Flow of a second grade fluid with convective boundary conditions

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Abstract: The flow and heat transfer in a second grade fluid over a stretching sheet subjected to convective boundary conditions are investegated. Similarity transformations have been used for the reduction of partial differential equation into the ordinary differential. Homotopy analysis method (HAM) has been utilized for the series solutions. Graphical results are displayed and analyzed. Computations for local Nusselt number have been carried out.
Keywords: Heat transfer, second grade fluid, convective boundary conditions, stretching surface.

Introduction

The boundary layer flows and heat transfer over a stretching sheet are quite useful in the engineering applications. Specific examples of such flows occur in the extrusion process, glass fiber and paper production, hot rolling, wire drawing, electronic chips, crystal growing, plastic manufactures and aerodynamic extrusion of plastic sheets. An extensive literature is available for boundary layer flows induced by a stretching sheet [1−10].

A variety of constitutive equations have been suggested to predict the behavior of non-Newtonian fluids in industry and engineering. Amongst these non-Newtonian fluids, there is one simplest model of differential type fluids which is known as second grade fluid [11−18]. This model can describe the normal stress effects even in steady flows. Convection flow has further practical engineering applications such as cooling of polymer films and metallic plates on conveyers. Recently Yao et al. [19] discussed the flow and heat transfer in a viscous fluid flow over a stretching/shrinking sheet with convective boundary conditions. The purpose of this work is to extend the analysis of reference [19] in two directions. Firstly to develop problem formulation for a second grade fluid and secondly to find the series solutions. This paper is arranged as follows. In the next section we present the problem formulation. Section three includes the solutions for the velocity and temperature fields. Homotopy analysis method (HAM) has been used for the derivation of solutions. This method is very powerful and several interesting problems have been solved by this method [20 – 35]. In section four, we discuss the convergence of the obtained solutions. The graphical results for the pertinent parameters are shown and analyzed in section five. In section six we present the concluding remarks.

Problem formulation

Consider the two-dimensional and steady flow of an incompressible second grade fluid bounded by a stretching sheet with heat transfer when the fluid remains stationary. The sheet is stretched with a velocity $u_w(x) = bx,$ where b is a real number. The constant mass transfer velocity is denoted by v_w with $v_w > 0$ for injection and $v_w < 0$ for suction, respectively. We choose $x-$ axis along the stretching surface and the $y-$ axis perpendicular to $x-$ axis. The present flow consideration is governed by the following expressions

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right],$$

where u and v denote the velocity components in the $x-$ and $y-$ directions, α_1 the second grade parameter, T the fluid temperature, c_p is the specific heat, $\nu = (\mu / \rho)$ the kinematic viscosity, ρ the density of the fluid.

The appropriate boundary conditions are considered in the following forms

$$u = u_w(x) = bx,\ v = v_w,\ \ -k\frac{\partial T}{\partial y} = h(T - T_f)\ \text{ at } y = 0,$$

$$u = 0,\ \ T = T_\infty\ \ \text{as } y \to \infty.$$

Here k is the thermal conductivity of fluid, h is the convective heat transfer coefficient, v_w is the wall heat transfer velocity and T_f is the

convective fluid temperature below the moving sheet.
Writing

$$u = axf'(\eta),\ v = -\sqrt{a\nu}\, f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty},\ \eta = y\sqrt{\frac{a}{\nu}},$$

Continuity equation is automatically satisfied and Eqs. $(2)-(5)$ reduce to

$$f''' + ff'' - f'^2 + K(2f'f''' - f''^2 - ff'''') = 0,$$

$$\theta'' + \Pr f\theta' + \Pr Ecf''^2 - \Pr EcK\left(f'f''^2 - ff''f'''\right) = 0,$$

$$f = S,\ \ f' = b/a = \alpha,\ \ \theta' = -\gamma(1-\theta(0)) \ \text{ at } \eta = 0,$$

$$f' = 0,\ \ \theta = 0 \ \text{ at } \eta \to \infty$$

where a is a constant, prime represents the differentiation with respect to η , $S > 0$ for suction and $S < 0$ for injection, $K = \frac{\alpha_1 a}{\mu}$, $\alpha = \frac{b}{a}$, $\Pr = \frac{\mu c_p}{k}$ the Prandtl number, $Ec = \frac{u_w}{c_p(T_f - T_\infty)}$ is Eckert number and $\gamma = \frac{h}{k}\sqrt{\frac{\nu}{a}}$ the Biot number.
The local Nusselt number Nu_x is

$$Nu_x = \frac{xq_w}{k(T_f - T_\infty)},$$

where heat transfer q_w is

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

Dimensionless expression of local Nusselt number is

$$Nu_x / \mathrm{Re}_x^{1/2} = -\theta'(0).$$

Homotopy solutions

We write the initial guesses and linear operators as

$$f_0(\eta) = S + \alpha\left(1 - e^{-\eta}\right),\ \ \theta_0(\eta) = \frac{\gamma\exp(-\eta)}{1+\gamma},$$

$$?_f = f''' - f',\ \ ?_\theta = \theta'' - \theta,$$

with

$$?_f(C_1 + C_2 e^{\eta} + C_3 e^{-\eta}) = 0,\ \ ?_\varphi(C_4 e^{\eta} + C_5 e^{-\eta}) = 0,$$

where C_i $(i = 1-5)$ denote the arbitrary constants.

Zeroth order deformation problems

The problems at this can be written as

$$(1-p)?_f\left[\hat{f}(\eta;p) - f_0(\eta)\right] = ph_f \mathbf{N}_f\left[\hat{f}(\eta;p), \hat{\theta}(\eta,p)\right],$$

$$(1-p)?_{\theta}\left[\hat{\theta}(\eta;p)-\theta_0(\eta)\right]=ph_{\theta}\mathbf{N}_{\theta}\left[\hat{f}(\eta;p),\hat{\theta}(\eta,p)\right],$$

$$\hat{f}(0;p)=S,\ \hat{f}'(0;p)=b/a=\alpha,\ \hat{f}'(\infty;p)=0,\ \hat{\theta}'(0,p)=-\gamma[1-\theta(0,p)],\ \hat{\theta}(\infty,p)=0,$$

$$\mathbf{N}_f[f(\eta,p),\theta(\eta,p)]=\frac{\partial^3 f(\eta,p)}{\partial\eta^3}-f(\eta,p)\frac{\partial^2 f(\eta,p)}{\partial\eta^2}-\left(\frac{\partial f(\eta,p)}{\partial\eta}\right)^2 +K\begin{bmatrix} 2\frac{\partial f(\eta,p)}{\partial\eta}\frac{\partial^3 f(\eta,p)}{\partial\eta 3}-\left(\frac{\partial f^2(\eta,p)}{\partial\eta^2}\right)^2 \\ -f(\eta,p)\frac{\partial^4 f(\eta,p)}{\partial\eta^4}\end{bmatrix},$$

$$\mathbf{N}_{\theta}[\theta(\eta,p),f(\eta,p)]=\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}+\Pr f(\eta,p)\frac{\partial\theta(\eta,p)}{\partial\eta}+\Pr Ec\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}- \Pr EcK\left[\frac{\partial f(\eta,p)}{\partial\eta}\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}-f(\eta,p)\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}\frac{\partial^3\theta(\eta,p)}{\partial\eta^3}\right],$$

where p is an embedding parameter, h_f and h_{ϕ} are the non-zero auxiliary parameters and N_f and N_{θ} are the nonlinear operators. For $p=0$ and $p=1$ we have

$$f(\eta;0)=f_0(\eta),\ \theta(\eta,0)=\theta_0(\eta)\text{ and } f(\eta;1)=f(\eta),\ \theta(\eta,1)=\theta(\eta)$$

and $f(\eta,p)$ and $\theta(\eta,p)$ vary from $f_0(\eta)$, $\theta_0(\eta)$ to $f(\eta)$ and $\theta(\eta)$ when p varies from 0 to 1.
By Taylor series expansion one has

$$f(\eta,p)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)p^m,$$

$$\theta(\eta,p)=\theta_0(\eta)\sum_{m=1}^{\infty}\theta_m(\eta)p^m,$$

$$f_m(\eta)=\frac{1}{m!}\frac{\partial^m f(\eta;p)}{\partial\eta^m}\bigg|_{p=0},\ \theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\theta(\eta;p)}{\partial\eta^m}\bigg|_{p=0}.$$

The series in Eqs. (23) and (24) is strongly dependent upon h_f and h_{θ}. The values of h_f and h_{θ} are selected by a processes that the series converge at $p=1$. Hence

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta),$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta).$$

mth-order deformation problems

At this stage problems are given by

$$?_f\,[\,f_m(\eta)-\chi_m f_{m-1}(\eta)] = h_f \mathbf{R}_f^m(\eta),$$

$$?_\theta[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)] = h_\theta \mathbf{R}_\theta^m(\eta),$$

$$f_m(0) = f_m'(0) = f_m'(\infty) = 0, \quad \theta_m'(0)-\gamma\theta_m(0) = \theta_m(\infty) = 0,$$

$$\mathbf{R}_f^m(\eta) = f_{m-1}'''(\eta) + \sum_{k=0}^{m-1}\left[f_{m-1-k} f_k'' - f_{m-1-k}' f_k'''\right]$$
$$+K\sum_{k=0}^{m-1}[2 f_{m-1-k}' f_k''' - f_{m-1-k}'' f_k'' - f_{m-1-k} f_k''''],$$

$$\mathbf{R}_\theta^m(\eta) = \theta_{m-1}'' + \Pr\sum_{k=0}^{m-1}\theta_{m-1-k}' f_k\,,$$

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & \end{cases}$$

The resulting series solutions are

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{\eta} + C_3 e^{-\eta}\,,$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^{\eta} + C_5 e^{-\eta}\,,$$

in which f_m^* and θ_m^* indicate the special solutions.

Convergence of homotopy solutions

Obviously the series solutions (26) and (27) contain the nonzero auxiliary parameters $\hbar_f$ and $\hbar_\theta$. Such parameters adjust and control the convergence of the series solutions. For range of admissible values of $\hbar_f$ and $\hbar_\theta$, the $\hbar$ curves have been displayed for $20th$ -order of approximations. Fig. 1 indicates that the range for admissible values of $\hbar_f$ and $\hbar_\theta$ are $-1.6 \le ?_f \le -0.1$ and $-1.8 \le ?_\theta \le -0.2.$ Further, the series also converge in the whole region of η when $?_f = ?_\theta = -1.0.$

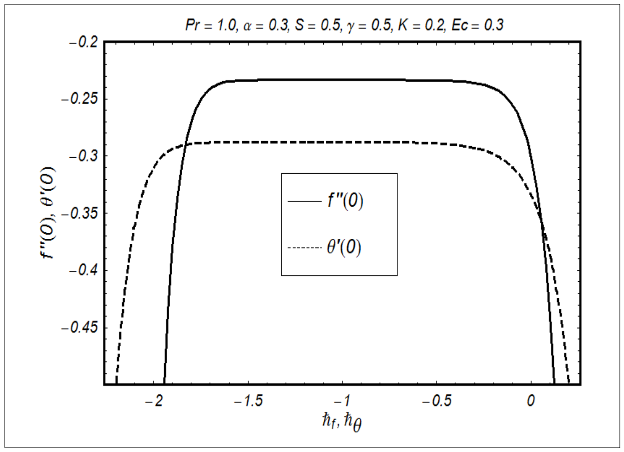

Fig. 1(a): ⊙✍curve for f and ♠.

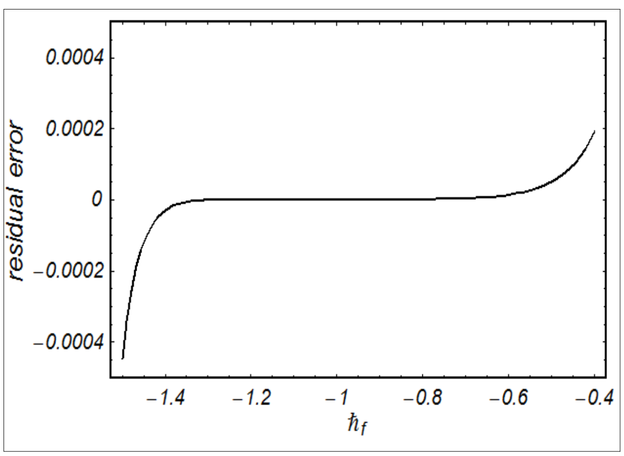

Fig. 1(b): Residual error for the function f.

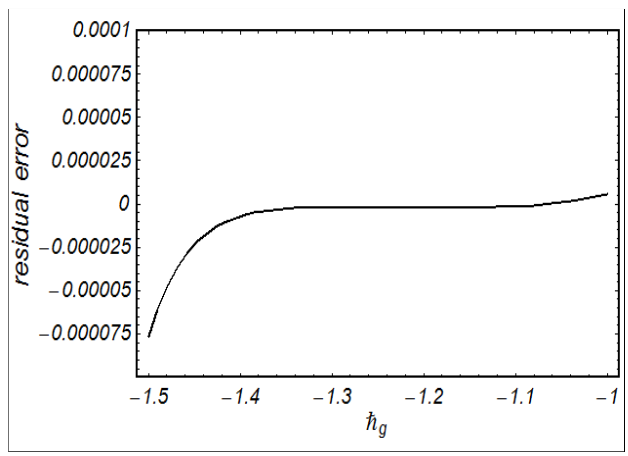

Fig. 1(c): Residual error for the function ♠.

Table: 1. Convergence of homotopy solution for different order of approximations when $K = 0.2,\quad \alpha = 0.3,\quad \Pr = 1.0\ ,\quad S = 0.5,\quad \gamma = 0.5$ and $?_f = ?_\theta = -1.0.$

Order of approximation	$-f''(0)$	$-\theta'(0)$
1	0.246001	0.308586
10	0.233492	0.287484
15	0.233472	0.287393
25	0.233472	0.287393
30	0.233472	0.287393
35	0.233472	0.287393

Graphical results and discussion

This section highlights the influence of different parameters on velocity field f'

and temperature profile θ. Figs. $2-8$ show the influence of different parameters α, K, S, $\Pr$ and γ on f' and θ. The effects of α on velocity profile f' are shown in Fig. 2. This Fig. indicates that there is no flow and the fluid velocity is zero when $\alpha = 0$. The fluid velocity f' increases when there is an increase in α . Fig. 3 is plotted for the variations of second grade parameter K on f' . The velocity profile f' increases and the boundary layer thickness decreases when K is increased. Fig. 4 shows the variation of suction parameter S on f'. As expected f' decreases by increasing S . The variations of S on f' are quite opposite when compared with K. Figs. $5-8$ have been displayed for the effects of $\Pr$, α, S and γ on the temperature profile θ. Fig. 5 depicts the variations of $\Pr$ on θ. The temperature profile θ decreases when $\Pr$ increases. Fig. 6 represents the effects of stretching parameter α on θ. The temperature profile θ is a decreasing function of α . The variations of S on θ is presented in Fig. 7. From Fig. 7 we see that S has similar effects on temperature profile θ when compared with velocity profile f'. Fig. 8 has been prepared for effects of γ on θ. We found that when $\gamma = 0$ then there is no heat transfer and the temperature is zero. The temperature profile θ clearly increases when γ increases. The influence of the Eckert number Ec is shown in Fig. 9. It is observed that θ is an increasing function of Ec. This is because heat energy is stored in fluid due to frictional heating. Thus the effect of increasing Ec, is to enhance the temperature at any point. The boundary layer thickness also increases when Ec increases. Table 2 depicts the variation of heat transfer at the wall $-\theta'(0)$ for some values of $\Pr$ Ec, γ and α when $K = 0.2$ and $S = 0$.

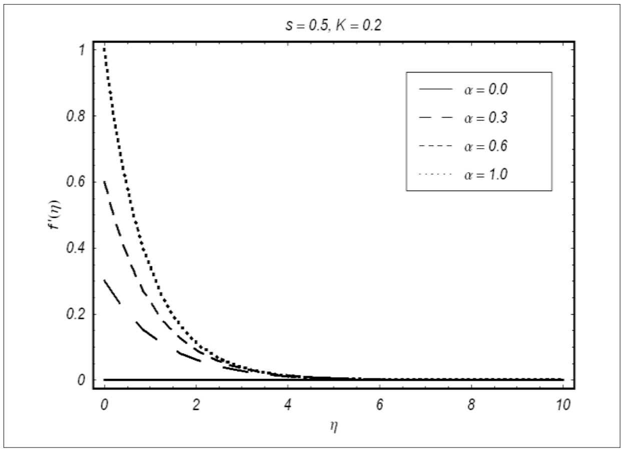

Fig. 2. Influence of ☺on *f*❖.

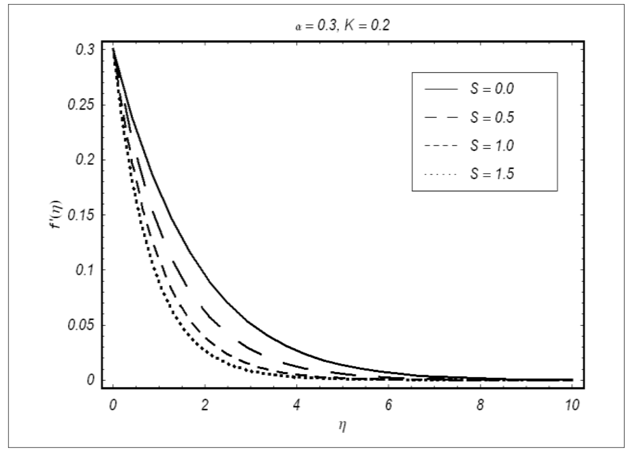

Fig. 3. Influence of *S* on *f*❖.

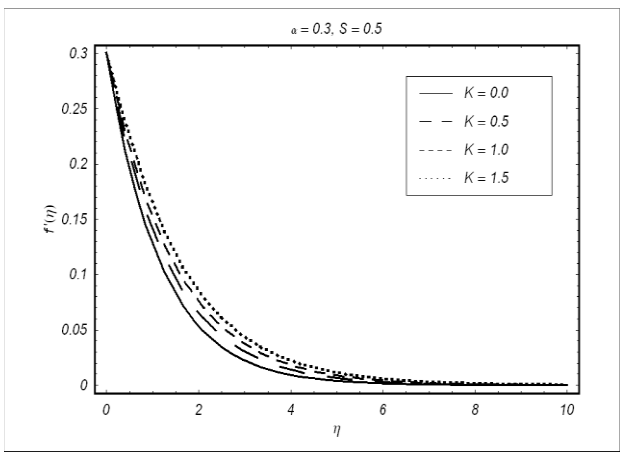

Fig. 4. Influence of *K* on *f*❖.

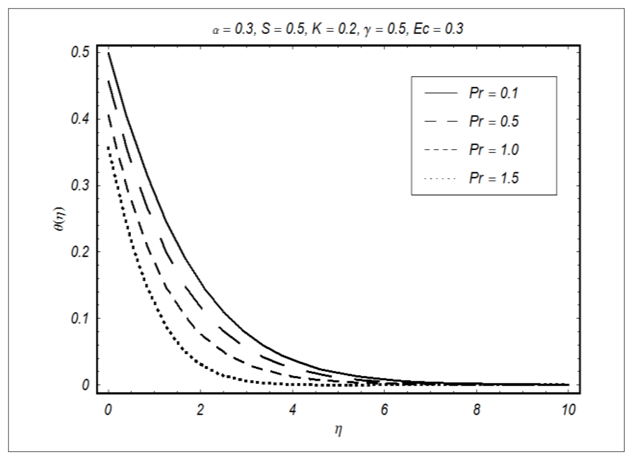

Fig. 5. Influence of Pr on ♠.

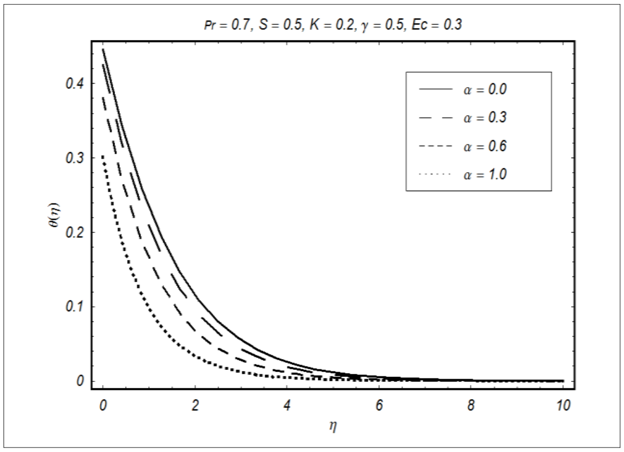

Fig. 6. Influence of α on θ.

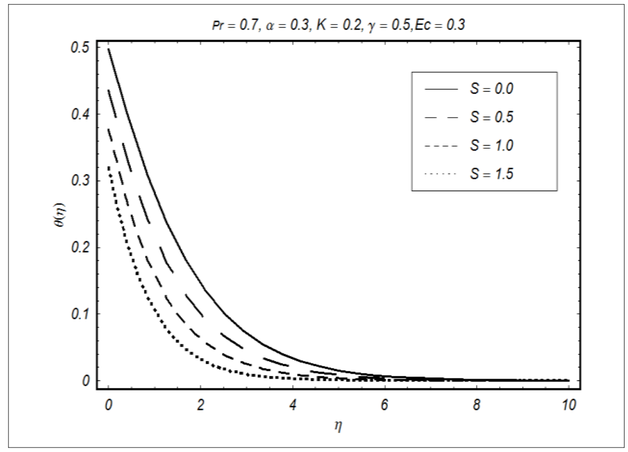

Fig. 7. Influence of S on θ.

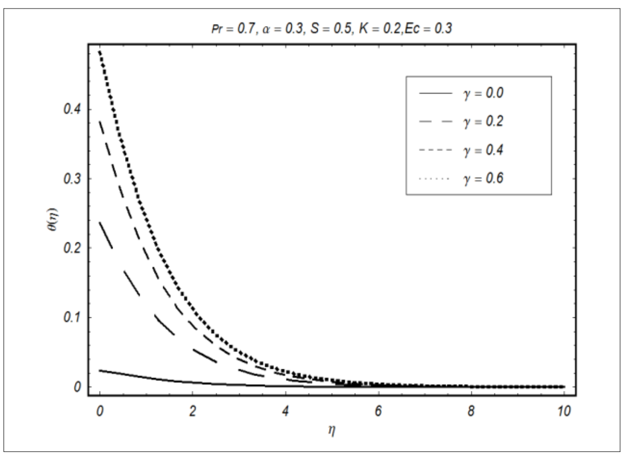

Fig. 8. Influence of γ on θ.

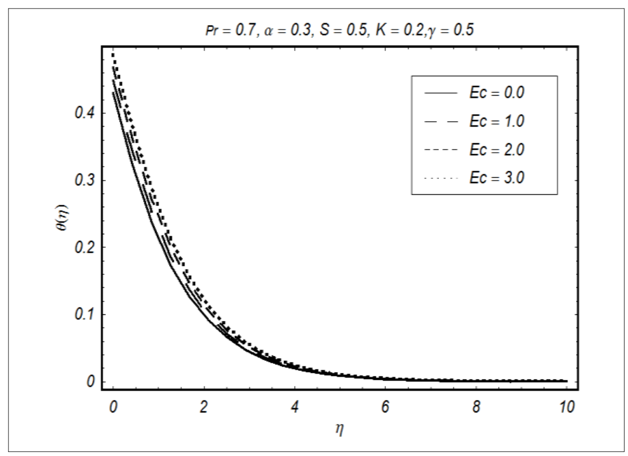

Fig. 9. Influence of Ec on θ.

Table 2: Values of local Nusselt number $Nu/\mathrm{Re}_x^{1/2}$ for parameters γ, $\Pr$, Ec and α when $S = 0.5$ and $K = 0.2$.

γ	Pr	α	Ec	$Nu/\mathrm{Re}_x^{1/2}$
0.1	0.7	0.3	0.2	0.082916
0.5				0.250173
1.0				0.334452
2.0				0.402237
0.5	0.5	0.3	0.2	0.214365
	0.7			0.250132
	1.0			0.288561
	2.0			0.356172
0.5	1.0	0.1	0.2	0.268673
		0.5		0.356192
		1.0		0.305945
		2.0		0.301107
0.5	0.7	0.2	0.0	0.242896
			0.2	0.242184
			0.5	0.241265

Concluding remarks

We studied the heat transfer analysis on the flow of a second grade fluid over a stretching wall with convective boundary conditions. The homotopy analysis method has been applied for the series solutions. The graphical results for emerging parameters are discussed. Numerical values of local Nusselt number are computed. The main results have been summarized as follows.

- The velocity field f' increases by increasing α and K.
- The effects of K and S on f' are quite opposite.
- The variations of Pr and α on θ are qualitatively similar.
- Behavior of Ec and Pr on the temperature θ are opposite.
- The heat transfer effects are absent when $\gamma = 0$.
- The local Nusselt number increases as Pr increases and decreases when Ec increases.

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Nomenclature	
a	constant parameter, $[s^{-1}]$
b	stretching sheet parameter, $[s^{-1}]$
K	second grade fluid parameter $= \frac{\alpha_1 a}{\mu}[-]$
Nu_x	Nusselt number, $[-]$
Pr	Prandtl number $(= \nu/\sigma)$,$[-]$
q_w	rate of heat transfer, $[Wm^{-2}]$
u, v	velocity components along $-x$ and $-y$ axes respectively,$[ms^{-1}]$
x, y	Cartesian coordianates along x- and y-axes
Greek Symbols	
θ	dimensionless temperature
κ	thermal conductivity, $[\mathrm{Wm}^{-1}\mathrm{K}^{-1}]$
μ	coefficient of viscosity $[kgms^{-1}]$
ν	kinematic viscosity $[ms^{-2}]$
γ	Biot number $= \frac{h}{k}\sqrt{\frac{\nu}{a}}$, $[-]$
ρ	density of fluid, $\left[kgm^{-3}\right]$
Superscripit	
$'$	differentiation with respect to η

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