}{2}X^2 + \left(\frac{Q}{2} - 1\right)X + 1 \quad (20)$$

For  $Q = 0$ , eq. (14) becomes identical to the slab without internal heat generation and its solution with the boundary conditions in eqs. (15) and (16) becomes:

$$\theta(X) = -X + 1 \quad (21)$$

### Entropy analysis

The local volumetric rate of entropy generation in the case of 1-D heat conduction with internal heat generation is:

$$\dot{S}''' = \frac{k \left( \frac{dT}{dx} \right)^2}{T^2} \quad (22)$$

Equation (22) can be integrated over the slab thickness in order to obtain the total entropy generation in the slab as:

$$\dot{S} = \int_0^L \dot{S}''' A dx \quad (23)$$

where  $A$  is the slab area normal to the heat flow direction.

The non-dimensional local volumetric rate of entropy generation can be obtained from eq. (22) as:

$$N_s = \frac{\dot{S}''' L^2}{k} = \frac{1}{(\theta + t)^2} \left( \frac{d\theta}{dX} \right)^2 \quad (24)$$

where the dimensionless temperature ratio  $t$  is defined as  $t = T_2 / (T_1 - T_2)$ .

The non-dimensional total entropy generation can be obtained from eq. (23) as:

$$N_T = \frac{\dot{S}L}{kA} = \int_0^1 N_s dX \quad (25)$$

### Results and discussion

The case of asymmetric convection results are presented in figs. 2-6 and the constant surface temperature case results are presented in figs. 7-12.

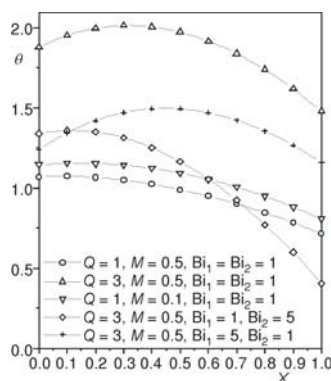


Figure 2. Temperature distribution as function of  $X$

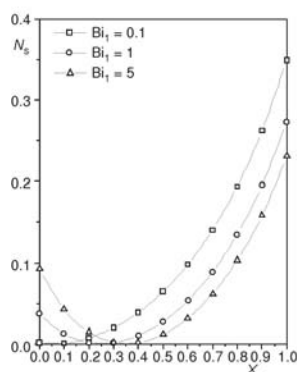


Figure 3. Variation of  $N_s$  with  $X$  for different  $Bi_1$  values ( $Bi_2 = 1$ )

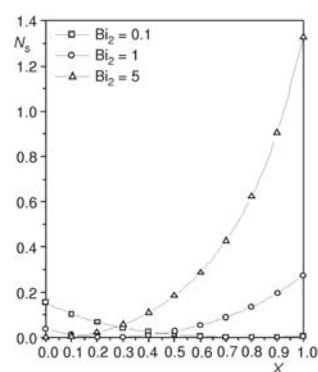


Figure 4. Variation of  $N_s$  with  $X$  for different  $Bi_2$  values ( $Bi_1 = 1$ )

The temperature distribution within the slab as function of axial co-ordinate is presented in fig. 2 for different heat transfer parameters for the case with asymmetric convection boundary conditions. The figure shows that there exists a maximum temperature. Figures 3, 4, and 5 are obtained for the following common parameters:  $M = 0.5$ ,  $Q = 2$ , and  $t = 1$ . The variation of local entropy generation rate with the axial co-ordinate is shown in fig. 3 for different  $Bi_1$  values. Figure 3 shows the local entropy generation rate has a minimum value at a certain  $X$  location, this minimum values is obtained because the temperature gradient is zero at that  $X$  location. Figure 3 shows that as  $Bi_1$  increases the local entropy generation rate decreases at higher  $X$  values but the situation is completely opposite at low  $X$  values. Figure 4 shows the variation of the local entropy generation rate with  $X$  for different  $Bi_2$ . The results of fig. 4 show that as  $Bi_2$  in-

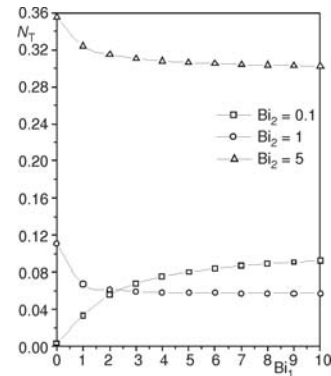


Figure 5. Variation of  $N_T$  with  $Bi_1$  for different  $Bi_2$  values

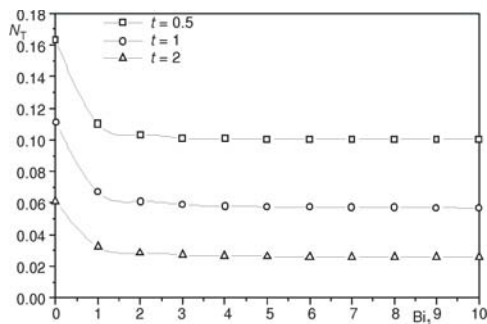


Figure 6. Variation of  $N_T$  with  $Bi_1$  for different  $t$  values ( $Bi_2 = 1$ )

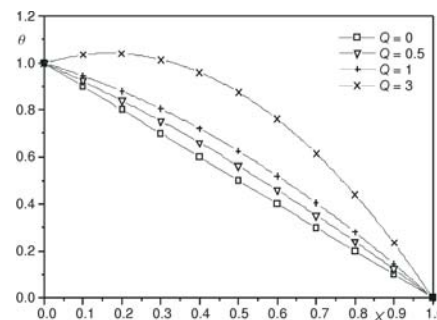


Figure 7. Temperature distribution as function of  $X$  for  $M = 0$  (Case 2)

creases the local entropy generation rate increases at higher  $X$  values. Moreover, the figure shows that there exists a minimum local entropy generation rate at certain  $X$  values at which the temperature gradient is zero. The variation of total entropy generation rate with  $Bi_1$  is presented in fig. 5 for different  $Bi_2$  values. The total entropy generation monotonically decreases as  $Bi_1$  increases for  $Bi_2 = 1$  and  $Bi_2 = 5$ , but it monotonically increases as  $Bi_1$  increases for  $Bi_2 = 0.1$ . Figure 6 presents the variation of the total entropy generation rate with  $Bi_1$  for different temperature ratio,  $t$  values at  $M = 0.5$ ,  $Q = 2$ , and  $Bi_2 = 1$ . The figure shows that the total entropy generation rate decreases as  $Bi_1$  increases, and the total entropy generation rate increases as the temperature ratio  $t$  increases for fixed  $Bi_1$ .

The temperature distribution for the constant slab surface temperature (Case 2) is presented in fig. 7 for different internal heat generation parameter  $Q$  at zero internal heat generation constant, *i. e.*  $M = 0$ . Figure 7 shows that the temperature is monotonically decreasing with the axial location for  $Q \leq 1$  but for  $Q = 3$  there is a maximum temperature. The figure also shows that as  $Q$  increases the temperature increases. Figure 8 shows the temperature distribution within the slab as function of axial location for different values of  $Q$  and  $M$ . In fig. 8, the temperature is decreasing as  $X$  increases. The figure also shows that increasing  $M$  values will result in a lower temperature for a fixed  $Q$  value. Figure 9 shows the distribution of dimensionless local entropy generation rate  $N_s$  as a function of  $X$  for different  $Q$  and temperature ratio  $t$  for the case  $M = 0$ .

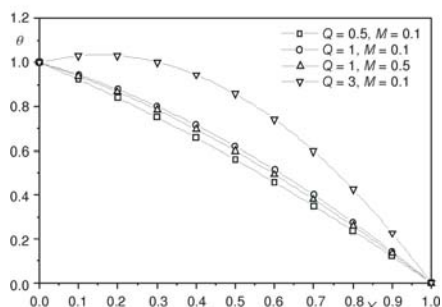


Figure 8. Temperature distribution as function of X (Case 2)

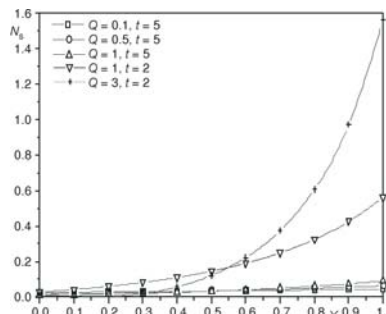


Figure 9. Variation of  $N_s$  with X for  $M = 0$  (Case 2)

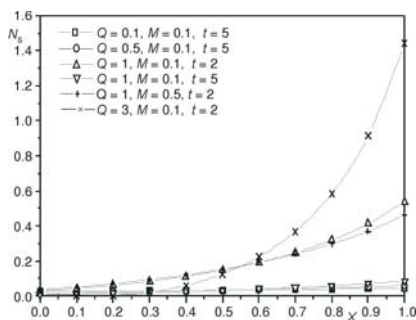


Figure 10. Variation of  $N_s$  with X (Case 2)

Figure 4 shows that the local entropy generation rate monotonically increases as  $X$  increases accept for  $Q = 3$ , the local entropy generation rate decreases and then increase with  $X$ , *i. e.*, a minimum  $N_s$  value exists closer to the left surface of the slab. Figure 10 shows the local entropy generation rate distribution as a function of  $X$  for different values of  $Q$ ,  $M$ , and  $t$ . Figure 10 shows that the local entropy generation rate is monotonically increasing with  $X$  accept for  $Q = 3$  there exists a minimum  $N_s$  values. The figure shows that the effect of the temperature ratio

$t$  on the local entropy generation is very strong. For lower  $t$  values the local entropy generation rate is higher than that for higher  $t$  values. The distribution of dimensionless total entropy generation rate  $N_T$  as function of temperature ratio  $t$  is given in fig. 11 for different  $Q$  values at  $M = 0$ . Figure 6 shows that the total entropy generation rate decreases sharply at low  $t$  values and then approaches an asymptotic value at high  $t$  values. Figure 11 shows increasing  $Q$  values results in higher total entropy generation rate at the same  $t$  value. Figure 12 presents the total entropy generation rate as function of temperature ratio  $t$  for different  $Q$  and  $M$  values. Figure 12 shows the total entropy generation rate decreases sharply at low  $X$  values and then reaches an asymptotic value at high  $t$  values.

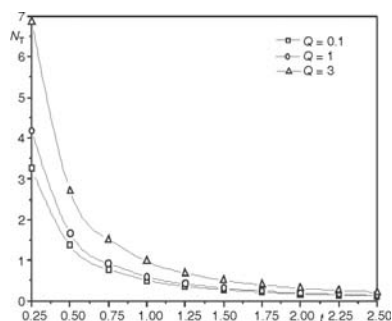


Figure 11. Variation of  $N_T$  with  $t$  for  $M = 0$  (Case 2)

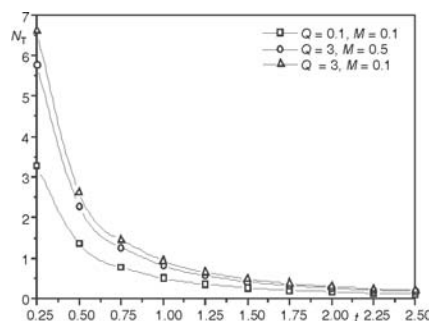


Figure 12. Variation of  $N_T$  with  $t$  (Case 2)

## Conclusions

In this work a basic analysis of local and total entropy generation rates of 1-D steady-state heat conduction in a slab with non-uniform internal heat generation has been carried out for two different cases of boundary conditions. In the first case, the slab external surfaces were subjected to asymmetric convection cooling and in the second case, the slab surfaces are maintained at fixed temperatures *i. e.* for infinite Bi. The temperature distribution in the slab was obtained analytically and presented graphically for different heat transfer parameters. The local entropy generation rate was also presented for different combination of heat transfer parameters. The results show that there exists a minimum local entropy generation rate for a certain combination of heat transfer parameters. The results also show that the total entropy generation rate does have a minimum value for the same set of heat transfer parameters. The results show that keeping the slab surface temperatures at fixed values do not result in obtaining a minimum local entropy generation rate as the case of asymmetric convective cooling for a certain combination of heat transfer parameters accept for  $Q = 3$ . The results of this work might be useful in engineering applications related to conduction heat transfer.

## Nomenclature

$A$	– slab area, [m <sup>2</sup> ]	$\dot{q}$	– internal heat generation, [Wm <sup>-3</sup> ]
$a$	– internal heat generation at $x = 0$ , [Wm <sup>-3</sup> ]	$\dot{S}$	– total entropy generation rate, [WK <sup>-1</sup> ]
Bi	– Biot number	$\dot{S}'''$	– volumetric entropy generation rate, [Wm <sup>-3</sup> K <sup>-1</sup> ]
$h$	– convection heat transfer coefficient, [Wm <sup>-2</sup> K <sup>-1</sup> ]	$T$	– temperature, [K]
$k$	– thermal conductivity, [Wm <sup>-1</sup> K <sup>-1</sup> ]	$t$	– temperature ratio
$L$	– slab thickness, [m]	$X$	– dimensionless axial co-ordinate
$M$	– dimensionless internal heat generation constant	$x$	– axial co-ordinate, [m]
$m$	– internal heat generation constant, [m <sup>-1</sup> ]	<i>Greek symbols</i>	
$N_s$	– dimensionless volumetric entropy generation rate	$\theta$	– dimensionless temperature
$N_T$	– dimensionless total entropy generation rate	<i>Subscripts</i>	
$Q$	– dimensionless internal heat generation parameter	1	– slab left side
		2	– slab right side

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