ASSESSMENT OF HOMOTOPY PERTURBATION METHOD IN NONLINEAR CONVECTIVE-RADIATIVE NON-FOURIER CONDUCTION HEAT TRANSFER EQUATION WITH VARIABLE COEFFICIENT

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Analytical solutions play a very important role in heat transfer. In this paper, the He's homotopy perturbation method (HPM) has been applied to nonlinear convective-radiative non-Fourier conduction heat transfer equation with variable specific heat coefficient. The concept of the He's homotopy perturbation method are introduced briefly for applying this method for problem solving. The results of HPM as an analytical solution are then compared with those derived from the established numerical solution obtained by the fourth order Runge-Kutta method in order to verify the accuracy of the proposed method. The results reveal that the HPM is very effective and convenient in predicting the solution of such problems, and it is predicted that HPM can find a wide application in new engineering problems.

Keywords: *Homotopy perturbation method (HPM), Non-Fourier conduction, Variable specific heat coefficient, Convective-radiative heat transfer, Numerical solution*

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Nomenclature			
A	Surface area, m^2	T_a	Ambient temperature, K
B	Boundary operator	T_i	Initial temperature, K
c	Specific heat, $Jkg^{-1}K^{-1}$	T_s	Effective sink temperature, K
c_a	Specific heat at the temperature T_a, $Jkg^{-1}K^{-1}$	u_0	Initial approximation
E	Surface emissivity	V	Volume, m^3
$f(r)$	A known analytic function	Ve	Vernotte number
Fo	Fourier number	**Greek symbols**	
h	Heat transfer coefficient, $Wm^{-2}K^{-1}$	β	Constant, volumetric thermal expansion coefficient, K^{-1}
$H(v, p)$	Homotopy expression	ε	Small parameter
L	Linear operator	Γ	The boundary of the domain Ω
N	Nonlinear operator	ρ	Mass density, kgm^{-3}
p	Homotopy parameter	σ	Stefan-Boltzmann constant
S	A general differential operator	τ	Thermal relaxation time, s
T	Temperature, K	θ	Dimensionless temperature
t	Temporal coordinate, s	Ω	Domain

1. Introduction

Nonlinear phenomena play a crucial role in applied mathematics and physics. We know that most engineering problems are nonlinear, and it is difficult to solve them analytically. Various powerful mathematical methods have been proposed for obtaining exact and approximate analytic solutions.

The homotopy perturbation method (HPM) was established by Ji-Huan He [1]. Also, recently he wrote a note on the HPM [2]. The method has been used by many authors to handle a wide variety of scientific and engineering applications to solve various functional equations. In this method, the solution is considered as the sum of an infinite series, which converges rapidly to accurate solutions. Considerable research work has recently been conducted in applying this method to a class of linear and non-linear equations.

The HPM was successfully applied to boundary-value problems [3], heat radiation equations [4, 5], generalized Hirota-Satsuma coupled KdV equations [6] and nonlinear coupled systems of reaction-diffusion equations [7]. Ganji et al. [8] solved three equations of a Fourier lumped system, temperature distribution of a fin and freezing of a saturated fluid through HPM and variational iteration method (VIM). Ganji et al. [9] developed the energy balance for a differential fin element and solved the resulting nonlinear differential equation by HPM and VIM. Khaleghi et al. [10] applied these methods to solve some nonlinear classical heat transfer equations with variable coefficients. Rajabi et al. [11] used homotopy perturbation method to solve the temperature distribution in Fourier lumped system of combined convection–radiation and also a nonlinear equation of the steady conduction in a slab with variable thermal conductivity. Ganji and Rajabi [12] applied HPM to the equation of a lumped system with combined convection and radiation and the equation of heat transfer with conduction in a slab with thermal dependent conductivity. Sajid and Hayat [13, 14] proved that the perturbation and homotopy perturbation solutions for the two problems namely unsteady convective–radiative equation and nonlinear convective–radiative conduction equation are only valid for weak non-linearity. Chowdhury et al. [15] compared the homotopy analysis method (HAM) with the homotopy perturbation method and the Adomian decomposition method (ADM) to determine the temperature distribution of a straight rectangular fin with power-law temperature dependent surface heat flux. Fathizadeh and Rashidi [16] solved convective heat transfer equations of boundary layer with pressure gradient over a flat plate using HPM.

More recently, Marinca and Herişanu [17] introduced the optimal homotopy perturbation method (OHPM) for solving strongly nonlinear differential equations. He [18] solved a problem of a cooling fin of thin rectangular section projecting from a hot plate held at a fixed temperature by various analytical methods. Also, homotopy perturbation method has been used to evaluate the temperature distribution of annular fin with temperature-dependent thermal conductivity and to determine the temperature distribution within the fin by Ganji et al. [19].

Many heat transfer researchers have attached much importance to the potentially feasible values of non-Fourier heat conduction in many applications, such as solidifying processes, surface thermal processing by lasers, temperature control of superconductors, laser surgery and freezing. Hence, non-Fourier heat conduction has become one of the noteworthy subjects in the field of heat transfer. The non-Fourier heat conduction equation has been experimentally validated for substances at very low temperatures, such as NaF at about 10 K [20] and Bi at 3.4 K [21].

The pursuit of analytical solutions for the non-Fourier heat conduction equation is of intrinsic scientific interest. To the best of the authors' knowledge, there is no paper that has solved convective-radiative nonlinear non-Fourier heat conduction by HPM. In this paper, the basic idea of HPM is described, then it is applied to the equation of convecting-radiating cooling of a lumped nonlinear non-Fourier heat conduction system. Moreover, it was assumed that the specific heat is a linear function of temperature, and we have made a comparison with the numerical solution. The fourth order Runge–Kutta method has been used and considered as the numerical solution for validity of this method.

2. Basic idea of homotopy perturbation method

To explain this method, let us consider the following function:

$$S(u) - f(r) = 0, r \in \Omega \tag{1}$$

With the boundary conditions of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \Gamma \tag{2}$$

Where S is a general differential operator, $f(r)$ is a known analytic function; B is a boundary operator and Γ is the boundary of the domain Ω. The operator S can be generally divided into two operators, L and N, where L is a linear and N a nonlinear operator. Eq. (1) can be, therefore, written as follows:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

Using the homotopy technique, we constructed a homotopy $v(r, p): \Omega \times [0,1] \to R$ which satisfies:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[S(v) - f(r)] = 0 \tag{4}$$

Or

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \tag{5}$$

Where $p \in [0,1]$, is called homotopy parameter, and u_0 is an initial approximation for the solution of Eq. (1), which satisfies the boundary conditions. Obviously from Eqs. (4) and (5) we will have:

$$H(v,0)=L(v)-L(u_0)=0 \tag{6}$$

$$H(v,1)=S(v)-f(r)=0 \tag{7}$$

We can assume that the solution of (4) or (5) can be expressed as a series in p, as follows:

$$v=v_0+pv_1+p^2v_2+\dots \tag{8}$$

Setting $p=1$ results in the approximate solution of Eq. (1)

$$u=\lim_{p\to 1}v=v_0+v_1+v_2+\dots \tag{9}$$

The above convergence is discussed in [22].

3. Problem statement

Consider the problem of combined convective-radiative cooling of a lumped system with low temperature. Let the system have volume V, surface area A, density ρ, specific heat c and initial temperature T_i. At $t=0$, the system is exposed to a convective environment at the temperature T_a with convective heat transfer coefficient of h. The system also loses heat through radiation, and the effective sink temperature is T_s. Assume that the specific heat c is a linear function of temperature in the form

$$c=c_a[1+\beta(T-T_a)] \tag{10}$$

where c_a is the specific heat at the temperature T_a and β is a constant. The cooling equation and the initial conditions using non-Fourier heat conduction are

$$\rho Vc\frac{dT}{dt}+\rho Vc\tau\frac{d^2T}{dt^2}+hA(T-T_a)+E\sigma A(T^4-T_s^4)=0 \tag{11}$$

$$t=0 \qquad T=T_i \tag{12a}$$

$$t=0 \qquad \frac{dT}{dt}=0 \tag{12b}$$

To solve Eq. (11) we do the following changes of parameters:

$$\theta=\frac{T}{T_i},\ \theta_a=\frac{T_a}{T_i},\ \theta_s=\frac{T_s}{T_i},\ Fo=\frac{t}{\rho Vc_a/(hA)},\ \varepsilon_1=\beta T_i,\ \varepsilon_2=\frac{E\sigma T_i^3}{h},\ Ve=\sqrt{\frac{\tau}{\rho Vc_a/(hA)}} \tag{13}$$

After parameter change, the heat transfer equation will result in the following:

$$[1+\varepsilon_1(\theta-\theta_a)]\frac{d\theta}{dFo}+[1+\varepsilon_1(\theta-\theta_a)]Ve^2\frac{d^2\theta}{dFo^2}+(\theta-\theta_a)+\varepsilon_2(\theta^4-\theta_s^4)=0 \tag{14}$$

$$Fo=0 \qquad \theta=1 \tag{15a}$$

$$Fo=0 \qquad \frac{d\theta}{dFo}=0 \tag{15b}$$

For simplicity, we assume the case of $\theta_a=\theta_s=0$. So we have

$$[1+\varepsilon_1\theta]\frac{d\theta}{dFo}+[1+\varepsilon_1\theta]Ve^2\frac{d^2\theta}{dFo^2}+\theta+\varepsilon_2\theta^4=0 \tag{16}$$

$$Fo=0 \qquad \theta=1 \tag{17a}$$

$$Fo=0 \qquad \frac{d\theta}{dFo}=0 \tag{17b}$$

3.1. Implementation of HPM

To solve Eq. (16) with the initial conditions (17), according to the well-known homotopy perturbation, we construct the following He's polynomials corresponding to Eq. (5):

$$L(\theta)=Ve^2\frac{d^2\theta}{dFo^2}+\frac{d\theta}{dFo}+\theta \tag{18a}$$

$$N(\theta)=Ve^2\varepsilon_1\theta\frac{d^2\theta}{dFo^2}+\varepsilon_1\theta\frac{d\theta}{dFo}+\varepsilon_2\theta^4 \tag{18b}$$

$$\begin{aligned}H(v,p)&=Ve^2\frac{d^2\theta}{dFo^2}+\frac{d\theta}{dFo}+\theta-Ve^2\frac{d^2u_0}{dFo^2}-\frac{du_0}{dFo}-u_0\\&+p\left(Ve^2\frac{d^2u_0}{dFo^2}+\frac{du_0}{dFo}+u_0\right)+p\left(Ve^2\varepsilon_1\theta\frac{d^2\theta}{dFo^2}+\varepsilon_1\theta\frac{d\theta}{dFo}+\varepsilon_2\theta^4\right)=0\end{aligned} \tag{19}$$

Substituting $\theta=\theta_0+p\theta_1+p^2\theta_2+\ldots$ into Eq. (19) and rearranging the resultant equation based on powers of p-terms, one has:

$$p^0: Ve^2\frac{d^2\theta_0}{dFo^2}+\frac{d\theta_0}{dFo}+\theta_0-Ve^2\frac{d^2u_0}{dFo^2}-\frac{du_0}{dFo}-u_0=0 \tag{20a}$$

$$p^1: Ve^2\frac{d^2\theta_1}{dFo^2}+\frac{d\theta_1}{dFo}+\theta_1+Ve^2\frac{d^2u_0}{dFo^2}+\frac{du_0}{dFo}+u_0+Ve^2\varepsilon_1\theta_0\frac{d^2\theta_0}{dFo^2}+\varepsilon_1\theta_0\frac{d\theta_0}{dFo}+\varepsilon_2\theta_0^4=0 \tag{20b}$$

$$p^2: Ve^2\frac{d^2\theta_2}{dFo^2}+\frac{d\theta_2}{dFo}+\theta_2+Ve^2\varepsilon_1\theta_0\frac{d^2\theta_1}{dFo^2}+Ve^2\varepsilon_1\theta_1\frac{d^2\theta_0}{dFo^2}+\varepsilon_1\theta_0\frac{d\theta_1}{dFo}+\varepsilon_1\theta_1\frac{d\theta_0}{dFo}+4\varepsilon_2\theta_0^3\theta_1=0 \tag{20c}$$

With the following conditions:

$$\theta_0(0)=1 \qquad \frac{d\theta_0}{dFo}(0)=0 \tag{21a}$$

$$\theta_i(0)=0 \qquad \frac{d\theta_i}{dFo}(0)=0 \qquad for \qquad i=1,2,\ldots \tag{21b}$$

With the effective initial approximation for θ_0 from the conditions (21a) the solutions of Eq. (20), if $-1+4Ve^2>0$, we obtain:

$$\theta_0(Fo)=\frac{e^{-\frac{1}{2}\frac{Fo}{Ve^2}}\sin\left(\frac{1}{2}\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)}{\sqrt{-1+4Ve^2}}+e^{-\frac{1}{2}\frac{Fo}{Ve^2}}\cos\left(\frac{1}{2}\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right) \tag{22a}$$

$$\theta_1(Fo)=\frac{1}{3}\frac{1}{(-1+4Ve^2)^2(-4+25Ve^2)(Ve^2+2)}\left(\varepsilon_2\left((-1+4Ve^2)^2 e^{-\frac{1}{2}\frac{Fo}{Ve^2}}\cos\left(\frac{1}{2}\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)\right.\right.$$

$$(15Ve^4+50Ve^2+8)+(-450Ve^8+72Ve^6+4\cos\left(\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)Ve^2(-4+25Ve^2)$$

$$\left.\left.(2Ve^4-7Ve^2+2)+\cos\left(\frac{2\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)(Ve^2+2)(10Ve^6-40Ve^4-4+25Ve^2))e^{\frac{-2Fo}{Ve^2}}\right)\right)+$$

$$\varepsilon_1\left(\frac{(18Ve^4-4Ve^2-\cos\left(\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)(-9Ve^2+2+6Ve^4))e^{\frac{-Fo}{Ve^2}}}{(-1+4Ve^2)(9Ve^2-2)}\right.$$

$$\left.-\frac{\cos\left(\frac{1}{2}\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)e^{\frac{-2Fo}{Ve^2}}(-1+4Ve^2)(2+3Ve^2)}{(-1+4Ve^2)(9Ve^2-2)}\right) \tag{22b}$$

$$+\frac{1}{3}\frac{1}{(-1+4Ve^2)^{3/2}(-4+25Ve^2)(9Ve^2-2)(Ve^2+2)}(\sin\left(\frac{1}{2}\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)e^{\frac{-Fo}{2Ve^2}}(-1+4Ve^2)$$

$$(48\varepsilon_1-1881\varepsilon_2Ve^6-16\varepsilon_2+975\varepsilon_1Ve^6-284Ve^4\varepsilon_2+1644Ve^4\varepsilon_1+228\varepsilon_2Ve^2-588\varepsilon_1Ve^2)$$

$$+\sin\left(\frac{\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)(-4+25Ve^2)(4\varepsilon_2e^{\frac{-2Fo}{Ve^2}}Ve^2(9Ve^2-2)(3Ve^2-2)-3e^{\frac{-Fo}{Ve^2}}\varepsilon_1(-1+4Ve^2)$$

$$(5Ve^2-2)(Ve^2+2))+\sin\left(\frac{2\sqrt{-1+4Ve^2}\,Fo}{Ve^2}\right)\varepsilon_2e^{\frac{-2Fo}{Ve^2}}(Ve^2+2)(9Ve^2-2)(14Ve^4-17Ve^2+4))$$

And if $-1+4Ve^2<0$

$$\theta_0(Fo)=\frac{e^{-\frac{1}{2}\frac{Fo}{Ve^2}}\sinh\left(\frac{1}{2}\frac{\sqrt{1-4Ve^2}\,Fo}{Ve^2}\right)}{\sqrt{1-4Ve^2}}+e^{-\frac{1}{2}\frac{Fo}{Ve^2}}\cosh\left(\frac{1}{2}\frac{\sqrt{1-4Ve^2}\,Fo}{Ve^2}\right) \tag{23a}$$

$$\theta_1(Fo)=\frac{1}{3}\frac{1}{(1-4Ve^2)^2(-4+25Ve^2)(Ve^2+2)}\left(\varepsilon_2\left((1-4Ve^2)^2 e^{-\frac{1}{2}\frac{Fo}{Ve^2}}\cosh\left(\frac{1}{2}\frac{\sqrt{1-4Ve^2}Fo}{Ve^2}\right)\right.\right.$$

$$(15Ve^4+50Ve^2+8)+(-450Ve^8+72Ve^6+4\cosh\left(\frac{\sqrt{1-4Ve^2}Fo}{Ve^2}\right)Ve^2(-4+25Ve^2)$$

$$\left.\left.(2Ve^4-7Ve^2+2)+\cosh\left(\frac{2\sqrt{1-4Ve^2}Fo}{Ve^2}\right)(Ve^2+2)(10Ve^6-40Ve^4-4+25Ve^2))e^{\frac{-2Fo}{Ve^2}}\right)\right)+$$

$$\varepsilon_1\left(\frac{-(18Ve^4-4Ve^2-\cosh\left(\frac{\sqrt{1-4Ve^2}Fo}{Ve^2}\right)(-9Ve^2+2+6Ve^4))e^{\frac{-Fo}{Ve^2}}}{(1-4Ve^2)(9Ve^2-2)}\right.$$

$$\left.-\frac{\cosh\left(\frac{1}{2}\frac{\sqrt{1-4Ve^2}Fo}{Ve^2}\right)e^{\frac{-2Fo}{Ve^2}}(1-4Ve^2)(2+3Ve^2)}{(1-4Ve^2)(9Ve^2-2)}\right) \tag{23b}$$

$$+\frac{1}{3}\frac{1}{(1-4Ve^2)^{3/2}(-4+25Ve^2)(9Ve^2-2)(Ve^2+2)}(\sinh\left(\frac{1}{2}\frac{\sqrt{1-4Ve^2}Fo}{Ve^2}\right)e^{\frac{-Fo}{2Ve^2}}(1-4Ve^2)$$

$$(48\varepsilon_1-1881\varepsilon_2Ve^6-16\varepsilon_2+975\varepsilon_1Ve^6-284Ve^4\varepsilon_2+1644Ve^4\varepsilon_1+228\varepsilon_2Ve^2-588\varepsilon_1Ve^2)$$

$$-\sinh\left(\frac{\sqrt{1-4Ve^2}Fo}{Ve^2}\right)(-4+25Ve^2)(4\varepsilon_2e^{\frac{-2Fo}{Ve^2}}Ve^2(9Ve^2-2)(3Ve^2-2)+3e^{\frac{-Fo}{Ve^2}}\varepsilon_1(1-4Ve^2)$$

$$(5Ve^2-2)(Ve^2+2))-\sinh\left(\frac{2\sqrt{1-4Ve^2}Fo}{Ve^2}\right)\varepsilon_2e^{\frac{-2Fo}{Ve^2}}(Ve^2+2)(9Ve^2-2)(14Ve^4-17Ve^2+4))$$

Having $\theta_i, i=0,1,2$, from Eq. (9), the solution $\theta(Fo)$ is as follows:

$$\theta(Fo)=\theta_0(Fo)+\theta_1(Fo)+\theta_2(Fo) \tag{24}$$

4. Results and discussion

In this section we present the results with HPM to show the efficiency of the method, described in the previous section for solving Eq. (16). We consider different values of ε_1, ε_2 and Ve, and we compare our results with numerical integration results, obtained using a fourth order Runge-Kutta method. It is easy to verify the accuracy of the obtained results if we graphically compare HPM solutions with the numerical ones.

Figure 1 shows the comparison between the present solution and the numerical integration results. Moreover, Figs. 2 and 3 illustrate the results of HPM for different values of ε_1, ε_2 and Ve.

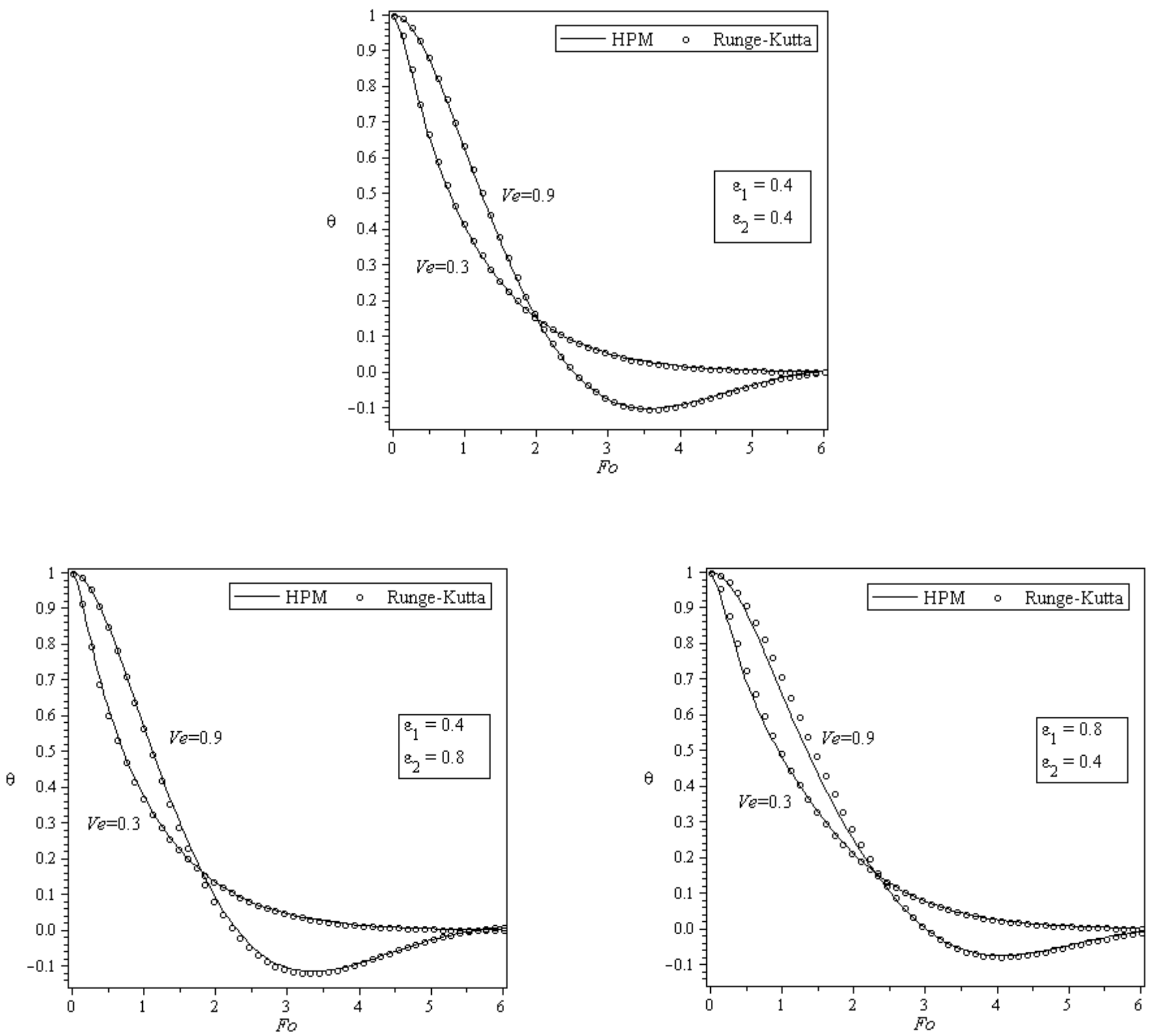

Fig. 1. Comparison between HPM and fourth order Runge-Kutta method for two Vernotte numbers

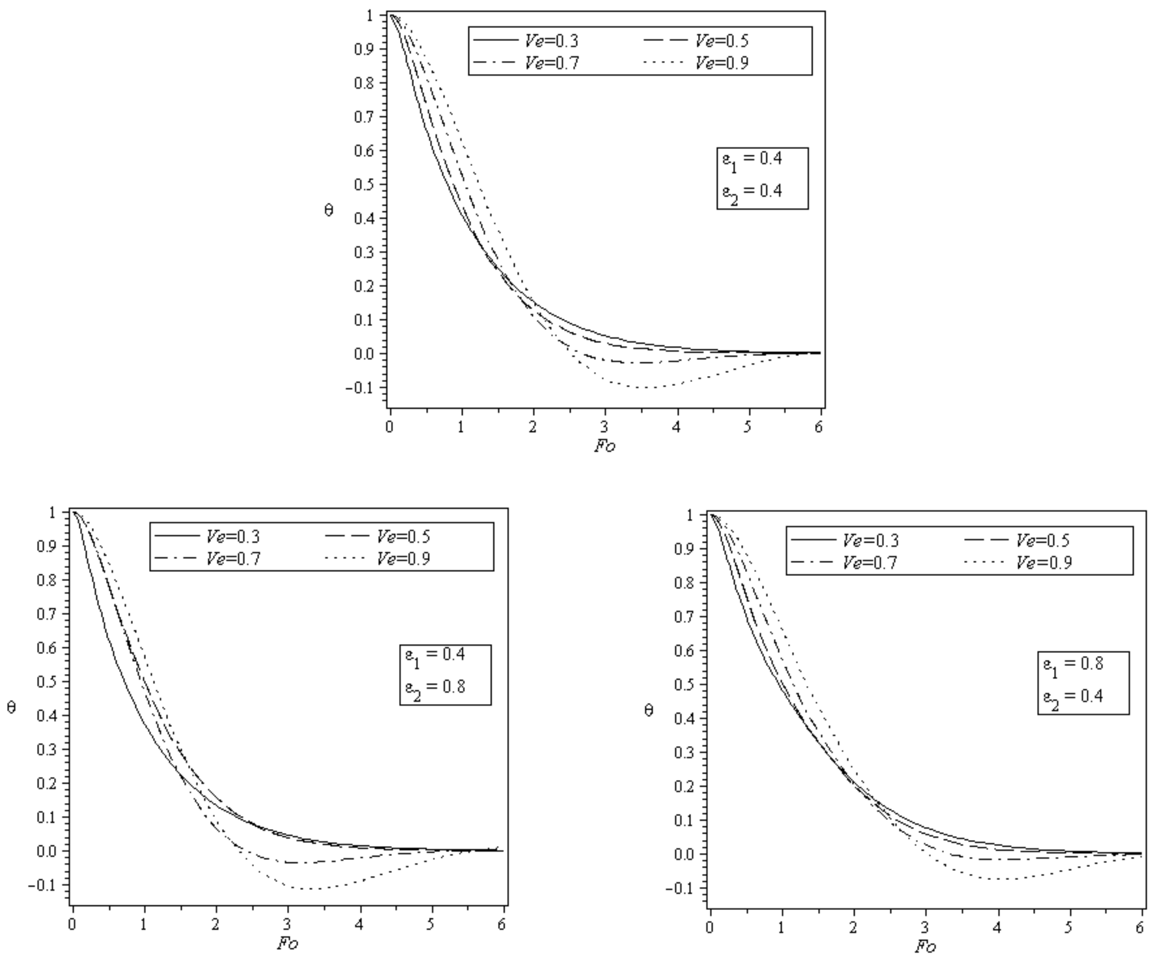

Fig. 2. Temperature θ for HPM solution with different Vernotte numbers

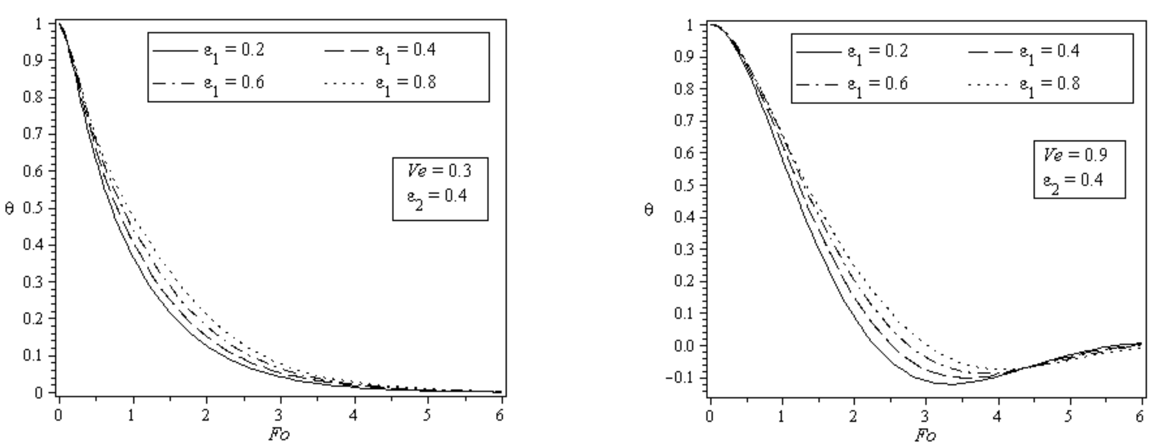

Fig. 3. Temperature θ for HPM solution with different ε_1

It can be deduced from Fig. 2 that, the object with a lower Vernotte number reaches the equilibrium temperature sooner than the object with a larger Vernotte number. It can be clearly seen from Fig. 3 as ε_1 increases, the object needs more time to reach the equilibrium temperature; and from Fig. 4 as ε_2 decreases, the object needs more time to reach the equilibrium temperature.

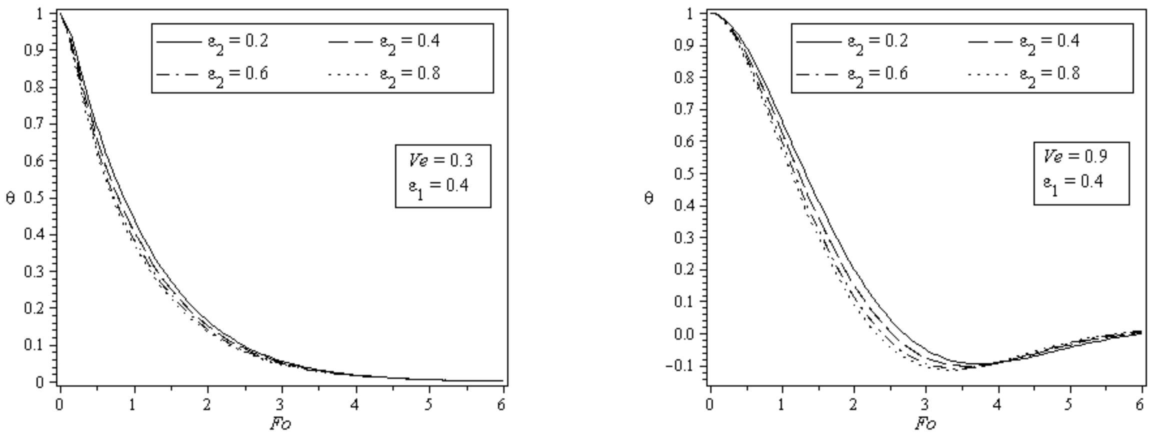

Fig. 4. Temperature θ for HPM solution with different ε_2

Accordingly, for the case of variable ε_1 and ε_2, results of the present analysis are tabulated against the numerical solution obtained by fourth-order Runge-Kutta in Tabs. 1 and 2.

Table 1
The results of HPM and fourth order Runge-Kutta method at $Fo = 1$, $Ve = 0.2$

ε_1	ε_2	$\theta(Fo)_{HPM}$	$\theta(Fo)_{RK}$	$Error = \lvert\theta(Fo)_{HPM} - \theta(Fo)_{RK}\rvert$
0.4	0.4	0.4064914537	0.4074278881	0.0009364344
0.4	0.6	0.3874255825	0.3863742390	0.0010513435
0.4	0.8	0.3753762106	0.3684492318	0.0069269788
0.8	0.4	0.4780999402	0.4804393217	0.0023393815
0.8	0.6	0.4496854865	0.4561975967	0.0065121102
0.8	0.8	0.4416217553	0.4353383524	0.0062834029

Table 2
The results of HPM and fourth order Runge-Kutta method at $Fo = 1$, $Ve = 0.8$

ε_1	ε_2	$\theta(Fo)_{HPM}$	$\theta(Fo)_{RK}$	$Error = \lvert\theta(Fo)_{HPM} - \theta(Fo)_{RK}\rvert$
0.4	0.4	0.5757975098	0.5777179513	0.0019204415
0.4	0.6	0.5442346204	0.5379133117	0.0063213087
0.4	0.8	0.5198995402	0.5010762419	0.0188232983
0.8	0.4	0.6153706338	0.6568938197	0.0415231859
0.8	0.6	0.5920333459	0.6214500824	0.0294167365
0.8	0.8	0.5759238673	0.5882388366	0.0123149693

In this case, a very interesting agreement between the results is observed, which confirms the excellent validity of the HPM. As indicated in Tab. 1, for small ε_1, as ε_2 increases, the difference between the HPM and numerical results is more remarkable. But for larger ε_1 and small ε_2, this difference is considerably large, and as ε_2 approximately reaches ε_1, the error of the obtained results are reduced.

For the case of variable Ve, results of the present analysis are tabulated against the numerical solution obtained by fourth-order Runge-Kutta in Tab. 3.

Table 3

The results of HPM and fourth order Runge-Kutta method at $Fo = 1$, $\varepsilon_1 = 0.4$ **and** $\varepsilon_2 = 0.4$

Ve	$\theta(Fo)_{HPM}$	$\theta(Fo)_{RK}$	$Error = \lvert\theta(Fo)_{HPM} - \theta(Fo)_{RK}\rvert$
0.1	0.4065551182	0.4073399330	0.0007848148
0.3	0.4076161091	0.4089328577	0.0013167486
0.5	0.4367405120	0.4390552260	0.0023147140
0.7	0.5244703097	0.5266888944	0.0022185847
0.9	0.6246971194	0.6262974076	0.0016002882

It is worth nothing that a higher accuracy can be obtained by evaluating some more terms of the solution $\theta(Fo)$. Here, in the wake of the large third term of the solution, the results for two terms of the series are shown; however, the obtained results are calculated using three terms.

5. Conclusion

He's homotopy perturbation method (HPM) has been successfully utilized to derive approximate explicit analytical solutions for nonlinear convective-radiative non-Fourier heat transfer problem with a small parameter. The results show that this perturbation scheme provides excellent approximations to the solution of this nonlinear equation with high accuracy and avoids linearization and physically unrealistic assumptions. This new method accelerated the convergence to the solutions. As shown in Eq. (9), the homotopy perturbation method does not need a small parameter. Finally, it has been attempted to show the capabilities and wide-range applications of the homotopy perturbation method in comparison with the forth-order-Runge-Kutta method in solving heat transfer problems.

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