

FREE CONVECTIVE OSCILLATORY FLOW AND MASS TRANSFER PAST A POROUS PLATE IN THE PRESENCE OF RADIATION FOR AN OPTICALLY THIN FLUID

by

Andreas RAPTIS

Department of Mathematics, University of Ioannina, Ioannina, Greece

Original scientific paper

UDC: 532.546:536.255:517.95/.96

DOI: 10.2298/TSCI101208032R

We study the two-dimensional free convective oscillatory flow and mass transfer of a viscous and optically thin gray fluid over a porous vertical plate in the presence of radiation. The governing partial differential equations have been transformed to ordinary differential equations. Numerical solutions are obtained for different values of radiation parameter, Grashof number, and Schmidt number.

Key words: free convection, mass transfer, radiation, oscillatory flow

Introduction

Many processes in engineering areas occur at high temperature making the knowledge of thermal radiation heat transfer becomes very important. Plasma physics, gas turbines, and the various propulsion devices for aircraft, missiles, satellites and space vehicles, flow through a porous medium in the presence of radiation and glass production are some examples of such engineering areas.

Seddeek *et al.* [1] studied the effects of radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux. The free convection flow in the presence of radiation has been investigated by Hossain *et al.* [2, 3] and Raptis *et al.* [4]. The flow through a porous medium in the presence of radiation has been studied by Raptis [5], Badruddin *et al.* [6] and Mukhopadhyay *et al.* [7]. The magnetohydrodynamics flow in the presence of radiation has been analyzed by Chamkha *et al.* [8] and Duwairi [9]. In all these studies the fluid was assumed to be optically thick.

England *et al.* [10] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. The hydromagnetic free convection flow with radiative heat transfer in a rotating and optically thin fluid has been investigated by Bestman *et al.* [11]. Raptis *et al.* [12] have studied the hydromagnetic free convection flow with radiative heat transfer for an optically thin fluid past a vertical plate when the induced magnetic field is taken into account. Raptis *et al.* [13] have investigated the unsteady flow of an optically thin fluid in the presence of free convection and mass transfer. Manivannan *et al.*

* Author's e-mail: araptis@uoi.gr

[14] have studied the free convection for one dimensional flow with radiative heat transfer for an optically thin fluid past a vertical oscillating plate in the presence of chemical reaction. Vijayalakshmi [15] studied the effects the radiation on free convection flow past an impulsively started vertical plate in a rotating fluid for optically thin fluid.

In this work we study the two-dimensional free convective oscillatory flow and mass transfer past a porous plate in the presence of radiation for an optically thin fluid. The fluid is a gray, absorbing-emitting radiation but non-scattering medium:

Analysis

We consider the unsteady two-dimensional flow of an incompressible viscous fluid past an infinite vertical porous plate, through which suction occurs with constant velocity. The x' -axis is along the plate in the upward direction and the y' -axis is normal to it. All the fluid properties are considered constant except the influence of the density variations with temperature and concentration. The radiation to the x' -direction is considered negligible as compared to the y' -direction. The equations governing the problem are

– continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

– momentum equation

$$\rho \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = -\frac{\partial p'}{\partial x'} - \rho g_{x'} + \nu \rho \frac{\partial^2 u'}{\partial y'^2} \quad (2)$$

– energy equation

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} \quad (3)$$

– diffusion equation

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

where u' and v' are the components of the velocity parallel and perpendicular to the plate, t' – the time, p' – the pressure, ρ – the fluid density, $g_{x'}$ – the acceleration due to gravity, T' – the fluid temperature, ν – the kinematic viscosity, c_p – the specific heat at constant pressure, k – the thermal conductivity, q_r – the radiative heat flux in the y' -direction, C' – the concentration, and D – the chemical diffusivity.

The boundary conditions are:

$$u' = 0, \quad v' = -v_0, \quad \frac{\partial T'}{\partial y'} = -\frac{q'}{k}, \quad C' = C'_w, \quad \text{at } y' = 0 \quad (5)$$

$$u' \rightarrow U' = U_0(1 + \varepsilon e^{i\omega t'}), \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty$$

where v_0 is the constant suction velocity and the negative sign indicates that it is towards the plate, q' – the constant heat flux, T'_∞ – the fluid temperature far away from the plate, C'_w – the

species concentration at the plate, C'_∞ – the species concentration far away from the plate, U_0 – the mean free stream velocity, ω' – the frequency of vibration of the fluid, and e ($e \ll 1$) – a constant quantity.

For the free stream, eq. (2) becomes:

$$\rho \frac{dU'}{dt'} = -\frac{\partial p'}{\partial x'} - \rho_\infty g_{x'} \quad (6)$$

On eliminating $\partial p'/\partial x'$ between (2) and (6) we get:

$$\rho \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = \rho \frac{dU'}{dt'} + g_{x'} (\rho_\infty - \rho) + \nu \rho \frac{\partial^2 u'}{\partial y'^2} \quad (7)$$

The state equation is

$$g_{x'} (\rho_\infty - \rho) = g_{x'} \rho \beta (T' - T'_\infty) + g_{x'} \rho \beta^* (C' - C'_\infty) \quad (8)$$

where β is the coefficient of thermal expansion and β^* – the coefficient of concentration expansion.

In the case of an optically thin gray fluid the local radiant absorption is expressed as:

$$-\frac{\partial q_r}{\partial y'} = 4d\sigma^* (T'^4_\infty - T'^4) \quad (9)$$

where d is the absorption coefficient and σ^* – the Stefan-Boltzman constant.

We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (10)$$

Equation (9) through (10) takes the form:

$$-\frac{\partial q_r}{\partial y'} = 16d\sigma^* T'^3_\infty (T'_\infty - T') \quad (11)$$

Equation (1) gives:

$$v' = -v_0 (v_0 > 0) \quad (12)$$

On substituting eqs. (8), (9), (11), and (12) in eqs. (3), (4), and (7) we take:

$$\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + g_{x'} \beta (T' - T'_\infty) + g_{x'} \beta^* (C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \quad (13)$$

$$\frac{\partial T'}{\partial t'} - v_0 \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + \frac{16d\sigma^* T'^3_\infty}{\rho c_p} (T'_\infty - T') \quad (14)$$

$$\frac{\partial C'}{\partial t'} - v_0 \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (15)$$

Using the transformations:

$$y = \frac{y'v_0}{\nu}, \quad t = \frac{t'v_0^2}{4\nu}, \quad T = \frac{T' - T'_\infty}{\frac{\nu q'}{kv_0}}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad u = \frac{u'}{U_0}, \quad U = \frac{U'}{U_0}, \quad \omega = \frac{4\nu\omega'}{v_0^2} \quad (16)$$

$Gr = g_x' \beta v^2 q' / k U_0 v_0^3$ (Grashof number), $Gc = (\nu g \beta^* (C'_w - C'_\infty) / U_0 v_0^2)$ (modified Grashof number), $Pr = \rho \nu c_p / k$ (Prandtl number), $S = 16 d \sigma^* T_\infty'^3 v^2$ (radiation parameter), $Ec = k U_0^2 v_0 / c_p \nu q'$ (Eckert number), $Sc = \nu / D$ (Schmidt number).

With the help of the non-dimensional quantities (16), eqs. (13)-(15) reduce to:

$$\frac{1}{4} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4} \frac{dU}{dt} + GrT + GcC + \frac{\partial^2 u}{\partial y^2} \quad (17)$$

$$Pr \left(\frac{1}{4} \frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} \right) = \frac{\partial^2 T}{\partial y^2} + Pr Ec \left(\frac{\partial u}{\partial y} \right)^2 - ST \quad (18)$$

$$Sc \left(\frac{1}{4} \frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} \right) = \frac{\partial^2 C}{\partial y^2} \quad (19)$$

with the boundary conditions

$$u = 0, \quad \frac{\partial T}{\partial y} = -1, \quad C = 1, \quad \text{at } y = 0 \quad (20)$$

$$u \rightarrow U(t) = 1 + \varepsilon e^{i\omega t}, \quad T \rightarrow 0, \quad C \rightarrow 0, \quad \text{as } y \rightarrow \infty$$

In order to solve the system of differential equations (17)-(19) we assume that:

$$u(y, t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \dots \quad (21)$$

$$T(y, t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) + \dots \quad (22)$$

$$C(y, t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) + \dots \quad (23)$$

On substituting eqs. (21)-(23) in eqs. (17)-(19) we get the following system of differential equations:

$$\frac{d^2 u_0}{dy^2} + \frac{du_0}{dy} = -GrT_0 - GcC_0 \quad (24)$$

$$\frac{d^2 T_0}{dy^2} + Pr \frac{dT_0}{dy} = -Pr Ec \left(\frac{du_0}{dy} \right)^2 + ST_0 \quad (25)$$

$$\frac{d^2C_0}{dy^2} + Sc \frac{dC_0}{dy} = 0 \quad (26)$$

$$\frac{d^2u_1}{dy^2} + \frac{du_1}{dy} - \frac{i\omega}{4}u_1 = -\frac{i\omega}{4} - GrT_1 - GcC_1 \quad (27)$$

$$\frac{d^2T_1}{dy^2} + Pr \frac{dT_1}{dy} - \frac{i\omega}{4}PrT_1 = -2PrEc \left(\frac{du_0}{dy} \right) \left(\frac{du_1}{dy} \right) + ST_1 \quad (28)$$

$$\frac{d^2C_1}{dy^2} + Sc \frac{dC_1}{dy} - \frac{i\omega}{4}ScC_1 = 0 \quad (29)$$

The corresponding boundary conditions (20) are:

$$u_0 = 0, \quad u_1 = 0, \quad \frac{dT_0}{dy} = -1, \quad \frac{dT_1}{dy} = 0, \quad C_0 = 1, \quad C_1 = 0 \quad \text{at } y = 0 \quad (30)$$

$$u_0 \rightarrow 1, \quad u_1 \rightarrow 1, \quad T_0 \rightarrow 0, \quad T_1 \rightarrow 0, \quad C_0 \rightarrow 0, \quad C_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

In order to solve the system of the differential eqs. (24)-(29) we put:

$$\begin{aligned} u_1(y) &= u_{11}(y) + iu_{12}(y) \\ T_1(y) &= T_{11}(y) + iT_{12}(y) \\ C_1(y) &= C_{11}(y) + iC_{12}(y) \end{aligned} \quad (31)$$

in this system. Equating terms which are independent of i and the coefficients of i we get:

$$\frac{d^2u_0}{dy^2} + \frac{du_0}{dy} = -GrT_0 - GcC_0 \quad (32)$$

$$\frac{d^2T_0}{dy^2} + Pr \frac{dT_0}{dy} = -PrEc \left(\frac{du_0}{dy} \right)^2 + ST_0 \quad (33)$$

$$\frac{d^2C_0}{dy^2} + Sc \frac{dC_0}{dy} = 0 \quad (34)$$

$$\frac{d^2u_{11}}{dy^2} + \frac{du_{11}}{dy} + \frac{\omega}{4}u_{12} = -GrT_{11} - GcC_{11} \quad (35)$$

$$\frac{d^2u_{12}}{dy^2} + \frac{du_{12}}{dy} - \frac{\omega}{4}u_{11} = -\frac{\omega}{4} - GrT_{12} - GcC_{12} \quad (36)$$

$$\frac{d^2T_{11}}{dy^2} + Pr \frac{dT_{11}}{dy} + \frac{\omega}{4}PrT_{12} = -2PrEc \left(\frac{du_0}{dy} \right) \left(\frac{du_{11}}{dy} \right) + ST_{11} \quad (37)$$

$$\frac{d^2 T_{12}}{dy^2} + \text{Pr} \frac{dT_{12}}{dy} - \frac{\omega}{4} \text{Pr} T_{11} = -2 \text{Pr} \text{Ec} \left(\frac{du_0}{dy} \right) \left(\frac{du_{12}}{dy} \right) + ST_{12} \quad (38)$$

$$\frac{d^2 C_{11}}{dy^2} + \text{Sc} \frac{dC_{11}}{dy} + \frac{\omega}{4} \text{Sc} C_{12} = 0 \quad (39)$$

$$\frac{d^2 C_{12}}{dy^2} + \text{Sc} \frac{dC_{12}}{dy} - \frac{\omega}{4} \text{Sc} C_{11} = 0 \quad (40)$$

The corresponding boundary conditions (30) become:

$$\left. \begin{array}{l} u_0 = 0, \quad u_{11} = 0, \quad u_{12} = 0, \\ \frac{dT_0}{dy} = -1, \quad \frac{dT_{11}}{dy} = 0, \quad \frac{dT_{12}}{dy} = 0, \\ C_0 = 1, \quad C_{11} = 0, \quad C_{12} = 0 \end{array} \right\} \text{at } y = 0 \quad (41)$$

$$\left. \begin{array}{l} u_0 \rightarrow 1, \quad u_{11} \rightarrow 1, \quad u_{12} \rightarrow 0, \\ T_0 \rightarrow 0, \quad T_{11} \rightarrow 0, \quad T_{12} \rightarrow 0, \\ C_0 \rightarrow 0, \quad C_{11} \rightarrow 0, \quad C_{12} \rightarrow 0 \end{array} \right\} \text{as } y \rightarrow 4$$

The solutions for the real part of the velocity, temperature, and concentration field are given, respectively, by the expressions:

$$u(y, t) = u_0 + \varepsilon(u_{11} \cos \omega t - u_{12} \sin \omega t) \quad (42)$$

$$T(y, t) = T_0 + \varepsilon(T_{11} \cos \omega t - T_{12} \sin \omega t) \quad (43)$$

$$C(y, t) = C_0 + \varepsilon(C_{11} \cos \omega t - C_{12} \sin \omega t) \quad (44)$$

When $\omega t = \pi/2$ the above expressions become:

$$u(y) = u_0 - \varepsilon u_{12} \quad (45)$$

$$T(y) = T_0 - \varepsilon T_{12} \quad (46)$$

$$C(y) = C_0 - \varepsilon C_{12} \quad (47)$$

where u_0 , T_0 , C_0 , u_{12} , T_{12} , and C_{12} derive from the numerical solution of the system of differential eqs. (32)-(40), under the boundary conditions (41), by using the shooting method.

Discussion

In order to understand the physical situation of the problem we have computed the numerical values of the non-dimensional velocity (45) and non-dimensional temperature (46) for different values of the physical parameters. The obtained numerical values are illustrated in figs. 1-5 with $\varepsilon = 0.02$ and $\omega t = \pi/2$.

Figure 1 demonstrates the effect of the radiation parameter S on the non-dimensional velocity $u(y)$, when $Gr = 5$, $Pr = 0.7$, $Gc = 2$, $Ec = 0.001$, $Sc = 0.22$, and $\omega = 0.8$. It is observed that the non-dimensional velocity decreases with the increase of the radiation parameter S .

The effect of the Grashof number on the non-dimensional velocity $u(y)$, is shown in fig. 2, when $Pr = 0.7$, $Gc = 2$, $Ec = 0.001$, $Sc = 0.22$, $S = 0.2$, and $\omega = 0.8$. It is noticed that when the Grashof number increases the non-dimensional velocity also increases.

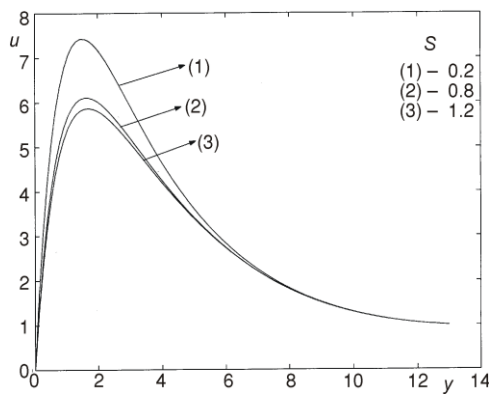


Figure 1. Velocity profiles for different values of radiation parameter S

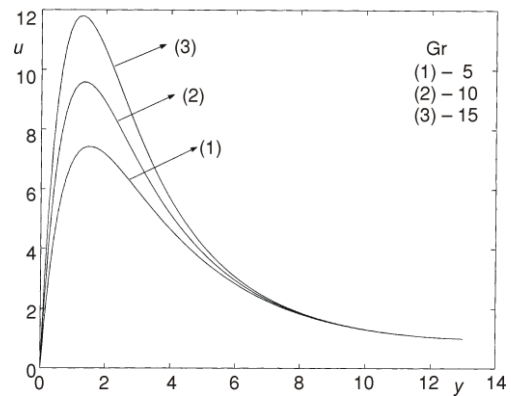


Figure 2. Velocity profiles for different values of Grashof number Gr

Figure 3 shows the effect of the Schmidt number on the non-dimensional velocity $u(y)$, when $Gr = 5$, $Pr = 0.7$, $Gc = 2$, $Ec = 0.001$, $S = 0.2$, and $\omega = 0.8$. It is observed that the non-dimensional velocity decreases with the increase of the Schmidt number.

Figure 4 demonstrates the effect of the radiation parameter S on the non-dimensional temperature field $T(y)$, when $Gr = 5$, $Pr = 0.7$, $Gc = 2$, $Ec = 0.001$, $S = 0.22$, and $\omega = 0.8$. It is observed that the non-dimensional temperature decreases with the increase of the radiation parameter S .

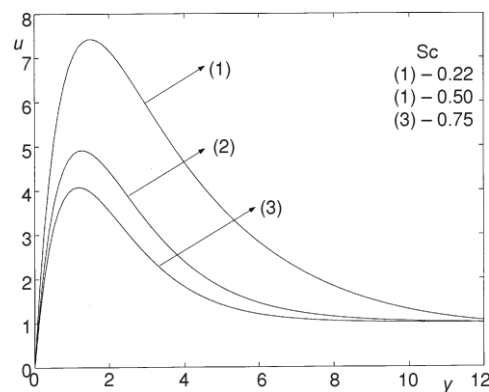


Figure 3. Velocity profiles for different values of Schmidt number Sc

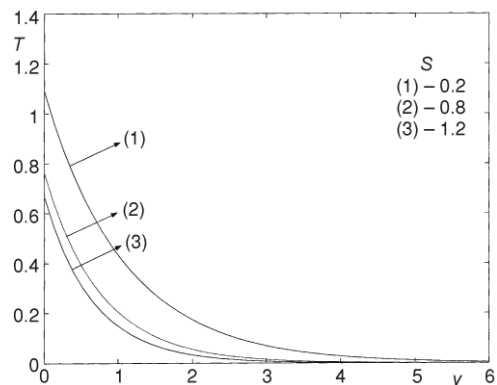


Figure 4. Temperature profiles for different values of radiation parameter S

Conclusions

An analysis is performed to study the free convective oscillatory flow and mass transfer past a porous plate in the presence of radiation for an optically thin fluid. The governing equations are solved numerically. The conclusions of the study are:

- The velocity decreases with the increase of the radiation parameter.
- The velocity increases with the increase of the Grashof number.
- The velocity decreases with the increase of the Schmidt number.
- The temperature decreases with the increase of the radiation parameter.

Nomenclature

C	– dimensionless concentration	U_0	– mean free stream velocity, [ms^{-1}]
C'	– concentration, [molm^{-3}]	u	– dimensionless velocity of the fluid at the x' -direction
C'_w	– species concentration at the plate, [molm^{-3}]	u'	– velocity of the fluid at the x' -direction, [ms^{-1}]
C'_∞	– species concentration far away from the plate, [molm^{-3}]	v'	– velocity of the fluid at the y' -direction
c_p	– specific heat at constant pressure, [$\text{Jkg}^{-1}\text{K}^{-1}$]	v_0	– suction velocity, [ms^{-1}]
D	– chemical diffusivity, [m^2s^{-1}]	x'	– co-ordinate axis along the plate
Ec	– Eckert number, [–]	y'	– co-ordinate axis normal to the plate
$g_{x'}$	– acceleration due to gravity, [ms^{-2}]	<i>Greek symbols</i>	
Gr	– modified Grashof number, [–]	α	– absorption coefficient, [m^{-1}]
Gr	– Grashof number, [–]	β	– coefficient of thermal expansion, [K^{-1}]
k	– thermal conductivity, [$\text{Wm}^{-1}\text{K}^{-1}$]	β^*	– coefficient of concentration expansion, [$(\text{molm}^{-3})^{-1}$]
Pr	– Prandtl number, [–]	ν	– kinematic viscosity, [m^2s^{-1}]
p'	– pressure, [$\text{kgm}^{-1}\text{s}^{-2}$]	ρ	– fluid density, [kgm^{-3}]
q'	– heat flux at the plate, [Wm^{-2}]	σ^*	– Stefan-Boltzman constant, [$\text{Wm}^{-2}\text{K}^{-4}$]
S	– radiation parameter	ω	– dimensionless frequency of vibration of the fluid
Sc	– Schmidt number, [–]	ω'	– frequency of vibration of the fluid, [rads^{-1}]
T	– dimensionless fluid temperature		
T'	– fluid temperature, [K]		
T'_∞	– temperature of the fluid far away from the plate, [K]		
t	– dimensionless time		
t'	– time, [s]		

References

- [1] Seddeek, M. A., Abdelmeguid, M. S., Effects of Radiation and Thermal Diffusivity on Heat Transfer over a Stretching Surface with Variable Heat Flux, *Physics Letters A*, 348 (2006), 3-6, pp. 172-179
- [2] Hossain, M. A., Alim, M. A., Rees, D. A. S., The Effect of Radiation on Free Convection from a Porous Vertical Plate, *International Journal of Heat and Mass Transfer*, 42 (1999), 1, pp.181-191
- [3] Hossain, M. A., Khanafer, K., Vafai, K., The Effect of Radiation on Free Convection Flow of Fluid with Variable Viscosity from a Porous Vertical Plate, *International Journal of Thermal Sciences*, 40 (2001), 2, pp.115-124
- [4] Raptis, A., Toki, C. J., Thermal Radiation in the Presence of Free Convective Flow Past a Moving Vertical Porous Plate – An Analytical Solution, *Int. J. of Applied Mechanics and Engineering*, 14 (2009), 4, pp. 1115-1126
- [5] Raptis, A., Radiation and Free Convection Flow through a Porous Medium, *Int. Comm. Heat Mass Transfer*, 25 (1998), 2, pp. 289-295
- [6] Badruddin, I. A., et al., Free Convection and Radiation for a Vertical Wall with Varying Temperature Embedded in a Porous Medium, *International Journal of Thermal Sciences*, 45 (2006), 5, pp. 487-493

- [7] Mukhopadhyay, S., Layek, G. C., Radiation Effect on Forced Convective Flow and Heat Transfer over a Porous Plate in a Porous Medium, *Meccanica*, 44 (2009), 5, pp. 587-597
- [8] Chamkha, A. J., *et al.*, Thermal Radiation Effects on MHD Forced Convection Flow Adjacent to a Non-Isothermal Wedge in the Presence of a Heat Source or Sink, *Heat and Mass Transfer*, 39 (2003), 4, pp. 305-312
- [9] Duwairi, H. M., Viscous and Joule Heating Effects on Forced Convection Flow from Radiate Isothermal Porous Surfaces, *International Journal of Numerical Methods for Heat and Fluid Flow*, 15 (2005), 5-6, pp. 429-440
- [10] England, W. G., Emery, A. F., Thermal Radiation Effects on the Laminar Free Convection Boundary Layer of an Absorbing Gas, *J. Heat Transfer*, 91 (1969), 1, pp. 37-44
- [11] Bestman, A. R., Adiepong, S. K., Unsteady Hydromagnetic Free-Convection Flow with Radiative Heat Transfer in a Rotating Fluid, *Astrophysics and Space Science*, 143 (1988), 1, pp.73-80
- [12] Raptis, A., Perdikis, C., Leontitsis, A., Effects of Radiation in an Optically thin Gray Gas Flowing Past a Vertical Infinite Plate in the Presence of a Magnetic Field, *Heat and Mass Transfer*, 39 (2003), 8-9, pp.771-773
- [13] Raptis, A., Perdikis, C., Thermal Radiation of an Optically thin Gray Gas, *Int. J. of Applied Mechanics and Engineering*, 8 (2003), 1, pp. 131-134
- [14] Manivannan, K., Muthucumaraswamy, R., Thangaraj, V., Radiation and Chemical Reaction Effects on Isothermal Vertical Oscillating Plate with Variable Mass Diffusion, *Thermal Science*, 13 (2009), 2, pp. 155-162
- [15] Vijayalakshmi, A. R., Radiation Effects on Free-Convection Flow Past an Impulsively Started Vertical Plate in a Rotating Fluid, *Theoret. Appl. Mech.*, 37 (2010) 2, pp. 79-95