Convection heat and mass transfer in a hydromagnetic Carreau fluid past a vertical porous plate in presence of thermal radiation and thermal diffusion

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Abstract

The paper presents the numerical investigation of the convection heat and mass transfer in a hydromagnetic Carreau fluid past a vertical porous plate in presence of thermal radiation and thermal diffusion. The non – linear partial differential equations governing the flow are transformed into ordinary differential equations using the usual similarity method and the resulting similarity equations are solved numerically using Runge – Kutta shooting method. The results are presented as velocity, temperature and concentration profiles for different values of parameters entering into the problem. The effects of suction, magnetic field, thermal radiation and thermal diffusion on the skin friction, rate of heat transfer and mass transfer are presented numerically in tabular form.

Keywords: *Carreau fluid, hydromagnetic, suction, convection heat, thermal radiation, thermal diffusion*

Introduction

Recently, researchers in engineering and scientific field have shown great interest in the study of non – Newtonian fluids due to its importance in industrial processes. The development of the theory of non – Newtonian fluid mechanics arose from the inadequacy of the theory of Newtonian fluids in predicting the behaviours of many fluids especially those of high molecular weight.

Many authors have examined the flow, heat and mass transfer in non – Newtonian fluid of different type, most especially in power law and higher order fluids. Abel et. al. [1] examined the effects of viscous dissipation and non-uniform heat source/sink on the boundary layer flow and heat transfer characteristics of a second grade, non-Newtonian fluid through a porous medium. Ahmad [2] carried out the mathematical analysis of heat transfer effects on the axisymmetric flow of a second grade fluid over a radially stretching sheet using the homotopy analysis method. Ahmed [3] presented Lie group analysis and the basic similarity reductions for the MHD aligned slowly flowing and heat transfer in second grade fluid with neglecting the inertial terms.

Bikash [4] studied the numerical solution of the laminar flow and heat transfer of an incompressible, third grade, electrically conducting fluid impinging normal to a plane in the presence of a uniform magnetic field.

Hayat and Sajid [5] examined the steady laminar flow and heat transfer in an axisymmetric flow of a second grade fluid is induced due to linear stretching of a sheet.

Hayat et. al [6] considered the laminar flow problem of convective heat transfer for a second grade fluid over a semi-infinite plate in the presence of species concentration and chemical reaction, they gave the boundary layer analysis of the solution obtained by homotopy analysis method. Hayat et. al [7] obtained the series solutions for the flow and heat transfer problem of an incompressible and electrically conducting second grade fluid film over an unsteady stretching sheet using the homotopy analysis method. Hayat et. al [8] obtained the series solution and analyzed the convergences for heat transfer on the flow of a fourth grade fluid past a porous plate using the homotopy analysis method. Hayat et. al. [9] examined the influences of the Hall parameter and porosity of the medium on the velocity and temperature profiles for the heat transfer on a rotating flow of a second grade fluid past a porous plate with variable suction. Hayat et. al. [10] presented analytical solutions of the equations of motion and energy of a electrically conducting fluid second grade fluid for the developed flow over a semi-infinite porous stretching sheet with slip condition.

Hayat et. al. [11] examined a two-dimensional mixed convection boundary layer magnetohydrodynamic (MHD) stagnation-point flow through a porous medium bounded by a stretching vertical plate with thermal radiation. They obtained exact solution using the method of the homotopy analysis. Kai-Long [12] studied the heat transfer on a stretching sheet cooled or heated by a high or low Prandtl number, the buoyancy parameter, the magnetic parameter, the radiation parameter, and conduction–convection coefficient for second-grade viscoelastic fluid.

Khani [13] presented an analytic approximate solution for the natural convective dissipative heat transfer of an incompressible, third grade, non-Newtonian fluid flowing past an infinite porous plate embedded in a Darcy–Forchheimer porous medium.

Olajuwon [14, 15, 16, 17, and 18] studied the convection heat and mass transfer in a non – Newtonian power law fluid with heat generation, thermal diffusion, thermo diffusion and thermal radiation past vertical plate. The analysis of results obtained showed that these parameters have significant influences on the flow, heat and mass transfer.

But little attention has been paid to the four-parameter Carreau inelastic model with the stress formulation;

$$\tau_{yx} = \mu_0 \left[1 + \lambda^2 \left(\frac{\partial u}{\partial y} \right)^2 \right]^{\frac{n-1}{2}} \frac{\partial u}{\partial y} \qquad (1)$$

frequently used in chemical engineering. It fits reasonably well with the suspensions of polymers behavior in many flow situations. This model describes the behavior of a purely viscous fluid whose viscosity changes with increasing rate of deformation. Unlike the power-law or Ostwald - De Waele model, it predicts a viscosity that remains finite as the shear rate approaches zero. For that reason, the Carreau constitutive equation suits well for free surface flows. Among the recent studies in the theory of Carreau fluid include peristaltic flow and heat transfer.

Sobh [19] presented a theoretical study of a peristaltic transport of a Carreau fluid in an asymmetric channel under zero Reynolds number long – wavelength approximation for both slip and non slip flows. Sobh [20] studied the interaction of peristalsis with heat transfer for the flow of a viscous fluid through a porous medium in uniform and non – uniform channels in the presence of a constant transverse magnetic field. Mekheimer and Abdelmaboud [21] investigated the influence of heat transfer and magnetic field on the peristaltic flow of a Newtonian fluid in a vertical annulus under a zero Reynolds number and long wavelength approximation. The flow in investigated in a wave frame of reference moving with velocity of the wave. Ali and Hayat [22] presented the analytic solution of the mathematical modeling for the flow of incompressible Carreau fluid in an asymmetric channel with sinusoidal wall variations. Hayat et. al. [23] examined the magnetohydrodynamic (MHD) peristaltic flow of a Carreau fluid in a channel with different waveforms.

Due to the non – linear dependence, the analysis of the behaviours of the non – Newtonian Carreau fluids tends to be more complicated and subtle in comparison with that of the non – Newtonian fluids. In general, the equations of motion for non – Newtonian fluids are of higher complexity than the Navier – Stokes equations and thus one needs some conditions in addition to the usual adherence boundary condition. Hence, there is a need for a method which provides a means of obtaining other conditions necessary for the solution. One of such methods is the Runge - kutta shooting method. In addition, to best of author's knowledge the combined effects of the suction, thermal radiation and thermal diffusion on the convection heat and mass transfer flow in a Carreau fluid have to not been studied.

Thus, the objective of this paper is to investigate numerically the convection heat and mass transfer in a hydromagnetic Carreau fluid past a vertical porous plate in presence of thermal radiation and thermal diffusion. The results are presented as velocity, temperature and concentration profiles for different values of parameters entering into the problem. The effects of suction, magnetic field, thermal radiation and thermal diffusion on the skin friction, rate of heat transfer and mass transfer are presented numerically in tabular form.

Mathematical Formulation

Consider an unsteady convection flow of a generalized Newtonian Carreau fluid past a moving porous plate. Let the x – axis be taken along the plate in the vertically upward direction and the y – axis be taken normal to it. Let u and v be the velocity component along the x and y directions, respectively. If x- axis is chosen along the plate and y – axis perpendicular to it, then the investigated flow does not depends on x. Hence, the continuity equation becomes

$$\frac{\partial v}{\partial y} = 0 \qquad (2)$$

The surface is maintained at a constant temperature T_w which is higher than the constant temperature T_∞ of the surrounding and concentration C_w is greater than the constant concentration C_∞. The fluid properties are assumed to be constant. Since the plate is vertically upward, the governing equations of continuity, momentum, energy and concentration for the unsteady flow can be written as;

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}\left[1+\lambda^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-1}{2}}+\left[2\lambda^2\mu_0\left(\frac{n-1}{2}\right)\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}\right]\left[1+\lambda^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-3}{2}}$$

$$-\frac{\sigma_0\beta_0{}^2 u}{\rho}+g\beta(T-T_\infty)+g\lambda^*(C-C_\infty) \quad (3)$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\alpha}{k}\frac{\partial q_r}{\partial y} \quad (4)$$

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y}+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \quad (5)$$

Where, the radiative heat flux term is simplified by making use of the Rosseland approximation as

$$q_r=\frac{-4\sigma^*}{3\delta}\frac{\partial T^4}{\partial y} \quad (6)$$

And the last term on the right-hand side of the concentration equation (5) signifies the thermal diffusion effect. The appropriate boundary conditions are;

$$u=U,\quad T=T_w,\quad C=C_w \quad \text{at} \quad y=0 \quad (7)$$

$$u\to 0,\quad T\to T_\infty,\quad C\to C_\infty \quad \text{as} \quad y\to\infty,\ t>0 \quad (8)$$

Where U (at the time $t = 0$ the plate is impulsively set into motion with the velocity U) is the plate characteristics velocity.

Method of solution

Introducing a dimensionless similarity variable,

$$\eta=\frac{Ay}{t^{\frac{1}{2}}} \quad (9)$$

Where A is constant and t is the time, such that,

$$u=Uf(\eta) \quad (10)$$

and define the dimensionless quantities,

$$\left.\begin{aligned}&\theta=\frac{T-T_\infty}{T_w-T_\infty},\quad R_d=\frac{16\sigma T_\infty^3}{3\delta k},\quad Sc=\frac{\upsilon t}{DA},\ \phi=\frac{C-C_\infty}{C_w-C_\infty},\\ &M=\frac{\sigma\beta_0 t}{\rho},\quad Sr=\frac{D_m K_T(T_w-T_\infty)}{T_m\upsilon(C_w-C_\infty)t}\end{aligned}\right\} \quad (11)$$

$$\Pr_n = \frac{\rho ct}{kA}, \; Gr = \frac{g\beta t(T_w - T_\infty)}{U}, \; Gc = \frac{g\lambda^* t(n+1)(C_w - C_\infty)}{U},$$

From equation (2), v is either constant or a function of time. Choosing,

$$v = -\frac{c}{At^{\frac{1}{2}}} \tag{12}$$

Where, $c > 0$ is the suction parameter. Using equations (6), (9), (10), (11) and (12) in equations (3), (4) and (5). The reduced governing equations read:

$$f''\left\{\left[1+\lambda_1 (f')^2\right]^{\frac{n-1}{2}} + \left[2\lambda_1\left(\frac{n-1}{2}\right)(f')^2\right]\left[1+\lambda_1(f')^2\right]^{\frac{n-3}{2}}\right\} + \left(\frac{\eta}{2}+c\right)f' + Gr_n\theta + Gc_n\phi - Mf = 0 \tag{13}$$

$$\upsilon(1-R_d)\theta'' + \Pr\left(\frac{\eta}{2}+c\right)\theta' = 0 \tag{14}$$

$$\phi'' + Sc\left(\frac{\eta}{2}+c\right)\phi + SrSc\theta'' = 0 \tag{15}$$

The new boundary conditions are:

$$f(0)=1, \theta(0)=1, \phi(0)=1, \; f(\infty)=0, \theta(\infty)=0, \; \phi(\infty)=0 \tag{16}$$

Where, Gr is the thermal Grashof number, Gc is the solutal Grashof number, $\Pr$ is the Prandlt number, Sc is the Schmidt number, Sr is the Soret number, M is the magnetic field parameter, R_d is the radiation parameter, $\lambda_1 = \frac{\lambda^2 A^2 U^2}{t}$ is the material parameter and the prime symbol denotes derivative with respect to η.

Reduce the boundary valued problems (12), (13) and (14) to an initial valued problem. Let, $x_1 = \eta, x_2 = f$, $x_3 = f', x_4 = \theta, x_5 = \theta'$, $x_6 = \phi$ and $x_7 = \phi'$. Then, the following system is obtained;

$$
\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \\ x_6' \\ x_7' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \dfrac{Mx_2 - Gr\, x_4 - Gc\, x_6 - (\frac{x_1}{2} + c)x_3)}{\left[1+\lambda_1\ (x_3)^2\right]^{\frac{n-1}{2}} + \left[2\lambda_1\left(\dfrac{n-1}{2}\right)(x_3)^2\right]\left[1+\lambda_1 (x_3)^2\right]^{\frac{n-3}{2}}} \\ x_5 \\ \dfrac{-\Pr(\frac{x_1}{2}+c)x_5}{(1-R_d)} \\ x_7 \\ \dfrac{x_5(\frac{x_1}{2}+c)Sc\Pr Sr - Sc(1-R_d)x_7(\frac{x_1}{2}+c)}{(1-R_d)} \end{pmatrix} \tag{17}
$$

with the initial conditions;

$$
\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \\ x_5(0) \\ x_6(0) \\ x_7(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \gamma \\ 1 \\ \Omega \\ 1 \\ \Gamma \end{pmatrix} \tag{18}
$$

Equation (17) together with the initial condition (18) is solved using Runge – Kutta shooting method. The values of γ, Ω and Γ are obtained such that the boundary conditions (16) are satisfied.

In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial valued problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy.

Infact, the essence of this method is to reduce the boundary value problem to an initial value problem and then solved using the fourth order Runge – Kutta shooting technique to find $f'(0)=\gamma$, $\theta'(0)=\Omega$ and $\phi'(0)=\Gamma$. It is observed from (15) that the velocity, temperature and concentration decrease with increase in the value of η.Theoretically, the width for the fluid flow is given as $\eta \in [0,\infty]$, but it can be assumed that the flow width has a theoretical maximum. Using this approximation, the flow width of the fluid flow is taken as $\eta \in [0,1]$

The numerical results are presented in table 1 and graphically in figures 1- 7.

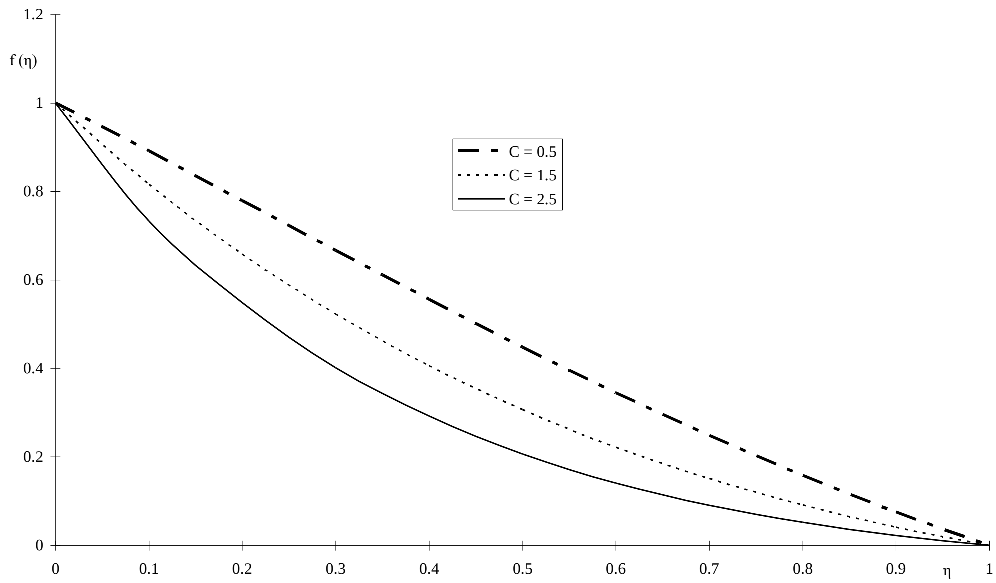

Figure 1: Velocity profile for various values of the Suction parameter, C

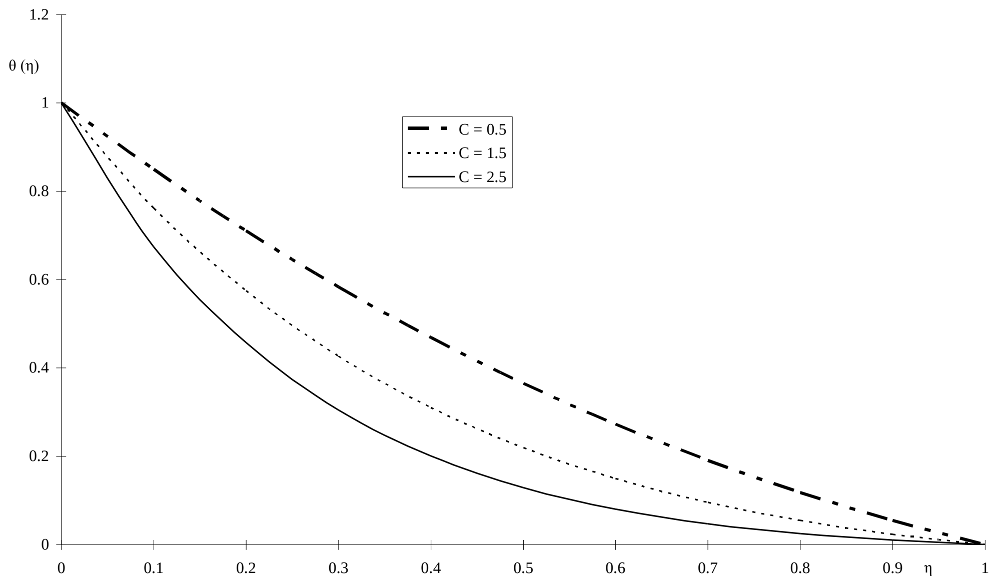

Figure 2: Temperature profile for different values of the suction parameter, C

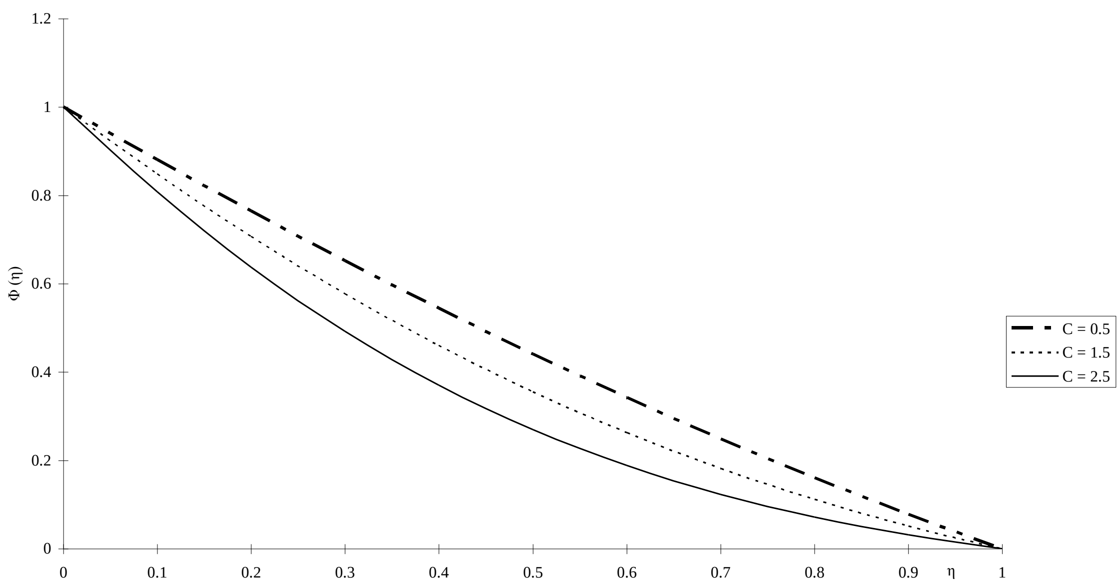

Figure 3: Concentration profile different values of the suction parameter, C

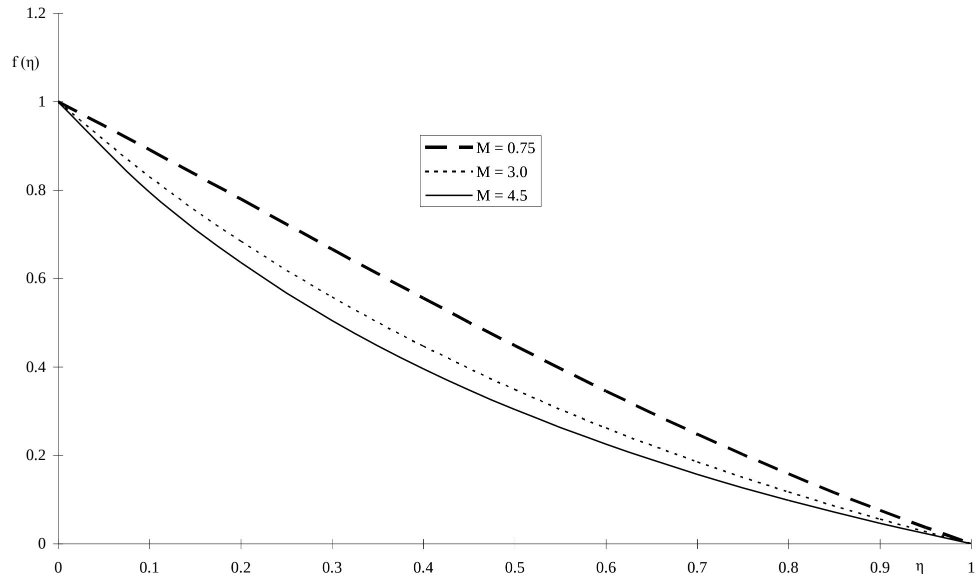

Figure 4: Velocity profile for various values of the Magnetic field parameter, M

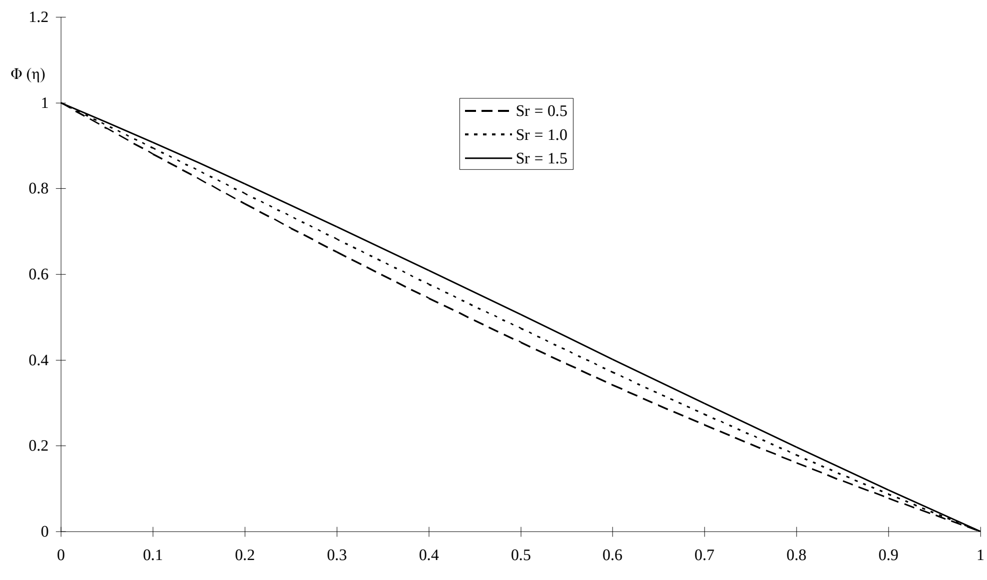

Figure 5: Concentration profile flor different values of the thermal diffusion parameter (Soret number), Sr

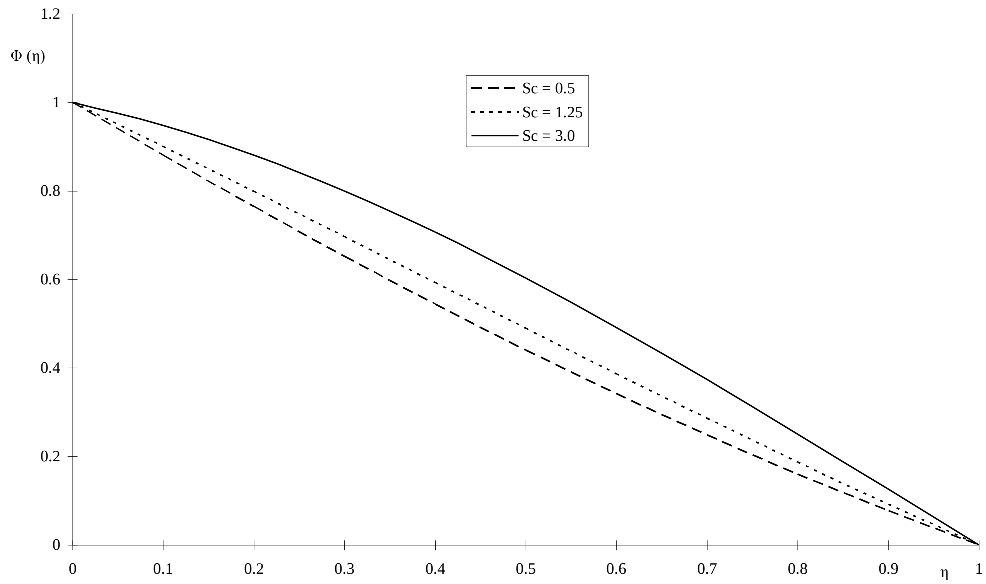

Figure 6: Concentration profile for different values of the Schmidt number, Sc

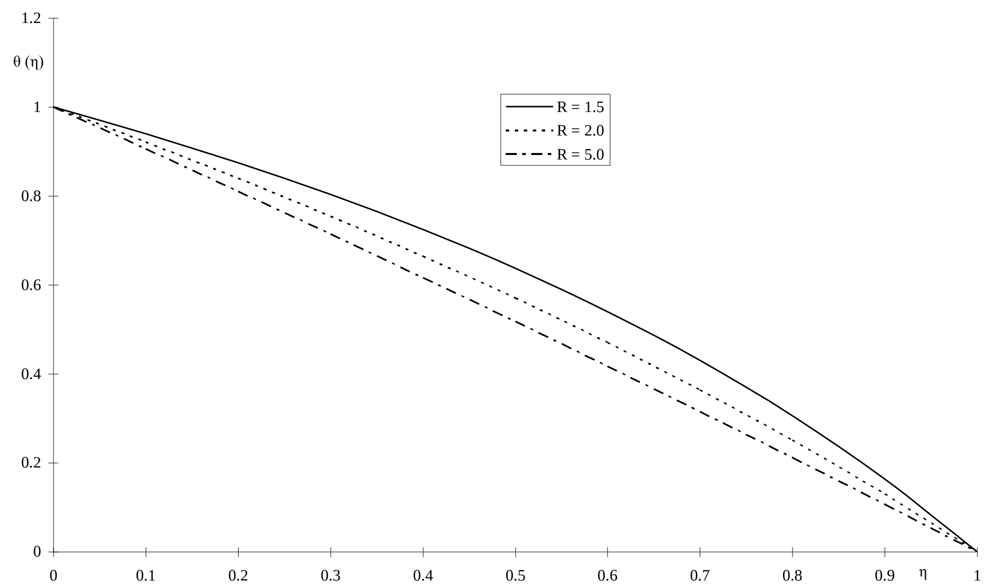

Figure 7: Temperature profile for different values of the Radiation parameter, R

Skin friction, rate of heat and mass transfer

We will now calculate the physical quantities of engineering primary interest, which are the local wall shear stress, local surface heat flux and the local mass flux respectively from the following definitions

$$\tau_w = \left(\mu_0 \left[1 + \lambda^2 \left(\frac{\partial u}{\partial y} \right)^2 \right]^{\frac{n-1}{2}} \frac{\partial u}{\partial y} \right)_{y=0} \qquad (19)$$

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \qquad (20)$$

$$M_w = -D_m \left(\frac{\partial C}{\partial y} \right)_{y=0} \qquad (21)$$

Where *n* is the flow index, *k* is the thermal conductivity and D_m is the diffusivity. And the dimensionless skin friction coefficient C_f, Nusselt number, *Nu*, and the Sherwood number, *Sh*, are given by;

$$C_f \operatorname{Re} = \left[1 + \lambda_1 \left(f'(0) \right)^2 \right]^{\frac{n-1}{2}} f'(0) \qquad (22)$$

$$Nu = \frac{t^{\frac{1}{2}} q_w}{k(T_w - T_\infty)} = -\theta'(0) \qquad (23)$$

$$Sh = \frac{t^{\frac{1}{2}} M_w}{D_m (C_w - C_\infty)} = -\phi'(0) \qquad (24)$$

Where,

$$\operatorname{Re}_n = \frac{t^{-\frac{1}{2}} \rho A U}{\upsilon} \qquad (25)$$

is the modified Reynolds number.

These dimensionless values of the local skin friction coefficient, local Nusselt number and local Sherwood number are obtained from the process of numerical computations and are presented in tables 1 – 7.

Discussion of result

The combined effect of the suction, thermal radiation, thermal diffusion and the magnetic field parameter on the convection heat and mass transfer in a Carreau fluid is considered for a shear thinning case. The numerical results were obtained for values of the power index, suction, Grashof number, material parameter, Prandtl number, magnetic field parameter, Schmidt number, thermal diffusion (Soret number), thermal radiation parameter; $n = 0.6, c = 0.5, G = 0.9, \lambda = 0.3, \Pr = 0.75, M = 0.75, Sc = 0.5, Sr = 0.5, R_d = 0.5$. These values are then varied to observe their effects on the heat and mass transfer problem. The numerical results are presented as the figures 1 – 7 and in tables 1 – 8.

Figures 1 to3 show the effect of the suction parameter on the velocity, temperature and concentration profiles. It is clearly shown that the velocity of the fluid flow decreases with increase in the suction parameter, the temperature and concentration of the fluid decrease with increase in the suction parameter. Figure 4 shows that velocity of the fluid flow decreases as the magnetic field parameter increases, this is an indication that the force which tends to oppose the fluid flow increases with increase in the magnetic field parameter. Figures 5, shows that the concentration of the fluid decreases with increase in thermal diffusion parameter. Figure 6 shows that the concentration of the fluid increases with increase in the Schmidt number, and figure 7 shows that temperature of the fluid decreases with increase in the thermal radiation parameter.

Table 1: Numerical result for different values of the power index

n	Nu	Sh	$C_f \operatorname{Re}$
0.3	1.5672	1.2108	- 0.95678236
0.6	1.5672	1.2108	- 0.99405718
0.9	1.5672	1.2108	- 1.03260556

From table 1, the skin friction decreases with increases in the power index, while the Nusselt and Sherwood number remain constant. The rate of the fluid flow increases with increase in the power index. Thus, a fluid with a higher power index flows faster.

Table 2: Effect of the Material parameter

λ	Nu	Sh	$C_f \operatorname{Re}$
0.3	1.5672	1.2108	- 0.99405718
0.8	1.5672	1.2108	- 0.95477154
1.5	1.5672	1.2108	- 0.87756947
2.5	1.5672	1.2108	- 0.82378404

It is clear from table 2 that as the Material parameter of the fluid increases the skin friction increases. The material parameter among other properties dictates the physical texture of the fluid, and it causes a slight increase in the rate of fluid flow.

Table 3: Effect of the Suction parameter

c	Nu	Sh	C_f Re
0.5	1.5672	1.2108	- 0.99405718
1.0	2.0941	1.3784	- 1.32621599
1.5	2.6886	1.5588	- 1.69732102
2.5	4.0117	1.9736	- 4.58123346

Table 3 shows that the skin friction decreases with increase in the suction parameter while the Sherwood number and the Nusselt number increase with increase in suction parameter. Hence, rate of heat and mass transfer increases with increase in the suction parameter.

Table 4: Effect of the Magnetic field parameter

M	Nu	Sh	C_f Re
0.75	1.5672	1.2108	- 0.99405718
1.5	1.5672	1.2108	- 1.22144897
3.0	1.5672	1.2108	- 1.61205399
4.5	1.5672	1.2108	- 1.93926121

From table 4, the skin friction decreases with increase in the Magnetic field parameter. The rate of heat and mass transfer are not affected by the Magnetic field parameter. Therefore, as the Magnetic field parameter increases the boundary layer thickness becomes thin.

Table 5: Effect of the Radiation parameter

R	Nu	Sh	C_f Re
1.5	0.5812	1.4809	- 0.93100548
2.0	0.7709	1.4271	- 0.94586424
3.0	0.88055	1.3965	- 0.95361394
5.0	0.9391	1.3803	- 0.95765410

Table 5 shows that the skin friction decreases slightly with increases in the thermal Radiation parameter, the Nusselt increases with increases in the thermal Radiation parameter, while the Sherwood number decreases with increases in the thermal Radiation parameter. Thus, rate of heat transfer increases with the thermal Radiation parameter and mass transfer decreases with increase in the thermal Radiation parameter.

Table 6: Effect of the Schimdt number

Sc	Nu	Sh	C_f Re
0.5	1.5672	1.2108	- 0.99405718
1.25	1.5672	0.9817	- 0.97812542

2.25	1.5672	0.6762	- 0.95670896
3.0	1.5672	0.4470	- 0.94068815

It is clear from table 6 that the skin friction increases with increase the Schmidt number, while the Sherwood number decreases with increase the Schmidt number. And this shows that the rate of mass transfer decreases with increase the Schmidt number.

Table 7: Effect of the Soret number

Sr	Nu	Sh	C_f Re
0.5	1.5672	1.2108	- 0.99405718
1.0	1.5672	1.0581	- 0.98341593
1.5	1.5672	0.9053	- 0.97274111
2.0	1.5672	0.7525	- 0.96203274

In table 7, the skin friction increases with increase in the thermal diffusion parameter, while the Sherwood decreases with increase in the thermal diffusion parameter. Hence, the rate of mass transfer decreases with increase in the thermal diffusion parameter.

Conclusion

A numerical study of the convection heat and mass transfer in a hydromagnetic Carreau fluid past a vertical porous plate in presence of thermal radiation and thermal diffusion has been carried out. The results obtained clearly shown the effects of the suction, thermal radiation, thermal diffusion, thermo - diffusion and the magnetic field parameter on the flow, heat and mass transfer. The study finds application in many industrial processes such as sanitary fluid transport, transport of corrosive fluids and blood pumps in heart lung machines. And the following conclusions are drawn;

- Increase in the power index, magnetic field and material parameters cause slight increase in the fluid flow rate.
- The rate of heat and mass transfer increase with increase in the suction parameter.
- The rate of heat transfer increases with the thermal Radiation parameter and mass transfer decreases with increase in the thermal Radiation parameter.
- Increase in Schimdt number and thermal diffusion (Soret number) results in decrease in the rate of mass transfer.

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Nomenclatures

T is the temperature

C is the fluid concentration

D_m is the coefficient of mass diffusivity

k is the thermal conductivity

υ is the kinematics viscosity

n is the flow index,

k is the thermal conductivity

D_m is the diffusivity.

C_f, the dimensionless skin friction Coefficient

Nu Nusselt number

Sh, Sherwood number,

Gr is the thermal Grashof number,

Gc is the solutal Grashof number,

Pr is the Prandlt number,

Sc is the Schmidt number,

Sr is the Soret number,

M is the magnetic field parameter,

R_d is the radiation parameter,

K_T is the thermal diffusion ratio

T_m is the mean fluid temperature

g is the acceleration due to gravity

C_∞ is the uniform concentration of the fluid far away from the plate

T_∞ is the uniform temperature of the fluid far away from the plate

C_w is the uniform concentration of the fluid at the plate surface

T_w is the uniform temperature of the fluid at the plate

Greek

α is the thermal diffusivity

σ^* is the Stefan Boltzmann constant

δ is the mean absorption coefficient.

ρ is the density

σ is the electrical conductivity

β_0 is the constant magnetic flux density

β is the volumetric expansion – coefficient due to temperature

λ^* is the volumetric expansion – coefficient due to concentration

λ_1 is the material parameter and the prime symbol denotes derivative with respect to η.

References

[1] Subhas Abel M, Mahantesh M. N, Sharanagouda B. M, Heat transfer in a second grade fluid through a porous medium from a permeable stretching sheet with non-uniform heat source/sink, *International Journal of Heat and Mass Transfer 53 (2010) 1788–1795.*

[2] Ahmad I, Sajid M, Hayat T, Heat transfer in unsteady axisymmetric second grade fluid, *Applied Mathematics and Computation 215 (2009) 1685–1695.*

[3] Ahmed A. A, Some new exact solutions for MHD aligned creeping flow and heat transfer in second grade fluids by using Lie group analysis, *Nonlinear Analysis 70 (2009) 3298–3306.*

[4] Bikash S, Hiemenz flow and heat transfer of a third grade fluid, *Communications in Nonlinear Science and Numerical Simulation 14 (2009) 811–826.*

[5] Hayat T, Sajid M, Analytic solution for axisymmetric flow and heat transfer of a second grade fluid past a stretching sheet, *International Journal of Heat and Mass Transfer 50 (2007) 75–84.*

[6] Hayat T, Abbas Z, Sajid M, Heat and mass transfer analysis on the flow of a second grade fluid in the presence of chemical reaction, *Physics Letters A, 372 (2008) 2400–2408.*

[7] Hayat T, Saif S, Abbas Z, The influence of heat transfer in an MHD second grade fluid film over an unsteady stretching sheet, *Physics Letters A 372 (2008) 5037–5045.*

[8] Hayat T, Noreen S, Sajid M, Heat transfer analysis of the steady flow of a fourth grade fluid, *International Journal of Thermal Sciences 47 (2008) 591–599.*

[9] Hayat T, Abbas Z, Asghar S, Effects of Hall current and heat transfer on rotating flow of a second grade fluid through a porous medium, *Communications in Nonlinear Science and Numerical Simulation 13 (2008) 2177–2192.*

[10] Hayat T, Javed T, Abbas Z, Slip flow and heat transfer of a second grade fluid past a stretching sheet through a porous space, *International Journal of Heat and Mass Transfer 51 (2008) 4528–4534.*

[11] Hayat T, Abbas Z, Pop I, Asghar S, Effects of radiation and magnetic field on the mixed convection stagnation-point flow over a vertical stretching sheet in a porous medium, *International Journal of Heat and Mass Transfer 53 (2010) 466–474.*

[12] Kai-Long H, Conjugate heat transfer of magnetic mixed convection with radiative and viscous dissipation effects for second-grade viscoelastic fluid past a stretching sheet, *Applied Thermal Engineering 27 (2007) 1895–1903.*

[13] Khani F, Farmany A, Ahmadzadeh R , Abdul Aziz, F. S, Analytic solution for heat transfer of a third grade viscoelastic fluid in non-Darcy porous media with thermophysical effects, *Commun Nonlinear Sci Numer Simulat 14 (2009) 3867–3878.*

[14] B.I Olajuwon, Convection heat and Mass Transfer in an electrically conducting power law flow over a heated vertical porous plate, *International Journal of Computational Methods in Engineering Science and Mechanics, Vol.* 11: 2, (2010) 100 - 108

[15] Olajuwon B. I**,** Flow and Natural Convection Heat Transfer in a power law fluid past a vertical plate with Heat generation, *International Journal of nonlinear Sciences, Vol.7,No.1* (2009),*pp 50 – 56,*

[16] Olajuwon B. I, Convection Heat and Mass Transfer in a power law fluid with Heat generation and Thermal diffusion past a vertical Plate, *Journal of Energy, Heat and Mass Transfer, Vol. 30 (1) ,* (2008) *pp 1 – 19*

[17] Olajuwon B. I, Convection heat and mass transfer in power law fluid with thermal radiation past a moving porous plate, *Progress in Computational fluid Dynamics: An International Journal Vol. 8, no. 6,* (2008) *pp 372 – 378*

[18] Olajuwon B.I (2007)**,** Thermal Radiation interaction with convection in a power law flow past a vertical plate with variable suction, *International Journal of Heat and Technology,vol.25(2), pp 57 – 65,* (2007)

[19] Sobh A. M., Slip flow in peristaltic transport of a Carreau fluid in an asymmetric channel, *Can.J.Phys.87,1-9,(2009)*

[20] ASobh A. M., Heat transfer in a slip flow of peristaltic transport of a magneto-Newtonian fluid through a porous medium, *Inter.J.Biomath., vol.2, No.3, (2009) 299-309*

[21] Mekheimer Kh.S, Abdelmaboud Y, The influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus: Application of an endoscope, *Phys.Lett.A 372, (2008), 1567-1665*.

[22] Ali N, Hayat T, Peristaltic motion of a Carreau fluid in an asymmetric channel, *Appl. Math. Comput.193, (2007) 535-552.*

[23] Hayat T, Saleem N, Ali N, Effect of induced magnetic field on peristaltic transport of a Carreau fluid, *Commun Nonlinear Sci Numer Simulat 15, (2010) 2407–2423*