SQUEEZED FLOW AND HEAT TRANSFER IN A SECOND GRADE FLUID OVER A SENSOR SURFACE

by

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An analysis has been carried out for the hydromagnetic flow and heat transfer over a horizontal surface located in an externally squeezed free stream. Mathematical formulation is developed by using constitutive equations of a second grade fluid. The resulting problems have been solved by a homotopy analysis method. In addition the skin friction coefficient and Nusselt number are tabulated. The physical quantities of interest are analyzed for various emerging parameters.

Keywords: *second grade fluid, squeezed flow, heat transfer, sensor surface, boundary layer flow, auxiliary parameters*

Introduction

The boundary layer flow and heat transfer inside thin films has practical relevance in lubrications, microchannels, and heat pipes. Hence various investigations have examined the flow in hydromagnetic or squeezed thin films. Langlois [1] discussed the hydromagnetic flow in isothermal squeezed thin films when density depends upon the pressure. The influence of moving boundary on pulsatile flow has been studied by Damodaran *et al.* [2]. The flow of dusty fluid inside squeezed thin films is studied by Bhattacharjee *et al.* [3]. Hamza [4] and Bhattacharyya *et al*. [5] examined the role of squeezing on the temperature profile inside the thin film. The variation of external but constant squeezing velocity on the temperature of squeezed fluid with fixed volume is reported by Debbaut [6]. Hydromagnetic squeezed flow and heat transfer over a sensor surface is studied by Khalid and Vafai [7]. They considered the time dependent transpiration velocity. Mahmood *et al*. [8] revisited the study [7] when transpiration velocity is uniform and time independent. Such considerations resulted locally non-similar formulation of the problem.

All the mentioned studies have been carried out for the viscous fluid. The formulation of the problems have been presented by employing the Navier-Stokes equations. However there are many fluids in engineering and industrial applications for which the Navier-Stokes equations are inadequate. The simplest model amongst such fluids is known as second grade. Having

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such importance in mind, the present article aims to extend the analysis of [7] into three directions. Firstly to consider a second grade fluid. Secondly to include the viscous dissipation effect. Thirdly to obtain the solutions for the velocity and temperature. Solution expression have been derived by homotopy analysis method (HAM) [9-20]. Finally the graphical results are reported and discussed.

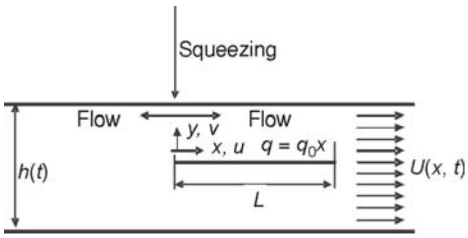

Figure 1. Geometry of the problem

Governing problem

We examine an incompressible flow of second grade fluid over a horizontal surface inside a squeezing channel. The x-axis is chosen along the length of the surface while the y-axis is normal to x-axis. We assume that the surface is enclosed inside a squeezed channel in such a manner that the $h(t)$ is greater than the boundary layer thickness. The squeezing in the free stream is assumed to start from the tip of the surface (see fig. 1 [6]). A time dependent magnetic field with strength B_m is exerted in the y-direction and thus the second grade fluid is electrically conducting with electrical conductivity σ_m. The induced magnetic field is neglected under the assumption of Reynolds number. The governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$
$$+\frac{\alpha_1}{\rho}\left(\frac{\partial^3 u}{\partial y^2\partial t}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)-\frac{\sigma_m\beta_m^2}{\rho}u \tag{2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\frac{\sigma_m\beta_m^2}{\rho}U \tag{3}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\beta\frac{\partial^2 T}{\partial T^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\alpha_1\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+q(T-T_\infty) \tag{4}$$

subject to the following boundary conditions

$$u(x,0,t)=0,\quad v(x,0,t)=v_0(t),\quad u(x,\infty,t)=U(x,t) \tag{5}$$

$$-k\frac{\partial T(x,0,t)}{\partial y}=q(x),\quad T(x,\infty,t)=T_\infty \tag{6}$$

In the equations α_1 is the second grade parameter, ρ – the density, σ_m – the electrical conductance, q – the heat flux, and k – the thermal conductivity.

From eqs. (2) and (3) we have:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial u}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma_m\beta_m^2}{\rho}(U-u)+$$
$$+\frac{\alpha_1}{\rho}\left(\frac{\partial^3 u}{\partial y^2\partial t}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right) \tag{7}$$

where the free stream velocity U and surface heat flux q are $U = ax$, $a = 1/(s + bt)$, $q = q_0x$ (where a is the strength of squeeze flow in which s is a constant and b – the squeezing parameter).

We now set:

$$\eta = y\sqrt{\frac{a}{\nu}}, \quad f(\eta) = \frac{\Psi}{x\sqrt{a\nu}}, \quad \theta = \frac{T - T_\infty}{\dfrac{q_0 x}{k}\sqrt{\dfrac{\nu}{a}}}, \quad q(x) = q_0(x) \tag{8}$$

$$\beta_m(t) = \beta_{m_0}\sqrt{a}, \quad v_0(t) = v_i\sqrt{a}, \quad u = axf'(\eta) \tag{9}$$

Note that eq. (1) is identically satisfied and eqs. (2), (3), (5), and (6) give:

$$\alpha\left(f + \frac{b}{2}\eta\right)f^{iv} + f'''(2b\alpha - 2\alpha f - 1) + \\ + f''\left(\alpha f'' - f - \frac{b}{2}\eta\right) + f'(f' - b + N) + b - N - 1 = 0 \tag{10}$$

$$\theta'' + \Pr\left(f - \frac{b}{2}\eta\right)\theta - \Pr\left(2f - \frac{b}{2} - \lambda_1\right)\theta + \text{Ec}\Pr[f''^2 + \alpha f''(ff'' - ff''')] \tag{11}$$

$$f'(0) = 0, \quad f(0) = -g_0, \quad f'(\infty) = 1 \tag{12}$$

$$\theta'(0) = -1, \quad \theta(\infty) = 0 \tag{13}$$

in which $N = \sigma_m\beta_{m0}^2/\rho$, $g_0 = v_i/(\nu)^{1/2}$, $\Pr = mc_p/\beta$, $\text{Ec} = a^2K/c_pq_0(a/\nu)^{1/2}$, $\alpha = \alpha_i a/\rho\nu$, $\lambda_1 = Q/a$.

The wall shear stress τ^* and the Nusselt number are given by:

$$\tau^* = \frac{\tau_w}{\mu a\sqrt{\text{Re}}} = f''(0), \quad \text{Nu} = \frac{x}{\theta(0)}\sqrt{\frac{a}{\nu}}$$

The next section comprises the solution of problems containing eqs. (10)-(13) by a homotopy analysis method.

Solution of the problems

We select the initial guesses and auxiliary linear operators as follows:

$$f_0(\eta) = g_0 + \lambda\eta + (1 + \lambda)[\exp(-\eta) - 1] \tag{14}$$

$$\theta_0(\eta) = \exp(-\eta) \tag{15}$$

$$L_f f(\eta) = \frac{\partial^3 f}{\partial\eta^3} + \lambda\frac{\partial^2 f}{\partial\eta^2} \tag{16}$$

$$L_\theta\theta(\eta) = \frac{\partial^2\theta}{\partial\eta^2} + \frac{\partial\theta}{\partial\eta} \tag{17}$$

$$L_f[c_1e^{-\eta} + c_2e^{\eta} + c_3] = 0 \tag{18}$$

$$L_\theta[c_4e^{-\eta} + c_5e^{\eta}] = 0 \tag{19}$$

where $c_1 - c_5$ are the arbitrary constants. Denoting $\hbar_f$ and $\hbar_\theta$ the non-zero auxiliary parameters and $p\in[0, 1]$ as an embedding parameter the zeroth-order deformation problems are:

$$(1-p)L_{\mathrm{f}}[\hat{f}(\eta;p)-f_0(\eta)]=p\hbar_{\mathrm{f}}N_{\mathrm{f}}[\hat{f}(\eta;p)] \tag{20}$$

$$(1-p)L_\theta[\theta(\eta;p)-\theta_0(\eta)]=p\hbar_\theta N_\theta[\theta(\eta;p)] \tag{21}$$

$$\hat{f}(0;p)=-g_0,\quad \frac{\partial\hat{f}}{\partial\eta}(0;p)=0,\quad \frac{\partial\theta}{\partial\eta}(0;p)=-1 \tag{22}$$

$$\frac{\partial\hat{f}}{\partial\eta}(\eta)=1,\quad \theta(\eta)\to 0\quad \text{as}\quad \eta\to\infty \tag{23}$$

$$N_f[\widetilde{f}(\eta,p)]=\alpha\frac{\partial^4\hat{f}}{\partial\eta^4}\left(f+\frac{b}{2}\eta\right)+\frac{\partial^3\hat{f}}{\partial\eta^3}(2b\alpha-2\alpha\hat{f}-1)+\frac{\partial^2\hat{f}}{\partial\eta^2}\left(\alpha\frac{\partial^2\hat{f}}{\partial\eta^2}-f-\frac{b}{2}\eta\right)+ \\ +\frac{\partial\hat{f}}{\partial\eta}\left(\frac{\partial\hat{f}}{\partial\eta}-b+N\right)+b-N-1 \tag{24}$$

$$N_\theta[\theta(\eta;p)]=\frac{\partial^3\theta}{\partial\theta^3}+\Pr\left(f-\frac{b}{2}\eta\right)\frac{\partial\theta}{\partial\theta}-\Pr\left(2\frac{\partial\hat{f}}{\partial\eta}-\frac{b}{2}-\lambda_1\right)\theta+ \\ +\mathrm{Ec}\Pr\left[\left(\frac{\partial^2\hat{f}}{\partial\eta^2}\right)^2+\alpha\frac{\partial^2\hat{f}}{\partial\eta^2}\left(\frac{\partial\hat{f}}{\partial\eta}\frac{\partial^2\hat{f}}{\partial\eta^2}-f\frac{\partial^3\hat{f}}{\partial\eta^3}\right)\right] \tag{25}$$

with

$$\hat{f}(\eta,0)=f_0(\eta),\quad\text{and}\quad \theta(\eta,0)=\theta_0(\eta) \tag{26}$$

$$\hat{f}(\eta,1)=f(\eta),\quad\text{and}\quad \theta(\eta,1)=\theta(\eta) \tag{27}$$

When p varies from 0 to 1, then $\hat{f}(\eta,p)$ and $\theta(\eta,p)$ vary from the initial guesses $f_0(\eta)$ and $\theta_0(\eta)$ to the final solutions $f(\eta)$ and $\theta(\eta)$, respectively. Considering that the auxiliary parameters $\hbar_{\mathrm{f}}$ and $\hbar_\theta$ are so properly chosen that the series of $f(\eta;p)$ and $\theta(\eta;p)$ converge at $p=1$ then:

$$f(\eta)=f_0(\eta)\sum_{m=1}^{\infty}f_m(\eta) \tag{28}$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta) \tag{29}$$

$$f_m(\eta)=\frac{1}{m!}\left.\frac{\partial^m\hat{f}(\eta,p)}{\partial p^m}\right|_{p=0} \tag{30}$$

$$\theta_m(\eta)=\frac{1}{m!}\left.\frac{\partial^m\theta(\eta;p)}{\partial p^m}\right|_{p=0} \tag{31}$$

The associated problems at the m^{th} order deformation are:

$$L_f[f_m(\eta)-\chi_m f_{m-1}(\eta)]=\hbar_f R_{f,m}(\eta) \tag{32}$$

$$L_\theta[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)]=\hbar_\theta R_{\theta,m}(\eta) \tag{33}$$

$$f_m(0)=0,\quad f'_m(0)=0,\quad \theta'_m(0)=0 \tag{34}$$

$$f'_m(\eta)=0,\quad \theta(\eta)=0,\quad \text{as}\quad \eta\to\infty \tag{35}$$

$$R_{f,m}(\eta)=\eta\frac{b}{2}f_{m-1}^{iv}+\alpha\sum_{i=0}^{m-1}f_i f_{m-i-1}^{iv}+2b\alpha f_{m-1}'''-2\alpha\sum_{i=0}^{m-1}f_i'\partial^3 f_{m-i-1}-f_{m-1}''+$$
$$+\alpha\sum_{i=0}^{m-1}f_i''f_{m-i-1}''\sum_{i=0}^{m-1}f_i f_{m-i-1}''-\eta\frac{b}{2}f_{m-1}''+\sum_{i=0}^{m-1}f_i'f_{m-i-1}'-$$
$$-bf_{m-1}'+Nf_{m-1}'+(b-N-1)(1-\chi_m) \tag{36}$$

$$R_{\theta,m}(\eta)=\theta_{m-1}''+\sum_{i=0}^{m-1}f_i\theta_{m-1-i}'-\Pr\frac{b}{2}\eta\theta_{m-1}'-2\Pr\sum_{i=0}^{m-1}\theta_i f_{m-1-i}'-\Pr\frac{b}{2}\theta_{m-1}-$$
$$-\lambda_1\theta_{m-1}+\mathrm{Ec}\Pr\sum_{i=0}^{m-1}f_i''f_{m-i-1}''+\alpha\Pr\mathrm{Ec}\sum_{k=0}^{m-1}f_{m-1-k}'\sum_{i=0}^{k}f_{k-1}''f_l''-$$
$$-\alpha\Pr\mathrm{Ec}\sum_{k=0}^{m-1}f_{m-1-k}'\sum_{l=0}^{k}f_{k-1}''f_l''' \tag{37}$$

$$\chi_m=\begin{cases}0, & m=1\\ 1, & m>1\end{cases} \tag{38}$$

The solutions of eqs. (32-35) through Mathematica can be expressed as:

$$f(\eta)=A_{0,0}+A_{1,0}\eta+\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}A_{k,m}\eta^k\exp(-m\eta) \tag{39}$$

$$\theta(\eta)=\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}B_{k,m}\eta^k\exp(-m\eta) \tag{40}$$

where the constants $A_{0,0}$, $A_{1,0}$, $A_{\mathrm{k,m}}$, and $B_{\mathrm{k,m}}$ have been computed easily.

Discussion

This section concerns with the effects of different parameters on the velocity field. Figure 2 displays the variation of magnetic parameter N. It is noticed that boundary layer thickness reduces as N increases. However the surface temperature decreases when the magnetic field increases (fig. 3).

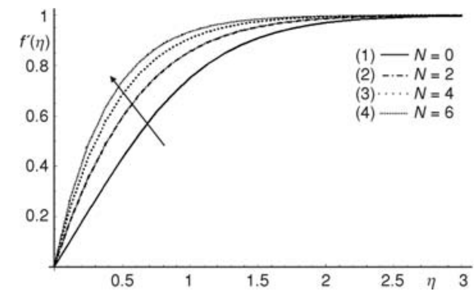

Figure 2. Variations f' with N

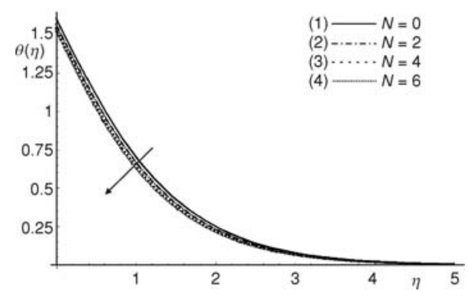

Figure 3. Variations θ with N

Figure 4 shows the effect of Prandtl number on the velocity profile. Figures 5 and 6 show the effects of the wall dimensionless permeable velocity g_0 for f' and θ. It is observed that the negative values of g_0 causes the fluid to be more attached to the surface when compared with the blowing conditions.

Figure 7 shows the variations in different values of the index of the squeezed flow. Here the velocity field decreases when there is an increase in b but it shows the reverse behavior

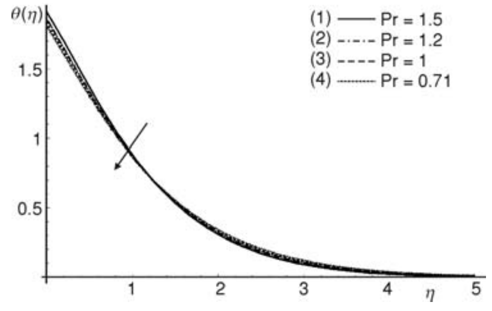

Figure 4. Variations of θ with Pr

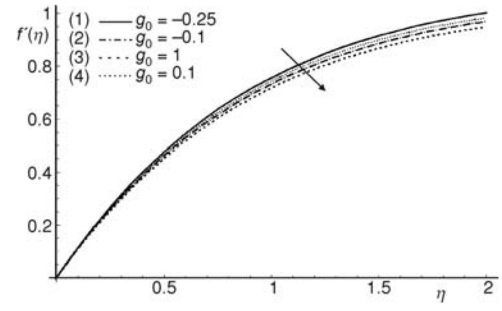

Figure 5. Variations of f' with g_0

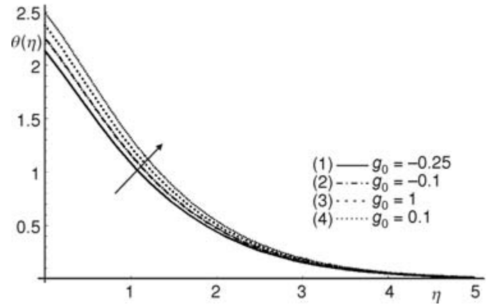

Figure 6. Variations of θ with g_0

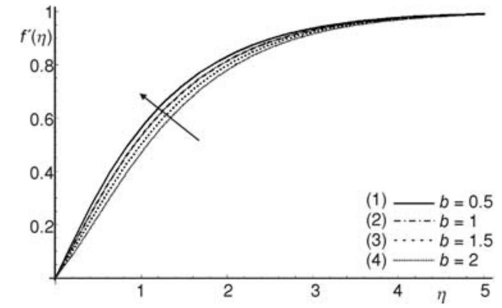

Figure 7. Variations of f' with b

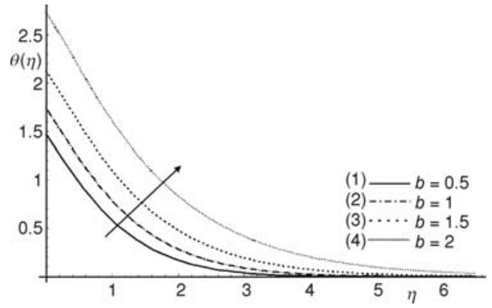

Figure 8. Variations of θ with b

for $\theta(\eta)$ (see fig. 9). The effect of λ_1 on the velocity is shown in fig. 9. It is found that an increase in λ_1 causes an increase in the velocity profile. The effects of Eckert number and fluid thermal diffusivity α are displayed in figs. 10 and 11. In these figures the dimensionless velocity profile increases when Ec and α are increased.

The convergence of the series solutions is obtained through $\hbar$-curves. The convergence of homotopy analysis solutions strongly depends

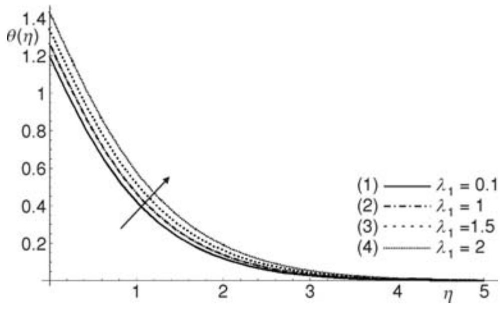

Figure 9. Variations of θ with λ_1

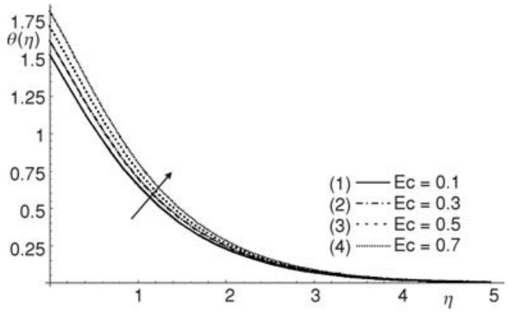

Figure 10. Variations of θ with Eckert number Ec

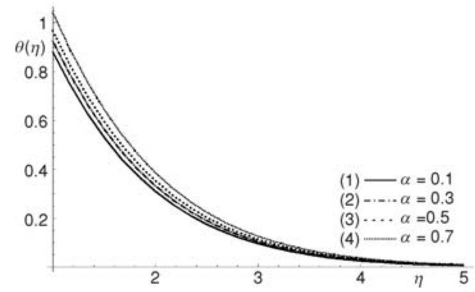

Figure 11. Variations of θ with α

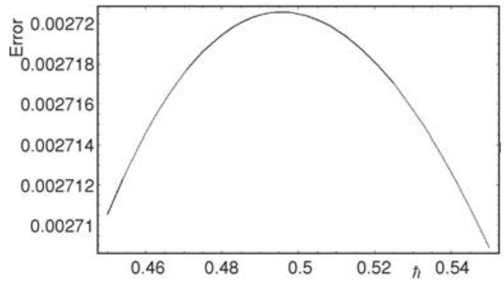

Figure 12. Residual error

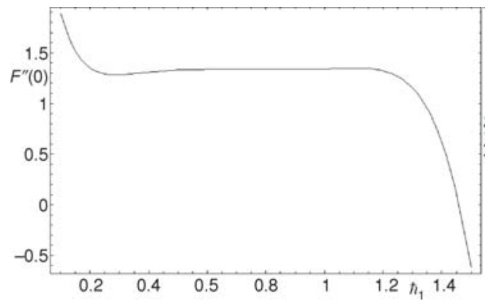

Figure 13. $\hbar_1$-curve for $f''(0)$ at 10th order of aproximation

Table. 1. Wall shear stress and local Nusselt number

Order of approximations $N = 1$, Pr = 1, Ec = 1/2 $\alpha = 1$, $\lambda_1 = 2$, $b = 2$	
$C_f\text{Re}x^{1/2}$	$-\text{Nu}xR^{-1/2}$
1.4975	0.71
1.13975	0.8435
1.19466	1.27573
1.27438	1.85253
1.31493	2.43864
1.32952	2.99042
1.33487	3.53174
1.33738	4.4663
1.33834	4.61019
1.33837	5.64364
1.33857	5.88523
1.33878	5.94620
1.33899	5.97462

upon the values of the auxiliary parameter $\hbar$ [14]. For the reasonable value of $\hbar$, the residual error is plotted (see fig. 12) which shows minimum error. The $\hbar$-curves are plotted in figs. 13 and 14. The range of $\hbar_1$ for f is $0.5 \leq \hbar_1 \leq 1$ and similarly for θ it is $-0.33 \leq \hbar_2 \leq -0.31$. From figs. 13 and 14 we can see that the values of $\hbar$-curves strongly depend on the values of different parameters involved in the physical problem. The wall shear stress and Nusselt number are shown through tab. 1. It is found that the skin friction coefficient and Nusselt number increases by increases the order of approximations.

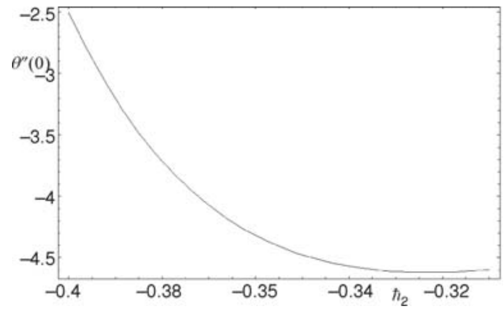

Figure 14. $\hbar_2$-curve for $\theta''(0)$ at 10th order

Closing remarks

The squeezed flow and heat transfer in a second grade fluid over a sensor surface are analyzed. The main findings have been summarized as follows.

- By increasing the magnetic parameter N the velocity $f'(0)$ increases but $\theta(\eta)$ decreases.

- For Prandtl number the temperature profiles first decreases and then increases.
- The effects of the permeable velocity parameter g_0 on $f'(\eta)$ increases for suction and decreases for blowing.
- Temperature $\theta(\eta)$ decreases for negative values of g_0 but increases for positive values.
- Increasing the value of indexed squeezed parameter b, $f'(\eta)$ decreases but $\theta(\eta)$ increases.
- Temperature profiles increase by increasing the values of λ_1, Ec and thermal diffusibility.

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