# THERMAL RADIATION AND HALL EFFECT ON MHD FLOW, HEAT AND MASS TRANSFER OVER AN INCLINED PERMEABLE STRETCHING SHEET

By

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*Of concern in this paper is an investigation of the combined effects of thermal radiation and Hall current on momentum, heat and mass transfer in laminar boundary-layer flow over an inclined permeable stretching sheet with variable viscosity. The sheet is linearly stretched in the presence of an external magnetic field and the fluid motion is subjected to a uniform porous medium. The effect of internal heat generation/absorption is also taken into account. The fluid viscosity is assumed to vary as an inverse linear function of temperature. The boundary-layer equations that governing the flow problem have reduced to a system of non-linear ordinary differential equations with a suitable similarity transformation. Then the transformed equations are solved numerically by employing a finite difference scheme. Thus the results obtained are presented graphically for the various parameters of interest.*

***Keywords:*** *Thermal radiation; Variable viscosity; MHD flow; Hall current; Heat and Mass Transfer*

## 1 Introduction

During the past few decades there has been a growing interest to investigate the boundary layer flows of viscous fluids in a continuous moving surface because of its many engineering and industrial applications [1-6]. Particularly, due to the engineering applications of magnetohydrodynamic flows with Hall currents in the areas of MHD power generators, flight magnetohydrodynamics as well as in the field of planetary magnetosphere. Also used in the extrusion of a polymer sheet from a die or in the drawing of plastic films, which are then seems to need the controlling of cooling temperature. Therefore, the rate of heat transfer at the stretching sheet play a vital role during final and better quality of the product.

Sakiadis [7] first explored the study of boundary-layer flow on a continuous moving surface and Crane [8] extended this problem to a stretching sheet whose surface velocity varies linearly with the distance $x$ from a fixed point O. Later on Gupta and Gupta [9] examined the heat and mass transfer over a stretching sheet subject to the suction or blowing. The chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [10] in the presence of a uniform transfer magnetic field. Recently, a new idea is added to the study of boundary-layer fluid flow and heat transfer is the consideration of the effect of thermal radiation and temperature dependent viscosity.

Many processes in engineering applications occur at high temperature and the radiate heat transfer becomes very important for the design of the pertinent equipment. Aziz *et al.* [11] analyzed the effects of variable viscosity on the laminar boundary layer flow and heat transfer of a non-Newtonian fluid over a stretching sheet with suction/injection. Mukhopadhaya *et al.* [12] investigated the problem of MHD boundary-layer flow over a heated stretching sheet with variable viscosity. However, Salem [13] investigated the effect of variable viscosity on MHD viscoelastic fluid flow and heat transfer over a stretching sheet without considering the effect of thermal radiation. But, no attempt is available in the existing scientific literatures for the consideration of the combined effects of thermal radiation and hall current on the study of MHD boundary-layer flow. Watanabe and Pop [14] carried out the magnetohydrodynamic boundary-layer flow over a continuously moving semi-infinite flat plate by taking into account the effect of Hall current. Moreover, Shit [15] investigated the Hall effects on MHD free-convective flow and mass transfer over a stretching sheet in the presence of chemical reaction. The analysis of the flow through porous media has become growing interest in several scientific and engineering applications. In view of this, Tak *et al.* [16] investigated the interaction of radiation with free convection in Darcian porous media by taking into account the Soret and Dufour's effects. The steady flow of a viscoelastic fluid past an infinite porous flat plate subject to suction or blowing with constant temperature has been studied by Reza and Gupta [17].

Owing to the above mentioned studies, we propose to investigate the combined effects of thermal radiation and Hall current on the hydromagnetic free-convective flow and mass transfer over an inclined permeable stretching surface with variable viscosity in the presence of heat generation/absorption. The present problem pertains to a situation in which the $n^{th}$ order chemical reaction takes place. Thus, the present study is not only applicable to the movement of gas or oil in the reservoir of an oil or gas field but also of great interest in the field of geomagnet with the fluid in the geothermal region.

## 2 Flow Analysis

Let us consider the steady free-convective flow and mass transfer of an incompressible, viscous and electrically conducting fluid past an inclined permeable stretching surface. A uniform strong magnetic field of strength $B_0$ is imposed along the perpendicular to the sheet (cf. Fig. 1) and the effect of Hall current is taken into account. The temperature and the species concentration are maintained at a prescribed constant values $T_w$, $C_w$ at the sheet and $T_\infty$ and $C_\infty$ are the fixed values far away from the sheet.

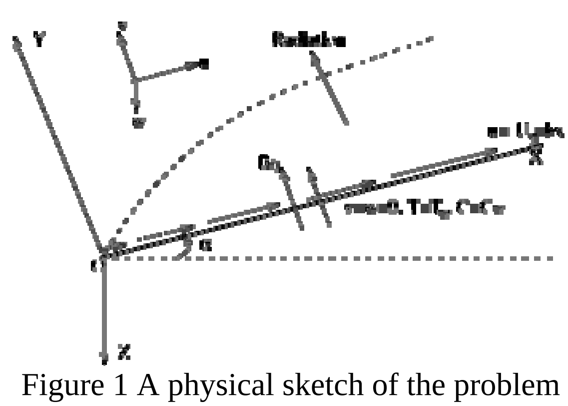



Figure 1 A physical sketch of the problem

Taking Hall effects into account the generalized Ohm's law (cf. Cowling [18]) may be put in the form:

$$\vec{J} = \frac{\sigma}{1+m^2}(\vec{E} + \vec{V} \times \vec{B} - \frac{1}{en_e}\vec{J} \times \vec{B}),$$

where $\vec{V}$, $\vec{E}$, $\vec{B}$, $\vec{J}$, $m = \frac{\sigma B_0}{en_e}$, $\sigma$, $e$, $n_e$ respectively represent the velocity vector, intensity vector of the electric field, magnetic induction vector, electric current density vector, Hall parameter, electrical conductivity, charge of the electron and number density of the electron. The effect of Hall current gives rise to a force in the $z$-direction which in turn produces a cross flow velocity in this direction and thus the flow becomes three-dimensional.

Following Lai and Kulacki [19], the fluid viscosity is assumed to vary as a reciprocal of a linear function of temperature given by

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma_0 (T - T_\infty)\right]. \tag{1}$$

By assuming Rosseland approximation for radiation, the radiative heat flux $q_r$ is given by

$$q_r = -\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y}, \tag{2}$$

where $\sigma^*$ is defined as the Stefan-Boltzman constant and $K$ the mean absorption coefficient.

Owing to the above mentioned assumptions, the boundary layer free-convection flow embedded in a porous medium with mass transfer and generalized Ohm's law is governed by the following system of equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \ , \tag{3}$$

$$\rho_\infty\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \rho_\infty g_0\left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty)\right] cos\alpha$$
$$-\frac{\sigma B_0^2}{1+m^2}(u + mw) - \frac{\mu}{k_1}u \ , \tag{4}$$

$$\rho_\infty\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) + \frac{\sigma B_0^2}{1+m^2}(mu - w) - \frac{\mu}{k_1}w \ , \tag{5}$$

$$\rho_\infty C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{1+m^2}(u^2 + w^2) + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$
$$-Q(T - T_\infty) - \frac{\partial q_r}{\partial y} \ , \tag{6}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_0\left(C-C_\infty\right)^n \;, \tag{7}$$

where $(u,v,w)$ are the velocity components along the $(x,y,z)$ directions respectively, $\mu$ is the coefficient of viscosity, $g_0$ the acceleration due to gravity, $\beta_t$ the coefficient of thermal expansion, $\beta_c$ the coefficient of expansion with concentration, $k_1$ the permeability of the porous media, $\alpha$ the angle of inclination of the stretching sheet with the horizontal line. $T$ and $C$ are the temperature and concentration variable respectively, $D$ the thermal molecular diffusivity, $k_0$ is the reaction rate constant, $C_p$ is the specific heat at constant pressure, $k$ is the thermal conductivity, $T_\infty$ and $\rho_\infty$ are the free stream temperature and density and $n$ denote the order of reaction. The term $Q(T-T_\infty)$ on the right hand side of equation (6) represents the amount of heat generated or absorbed per unit volume. The source term represents the heat source when $Q<0$ and heat sink when $Q>0$.

The boundary conditions for this problem can be written as

$$u=U_w=bx,\; v=w=0,\; T=T_w,\; C=C_w \;\; at \;\; y=0 \tag{8}$$

$$u\to 0,\; w\to 0,\; T\to T_\infty,\; C\to C_\infty \;\; as \;\; y\to\infty \tag{9}$$

where $b(>0)$ is the constant stretching rate and $U_w$ the surface velocity of the sheet. The boundary conditions on the velocity in (8) are the no-slip condition at the surface $y=0$, while the boundary conditions on velocity at $y\to 0$ follow from the fact that there is no flow far way from the stretching sheet.

To examine the flow regime adjacent to the sheet, the following transformations are invoked

$$u=bxf'(\eta);\; v=-\sqrt{b\nu}\, f(\eta);\; w=bxg(\eta);\; \eta=\sqrt{\frac{b}{\nu}}y;\; \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty};\; \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{10}$$

where $f$ is a dimensionless stream function, $\nu=\dfrac{\mu}{\rho_\infty}$ the kinematic viscosity, $\eta$ is a similarity space variable, $\theta$ and $\phi$ are the dimensionless temperature and concentration. Clearly, the continuity equation (3) is satisfied by $u$ and $v$ defined in equation (10). Substitution of (10) in the equations (4) - (7) yields

$$\left(\frac{\theta-\theta_r}{\theta_r}\right)\left(f'^2-ff''\right)+f'''-\left(\frac{\theta'}{\theta-\theta_r}\right)f''-\left(\frac{\theta-\theta_r}{\theta_r}\right)(Gr\theta+Gc\phi)cos\alpha$$
$$+M\left(\frac{\theta-\theta_r}{\theta_r}\right)\left(\frac{f'+mg}{1+m^2}\right)+A1\left(\frac{\theta-\theta_r}{\theta_r}\right)f'=0, \tag{11}$$

$$\left(\frac{\theta-\theta_r}{\theta_r}\right)\left(f'g-fg'\right)+g''-\left(\frac{\theta'}{\theta-\theta_r}\right)g'-M\left(\frac{\theta-\theta_r}{\theta_r}\right)\left(\frac{mf'-g}{1+m^2}\right)+A1\left(\frac{\theta-\theta_r}{\theta_r}\right)g=0, \tag{12}$$

$$(3Nr+4)\theta''+3NrPrf\theta'-3NrPr\lambda\theta+3NrPrEcM\left(\frac{f'^2+g^2}{1+m^2}\right)$$
$$+3NrPrEc\left(f''^2+g'^2\right)=0, \tag{13}$$

$$\phi''+Sc\left(f\phi'-\gamma\phi^n\right)=0 \tag{14}$$

and the transformed boundary conditions are given by

$$f'(\eta)=1,\ f(\eta)=0,\ g(\eta)=0,\ \theta(\eta)=1,\ \phi(\eta)=1,\ at\ \eta=0. \tag{15}$$

$$f'(\eta)\to 0,\ g(\eta)\to 0,\ \theta(\eta)\to 0,\ \phi(\eta)\to 0,\ as\ \eta\to\infty. \tag{16}$$

where primes denote the differentiation with respect to $\eta$ only and the dimensionless parameter $\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}=-(\frac{1}{\gamma_0(T_w-T_\infty)})$ is defined as the viscosity parameter, $M=\frac{\sigma B_0^2}{\rho_\infty b}$ the magnetic parameter, $A1=\frac{\mu}{k_1 b\rho_\infty}$ the permeablity parameter, $P_r=\frac{\rho C_p \nu}{k}$ the Prandtl number, $m=\frac{\sigma B_0}{en_e}$ is the Hall current parameter, $\gamma=\frac{k_0}{b}\left(C_w-C_\infty\right)^{n-1}$ the non-dimensional chemical reaction parameter, $G_r=\frac{g_0\beta_t(T_w-T_\infty)}{b^2x}$ the local Grashof number, $G_c=\frac{g_0\beta_c(C_w-C_\infty)}{b^2x}$ the local modified Grashof number, $Nr=\frac{kK}{4T_\infty^3\sigma^*}$ the thermal radiation parameter, $Ec=\frac{U_w^2}{C_p(T_w-T_\infty)}$ the Eckert number, $\lambda=\frac{Q}{b\rho_\infty C_p}$ the heat generation or absorption parameter and $Sc=\frac{\mu}{\rho_\infty D}$ the Schmidt number.

It is worthwhile to mention here that $\theta_r$ is negative for liquids and positive for gases. The elimination of $\gamma_0$ between the relation (1) and the definition of $\theta_r$ gives rise to $\mu=\frac{\mu_\infty}{(1-\frac{1}{\theta_r})}$.

From this relation it is obvious that when $-\theta_r\to\infty$, then $\mu\to\mu_\infty$, i.e, the viscosity variation in the boundary layer is negligible. However, the viscosity variation is more significant as $-\theta_r\to 0$.

When $M=m=0$ and $G_r=G_c=0$, present flow problem becomes hydrodynamic boundary-layer flow past a stretching sheet whose analytical solution put forwarded by Crane [8] as follows:

$$f(\eta)=1-e^{-\eta}\quad i.e,\quad f'(\eta)=e^{-\eta}. \tag{17}$$

With an aim to test the accuracy of our numerical results for axial velocity $f'(\eta)$, we have compared our results with this analytical solution.

## 3 Computational Results and Discussion

The system of coupled and non-linear ordinary differential equations (11) - (14) along with the boundary conditions (15) and (16) have been solved numerically by employing a finite difference scheme. We used Newton's linearization method (cf. Cebeci and Cousteix [20], Misra and Shit [21, 22]) to linearize the discretized equations. The essential features of this technique is that it is based on a finite difference scheme, which has better stability, simple, accurate and more efficient. Finite difference technique leads to a system which is tri-diagonal and therefore speedy convergence as well as economical memory space to store the coefficients. The computational work has been carried out by taking $\delta\eta$ = 0.0125 and further reduction in $\delta\eta$ does not bring about any significant change. Fig. 2 shows that our numerical results are complete agreement with the analytical results of Crane [8].

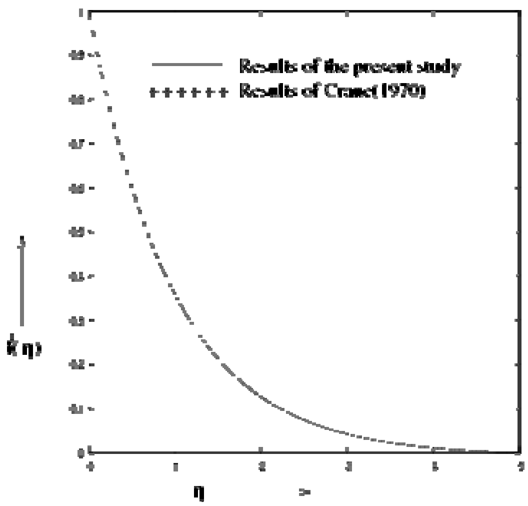



**Figure 2** Variation of $f'(\eta)$ with $\eta$ for hydrodynamic case ($M = m = Gr = Gc = 0$ $Sc = Pr = Nr = A1 = \alpha = 0$ and $-\theta_r \to \infty$ )

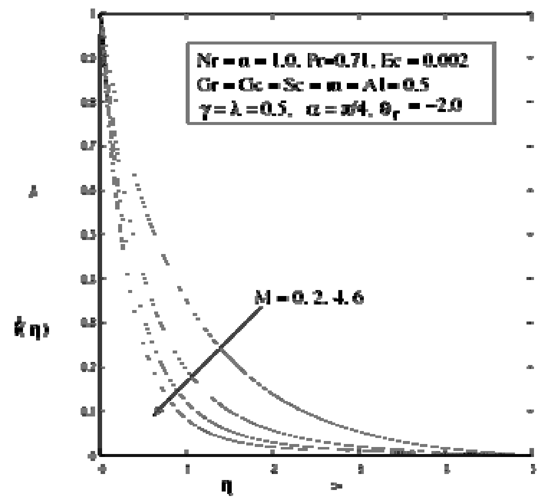



**Figure 3** Variation of $f'(\eta)$ with $\eta$ for different values of $M$

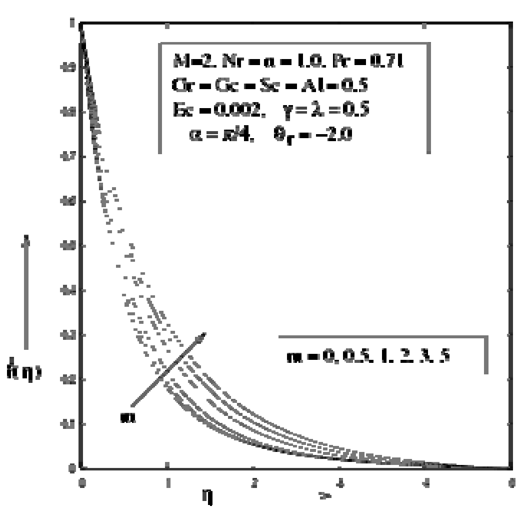



**Figure 4** Variation of $f'(\eta)$ with $\eta$ for different values of $m$

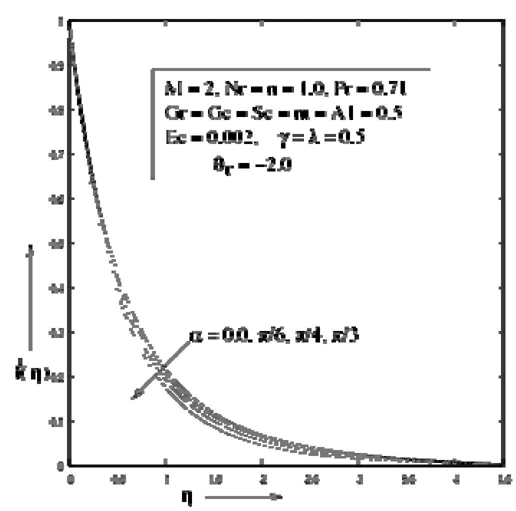



**Figure 5** Variation of $f'(\eta)$ with $\eta$ for different values of $\alpha$

Figs. 3-5 illustrate the variation of axial velocity for different values of the dimensionless parameters that involved in the present study. Fig. 3 shows that the axial velocity decreases with the increase of the magnetic parameter $M$, whereas from Fig. 4 we observe that the axial velocity increases with the increase of Hall parameter $m$. This is due to the fact that as $M$ increases, the Lorentz force which oppose the flow and leads to a deceleration of the fluid motion. Fig. 5 depicts that the axial velocity $f'(\eta)$ decreases with increasing the inclination angle $\alpha$. Hence we conclude that the boundary layer flow can be controlled by the application of magnetic field as well as by making use of inclination of the sheet.

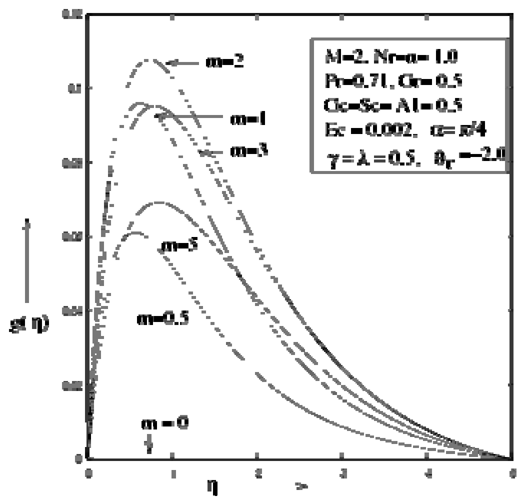



**Figure 6** Variation of $g(\eta)$ with $\eta$ for different values of $m$

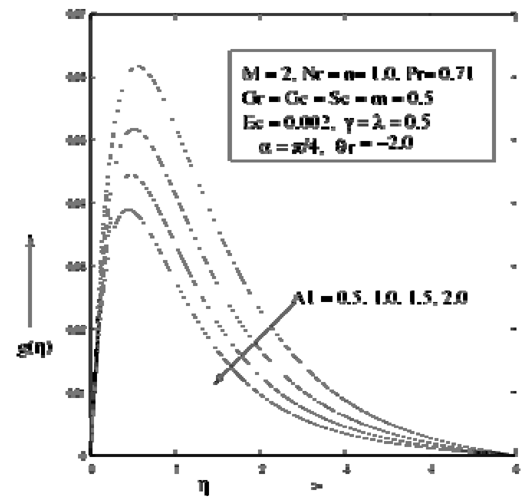
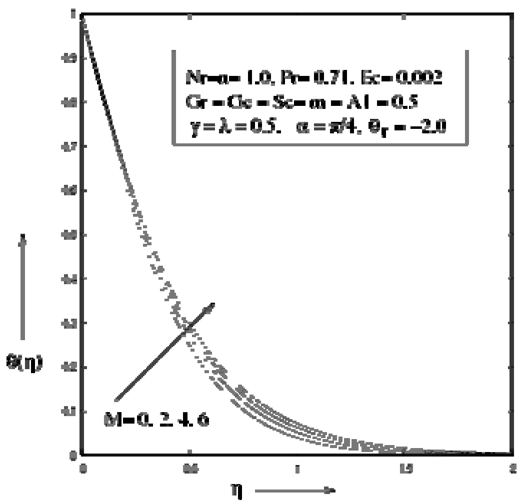



**Figure 7** Variation of $g(\eta)$ for different values of permablity parameter $A1$

Figs. 6 and 7 give the distribution of the $z$-component of velocity, which is induced due to the presence of Hall current. All these figures show that for any particular value of the physical parameters $g(\eta)$ reaches a maximum at a certain height $\eta$ above the sheet and beyond which it decreases gradually in asymptotic nature. Fig. 6 reveals that in the absence of magnetic parameter $M$ (=0) cross-flow velocity vanishes. This is due to fact that when there is no applied magnetic field, the Hall current effect does not exists and hence the cross-flow velocity becomes zero. An interesting result observed from this figure that the cross-flow velocity gradually increases with the increase of $m \le 2$ and the trend is opposite when $m > 2$. However, the values of $m$ beyond which the flow behaviour changes is considerably depend upon the choice of the magnetic field strength. Fig. 7 indicates that as the permeability parameter $A1$ increases, the cross-flow velocity diminishes.

Figs. 8-10 illustrate the distribution of dimensionless temperature $\theta(\eta)$ along the height from the stretching sheet for different values of the dimensionless parameters $M$, $m$ and $Nr$. Fig. 8 indicates that the dimensionless temperature increases with increasing values of $M$, while the thermal radiation parameter $Nr$ as well as Hall parameter $m$ have reducing effects on the dimensionless temperature $\theta(\eta)$ shown in Figs. 9 and 10. It is appear from Fig. 9 that the temperature is insignificantly affected by the Hall parameter $m$.



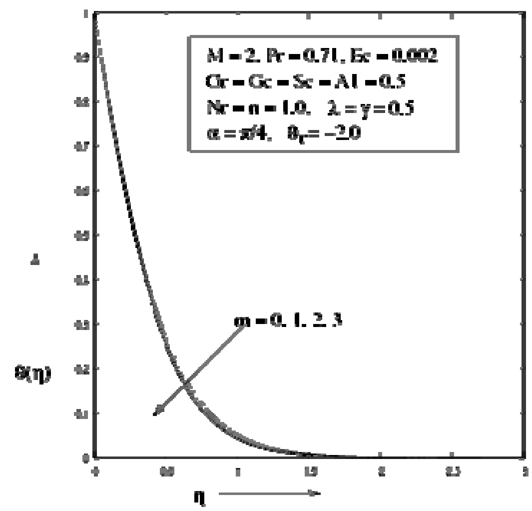
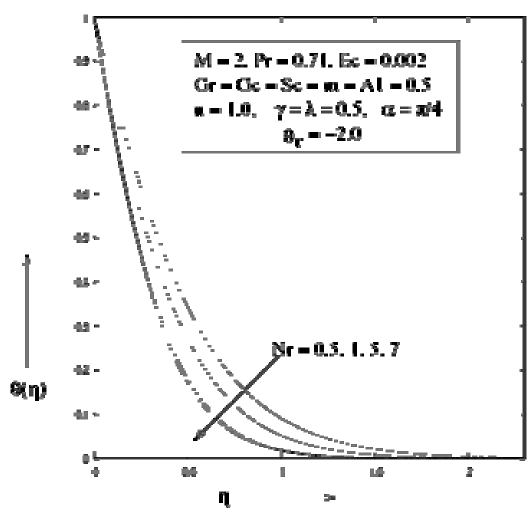




**Figure 8** Distribution of dimensionless temperature $\theta(\eta)$ for different values of $M$

**Figure 9** Distribution of dimensionless temperature $\theta(\eta)$ for different values of Hall parameter $m$



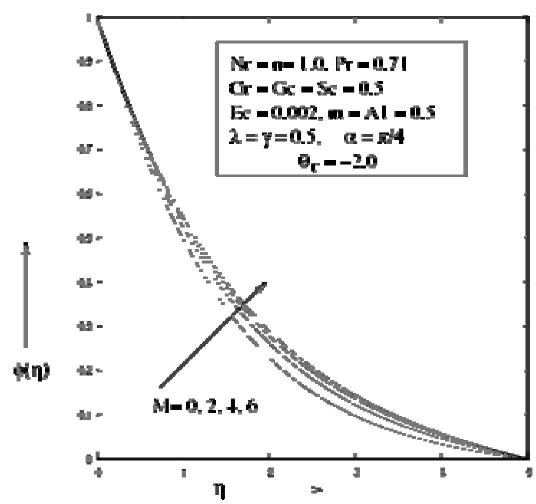




**Figure 10** Distribution of dimensionless temperature $\theta(\eta)$ for different values of $Nr$

**Figure 11** Concentration profiles for different values of $M$

Figs. 11-13 give the distribution of concentration species for different values of the magnetic parameter $M$, Schmidt number $Sc$ and the permeable parameter $A1$. It is observed from Fig. 11 that, $\phi$ increases with the increase of $M$, but the trend is reversed in the case of Schmidt number $Sc$. It is further observed from Fig. 13 that the concentration increases with the increase of the permeable parameter $A1$. Thus the permeability of the stretching sheet reduces the thickness of the concentration boundary-layer.

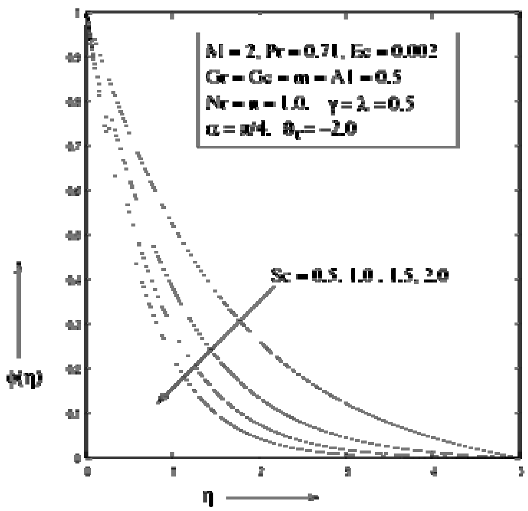



**Figure 12** Concentration profiles for different values of $Sc$

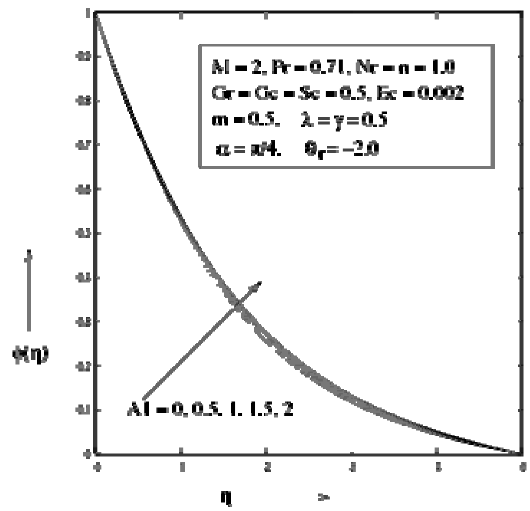



**Figure 13** Concentration profiles for different values of the permeable parameter $A1$

# 4 Concluding Remarks

The present study describes the numerical solution of the combined effects of thermal radiation and Hall current on the MHD flow over an inclined permeable stretching sheet in the presence of variable fluid properties. The effects of various key parameters including the angle of inclination $(\alpha)$, magnetic parameter $(M)$, Hall parameter $(m)$, thermal radiation parameter $(Nr)$, permeability parameter $(A1)$ are examined. The main findings of the present study are as follows: (i) The velocity components are decreasing with increasing in $\alpha$. It is also noted that the velocity profiles are strongly affected by the inclination of the sheet, whereas the temperature and concentration profiles are weakly affected. (ii) The cross flow velocity increases or decreases for a certain range of $m$. (iii) The increasing values of the thermal radiation parameter leads to the decreasing of thermal boundary layer thickness. Thus it may be used to increase the rate of cooling of the sheet. (iv) The increasing values of the permeability parameter has a reducing effect on the velocity profiles.

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