

## CONVECTIVE HEAT AND MASS TRANSFER IN A NON-NEWTONIAN FLOW FORMATION IN COUETTE MOTION IN MAGNETOHYDRODYNAMICS WITH TIME-VARING SUCTION

by

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*An analysis is carried out to study the effect of heat and mass transfer on a non-Newtonian fluid between two infinite parallel walls, one of them moving with a uniform velocity under the action of a transverse magnetic field. The moving wall moves with constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. Time-dependent wall suction is assumed to occur at permeable surface. The governing equations for the flow are transformed in a system of non-linear ordinary differential equations by perturbation technique and are solved numerically by using the shooting technique with fourth order Runge-Kutta integration scheme. The effect of non-Newtonian parameter, magnetic pressure parameter, Schmidt number, Grashof number, and modified Grashof number on velocity, temperature, concentration, and the induced magnetic field are discussed. Numerical results are given and illustrated graphically for the considered problem.*

Key words: *heat and mass transfer, non-Newtonian flow formation, Couette motion magnetohydrodynamics*

### Introduction

The Couette-type flow with heat and mass transfer problems have many practical applications in technological industrial manufacturing process, the steady and unsteady Couette flow in hydrodynamics and hydromagnetics that are subject to wall suction have been discussed by several authors [1-5]. Katagiri [6] studied the Couette-flow formation in magnetohydrodynamics (MHD). Neglecting the induced field, he solved the momentum equation by the method of Laplace transformation. Eldabe *et. al.* [7] examined the non-Newtonian flow formation in Couette motion in MHD with time-varying suction. Muhuri [8] considered the more general case wherein the velocity of the moving wall varies as  $(\text{time})^n$  and where the walls are subjected to uniform suction or injection.

Flow with heat and mass transfer problems over a continuous moving flat surface have many practical applications in technological and industrial manufacturing processes.

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Eldabe [9] studied the unsteady free convection flow of an incompressible, electrically conducting, viscous liquid through a porous medium past a hot, vertical porous plate in the presence of transverse magnetic field. Char [10] investigates the more complicated problem which involves both heat and mass transfer in the hydromagnetic flow of a viscoelastic fluid over a stretching sheet. Also, Elbashbeshy [11] studied heat and mass transfer along a vertical plate in presence of a magnetic field. Some researches have been carried out to include various physical aspects of the problem of combined heat and mass transfer. Chamkha *et al.* [12] investigated the problem of coupled heat and mass transfer by MHD free convection from an inclined plate in the presence of internal heat generation or absorption. Eldabe *et al.* [13] studied the unsteady flow of an electrically conducting fluid with MHD convection heat and mass transfer over an infinite solid surface. Eldabe *et al.* [14] also studied the mixed convective heat and mass transfer in non-Newtonian fluid at a peristaltic surface with temperature-dependent viscosity. The unsteady MHD convection heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption was studied in [15]. Seddeek *et al.* [16] studied the effects of variable viscosity and thermal conductivity on a 2-D laminar flow of viscous incompressible conducting fluid past a semi-infinite vertical porous moving plate, taking into account, the effect of magnetic field. Salem [17] investigated the simultaneous effects of coupled heat and mass transfer in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid-saturated porous medium under the effects of radiation and viscous dissipation. Recently, Anwer *et al.* [18] considered unsteady MHD Hartmann-Couette flow and heat transfer in a Darcian channel with hall current, ion slip, viscous and Joule heating effects.

This paper treats numerically the flow of an elastic-viscous, incompressible, and electrically conducting fluid between two infinite parallel walls one of them moving with a uniform velocity, taking into account the effects of heat and mass transfer. The system is stressed by a magnetic field of constant strength acting perpendicular to the walls.

### Mathematical formulation

Let us consider an unsteady, incompressible, viscoelastic fluid flowing between two infinite parallel walls at distance  $d$  apart and subject to a uniform magnetic field  $H_0$  normal to the walls in the presence of thermal and concentration buoyancy effect. A constant magnetic field produces an induced magnetic field  $\vec{h}$  and induced electric field  $\vec{E}$ . The lower wall moves with uniform velocity  $U$  while the other is at rest. The fluid is assumed to be elasto-viscous formulation by Walters [19] and electrically conducting whereas the walls are taken to be non-conducting. The temperature and the species concentration at the lower wall are  $T_0$  and  $C_0$  and at the upper wall  $T_1$  and  $C_1$ , respectively. The x-axis is taken along the lower wall and the y-axis is taken to be normal to the wall.

The electro-magnetic quantities satisfy Maxwell's equations when the displacement currents and free charges are neglected [20]:

$$\nabla \vec{H} = 0 \quad (1)$$

$$\nabla \times \vec{H} = \vec{J} \quad (2)$$

$$\nabla \times \vec{E} = -\mu_e \frac{\partial \vec{H}}{\partial t} \quad (3)$$

$$\nabla \vec{E} = 0 \quad (4)$$

where  $\vec{J}$  is the electric current density, and  $\mu_e$  – the magnetic permeability.

These equations are supplemented by Ohm's law:

$$\vec{J} = \sigma(\vec{E} + \mu_e \vec{V} \times \vec{H})$$

where  $\sigma$  is the electric conductivity.

As mentioned above the applied field  $\vec{H}$  has component  $(0, H_0, 0)$ . It can easily be seen from the above equations that the induced magnetic field  $\vec{h}$  has component  $(h, 0, 0)$ .

The vectors  $\vec{E}$  and  $\vec{J}$  will have non-vanishing components only in the  $z$ -direction, *i. e.*:

$$\vec{E}(0,0,E) \text{ and } \vec{J}(0,0,J), \text{ where } J = \sigma[E + \mu_e(H_0u - hv)] \quad (5)$$

The vector eqs. (2) and (3) reduced to the following scalar equations:

$$\frac{\partial H}{\partial y} = -J \quad (6)$$

$$\frac{\partial E}{\partial y} = -\mu_e \frac{\partial H}{\partial t} \quad (7)$$

Eliminating  $J$  between (5) and (6), we obtain:

$$\frac{\partial H}{\partial y} = -\sigma[E + \mu_e(Hu - hv)] \quad (8)$$

Eliminating  $E$  between eqs. (7) and (8), we obtain:

$$\frac{\partial H}{\partial t} + v \frac{\partial H}{\partial y} = \eta \frac{\partial^2 H}{\partial y^2} + H_0 \frac{\partial u}{\partial y} \quad (9)$$

where  $\eta = (\mu_e \sigma)^{-1}$  is the magnetic diffusivity.

The continuity and momentum equations governing the flow of MHD incompressible non-Newtonian fluid are:

$$\nabla \vec{V} = 0 \quad (10)$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \nabla) \vec{V} = \frac{1}{\rho} \nabla \bar{\tau} + \frac{\mu_e}{\rho} \vec{J} \times \vec{H} + \beta_T \bar{g}(T - T_o) + \beta_C \bar{g}(C - C_o) \quad (11)$$

where  $\bar{\tau}$  is defined by [19],  $\rho$ ,  $\bar{g}$ ,  $\beta_T$ , and  $\beta_C$  denote, respectively, the density, gravitational acceleration, the thermal and expansion coefficient and the concentration expansion coefficient.

The governing equations for energy and mass concentration are:

$$\frac{\partial T}{\partial t} + (\vec{V} \nabla) T = \alpha \nabla^2 T \quad (12)$$

$$\frac{\partial C}{\partial t} + (\vec{V} \nabla) C = D \nabla^2 C \quad (13)$$

where  $T$  is the temperature of the fluid,  $\alpha$  – the thermal diffusivity of the fluid medium,  $C$  – the mass concentration, and  $D$  – the diffusion coefficient.

Under the usual boundary layer approximation, the governing equations for this problem can be written as:

$$\frac{\partial v}{\partial y} = 0 \quad (14)$$

$$\rho \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + \mu_e H_0 \frac{\partial h}{\partial y} - k_0 \left( \frac{\partial^3 u}{\partial y^2 \partial t} + v \frac{\partial^3 u}{\partial y^3} \right) + \rho g \beta_T (T - T_0) + \rho g \beta_c (C - C_0) \quad (15)$$

$$\frac{\partial h}{\partial t} + v \frac{\partial h}{\partial y} = \eta \frac{\partial^2 h}{\partial y^2} + H_0 \frac{\partial u}{\partial y} \quad (16)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (17)$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (18)$$

Subject to the boundary conditions

$$Y = 0 \quad u = U, \quad h = 0, \quad T = T_0, \quad C = C_0 \quad (19)$$

$$Y = d \quad u = 0, \quad h = 0, \quad T = T_1, \quad C = C_1 \quad (20)$$

where  $u$ ,  $v$ , and  $t$  are the components of dimensional velocities along x and y-directions and dimensional time, respectively.  $\mu$  is the viscosity of the fluid and  $k_0$  – the elastic constant.

The continuity equation gives:

$$v(t) = -U(1 + \varepsilon e^{i\omega t}) \quad \varepsilon \ll 1 \quad (21)$$

which is the velocity of suction that consists of a basic steady value  $U$  with a weak time-varying component. Let us introduce the non-dimensional quantities as:

$$y = \frac{v}{U} y_1, \quad t = \frac{4v}{U^2} t_1, \quad w = \frac{U^2}{4v} w_1, \quad u = Uu_1, \quad h = H_0 h_1$$

$$\theta = \frac{T - T_0}{T_1 - T_0}, \quad \Gamma = \frac{C - C_0}{C_1 - C_0} \quad (22)$$

In view of eqs. (21) and (22), the governing eqs. (15)-(18) reduce to the following non-dimensional form after dropping the suffix 1:

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - k \left[ \frac{1}{4} \frac{\partial^3 u}{\partial y^2 \partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial^3 u}{\partial y^3} \right] + M^2 \frac{\partial h}{\partial y} + Gr\theta + Gc\Gamma \quad (23)$$

$$\frac{1}{4} \frac{\partial h}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial h}{\partial y} - \frac{\partial u}{\partial y} = \frac{1}{R_m} \frac{\partial^2 h}{\partial y^2} \quad (24)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (25)$$

$$\frac{1}{4} \frac{\partial \Gamma}{\partial t} - (1 + \varepsilon e^{i\omega t}) \frac{\partial \Gamma}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \Gamma}{\partial y^2} \quad (26)$$

where  $k = k_0 U^2 / \rho \nu^2$  is the visco-elastic parameter,  $Gr = [g\beta T(T_1 - T_0)/U^3] \nu$  – the Grashof number,  $Gc = [g\beta_c(C_1 - C_0)/U^3] \nu$  – the modified Grashof number,  $M = (\mu_e H_0^2 / \rho U^2)^{1/2}$  – the magnetic-pressure number,  $Pr = \nu/\alpha$  – the Prandtl number,  $Sc = \nu/D$  – the Schmidt number and,  $R_m = \nu/\eta$  – the magnetic Reynolds number.

The boundary conditions (19) and (20) are given in the following dimensionless form:

$$y=0, \quad u=1, \quad h=0, \quad \Gamma=0, \quad \theta=0 \quad (27)$$

$$y=1, \quad u=0, \quad h=0, \quad \Gamma=1, \quad \theta=1 \quad (28)$$

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we may represent the velocity, temperature, concentration and induced magnetic field as

$$\begin{aligned} u(y,t) &= u_1(y) + \varepsilon u_2(y) e^{i\omega t} + O(\varepsilon^2) \\ \theta(y,t) &= \theta_1(y) + \varepsilon \theta_2(y) e^{i\omega t} + O(\varepsilon^2) \\ \Gamma(y,t) &= \Gamma_1(y) + \varepsilon \Gamma_2(y) e^{i\omega t} + O(\varepsilon^2) \\ h(y,t) &= h_1(y) + \varepsilon h_2(y) e^{i\omega t} + O(\varepsilon^2) \end{aligned} \quad (29)$$

Substituting in eqs. (23)-(26) and comparing the harmonic and non-harmonic terms, neglecting terms of  $O(\varepsilon)^2$ , we get the following equations:

$$ku_1''' + u_1'' + u_1' + s^2 h_1' + Gr\theta_1 + Gc\Gamma_1 = 0 \quad (30)$$

$$ku_2''' - \frac{1}{4} i\omega ku_2'' + u_2' - \frac{1}{4} i\omega u_2 + s^2 h_2' + Gr\theta_2 + Gc\Gamma_2 = -ku_1''' - u_1' \quad (31)$$

$$h_1'' + R_m u_1' + R_m h_1' = 0 \quad (32)$$

$$h_2'' + R_m h_2' - \frac{1}{4} i\omega R_m h_2 + R_m u_2' = -R_m h_1' \quad (33)$$

$$\theta_1'' = Pr \theta_1' \quad (34)$$

$$\theta_2'' = \text{Pr} \left( \frac{1}{4} i w \theta_2 - \theta_2' - \theta_1' \right) \quad (35)$$

$$\Gamma_1'' = \text{Sc} \Gamma_1' \quad (36)$$

$$\Gamma_2'' = \text{Sc} \left( \frac{1}{4} i w \Gamma_2 - \Gamma_2' - \Gamma_1' \right) \quad (37)$$

Here primes denote differentiation with respect to  $y$ . The corresponding boundary conditions can be written as:

$$\begin{aligned} y=0: \quad u_1=1, \quad u_2=0, \quad \theta_1=\theta_2=0, \quad \Gamma_1=\Gamma_2=0, \quad h_1=h_2=0 \\ y=1: \quad u_1=u_2=0, \quad \theta_1=1, \quad \theta_2=0, \quad \Gamma_1=1, \quad \Gamma_2=0, \quad h_1=h_2=0 \end{aligned} \quad (38)$$

## Results and discussion

In order to get an insight in the physical situation of the problem, the system of ordinary differential equations (30)-(37) along with the boundary conditions (38), are solved numerically by using the modified fourth-order Runge-Kutta method with shooting technique. The numerical computation have been carried out for various values of non-Newtonian parameter ( $k$ ), magnetic pressure number ( $s$ ), Grashof number ( $Gr$ ), modified Grashof number ( $Gc$ ), Prandtl number ( $Pr$ ), and Schmidt number ( $Sc$ ). Such a possible combination of values for the parameters  $k$ ,  $s$ ,  $Gr$ ,  $Gc$ ,  $Pr$ , and  $Sc$  are shown in figs. 1-10. The numerical calculations were performed by taking values of some constant parameters as  $R_m = 1$ ,  $w = 1$ , and  $t = \pi$ .

Figures 1 and 2 illustrate the influence of non-Newtonian parameter  $k$  on the velocity and the induced magnetic field distribution, respectively. As shown, the velocity and

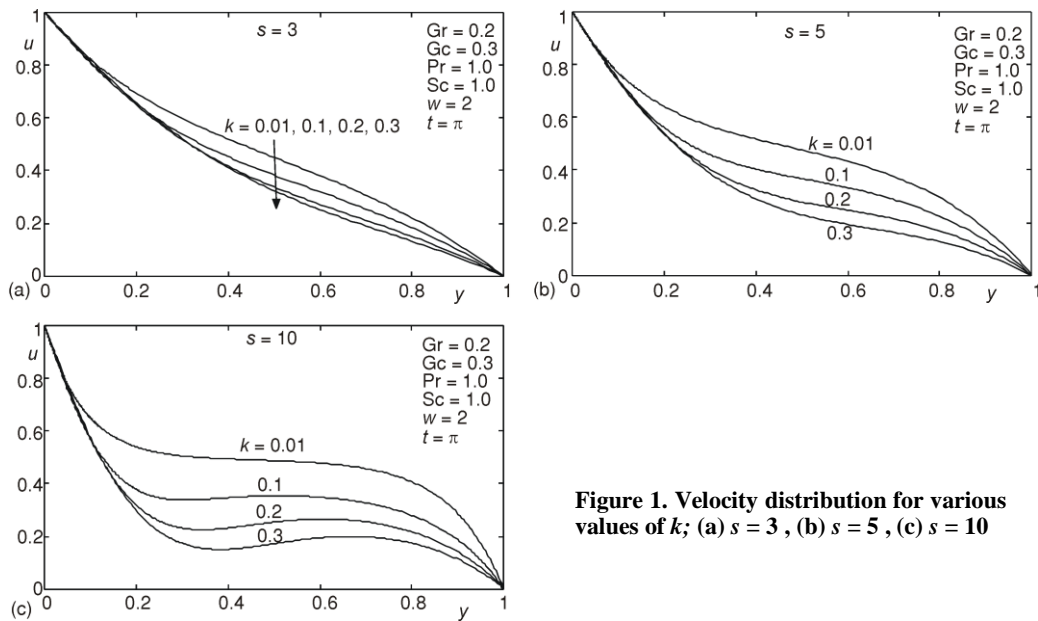


Figure 1. Velocity distribution for various values of  $k$ ; (a)  $s = 3$ , (b)  $s = 5$ , (c)  $s = 10$

the induced magnetic field are decreasing with increasing the non-Newtonian parameter  $k$ . This is because of the fact that the introduction of tensile stress due to viscoelasticity causes transverse contraction of the boundary layer. With fig. 1 it is interesting to note that the effect of  $k$  on the velocity is more pronounced at higher values of the magnetic pressure number  $s$ . Also, fig. 2 demonstrates that the peak of the profiles decreases near the stationary plate with increasing  $k$ .

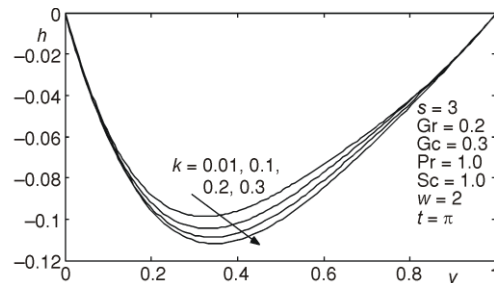


Figure 2. Induced magnetic field distribution for various values of  $k$

Figures 3 and 4 represent the effect of the magnetic-pressure number  $s$  on the velocity and the induced magnetic field profiles, respectively. From fig. 3, it is shown that the velocity distribution decreases near the stationary plate with increase in  $s$ , whereas it increases near the moving plate with the increase in  $s$ . This effect is more pronounced near the stationary plate. From fig. 4 we notice that the induced magnetic field has parabolic profiles and the peak of the profiles indicates the minimum at the central region. We also note that the effect of magnetic-pressure is to increase the induced magnetic field profiles.

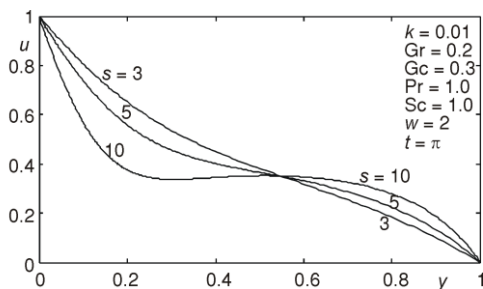


Figure 3. Velocity distribution for various values of  $s$

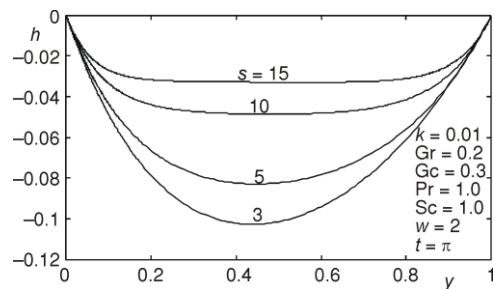


Figure 4. Induced magnetic field distribution for various values of  $s$

Figures 5 and 7 show the effect of Grashof number  $Gr$  and modified Grashof number  $Gc$  on the velocity distribution. Physically  $Gr > 0$  means heating of the fluid or cooling of the boundary surface,  $Gr < 0$  means cooling of the fluid or heating of the boundary surface and  $Gr = 0$  corresponding to the absence of free convection current. As shown, the velocity distribution increase with increase of  $Gr$  or  $Gc$ . Increase of  $Gr$  means increase of temperature gradients  $T_1 - T_0$ , which leads to the increase of velocity distribution but increase of  $Gc$  means the drag forces created by the solid matrix increases and as a result the velocity increases. From figs. 6 and 8 we observe that the induced magnetic field profile increases near

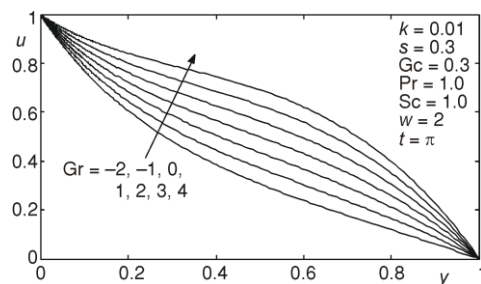


Figure 5. Velocity distribution for various values of  $Gr$

the stationary plate with increase in Gr or Gc, whereas it decreases near the moving plate with the increase in Gr or Gc.

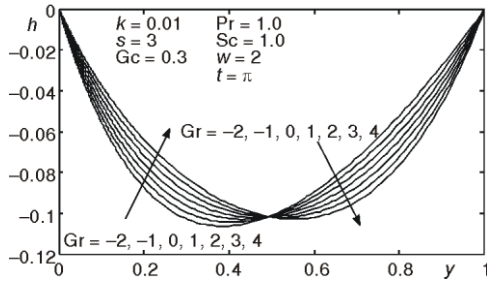


Figure 6. Induced magnetic field distribution for various values of Gr

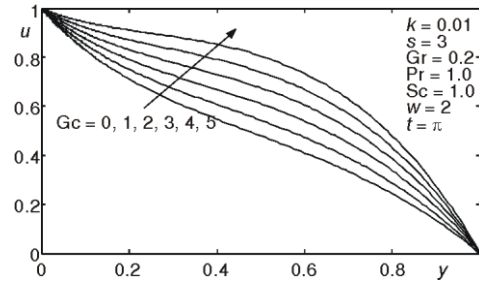


Figure 7. Velocity distribution for various values of Gc

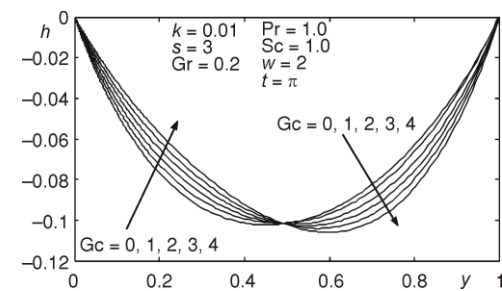


Figure 8. Induced magnetic field distribution for various values of Pr

Figures 9 and 10 give the effect of Prandtl number Pr on the velocity and temperature distributions, respectively. It is seen, as expected that the velocity and temperature distributions decrease with increasing the parameter Pr. This is due to the fact that there would be decrease of thermal boundary layer thickness with the increasing values of Pr.

Figures 11-12 display the effects of the Schmidt number Sc on the velocity and concentration profiles, respectively. We notice that the effect of increasing values of Sc is to decrease the concentration profile in the flow field. Physically, the increase of Sc means decrease of molecular diffusivity  $D$ . That results in decrease of concentration boundary layer. Hence, the concentration of species is higher for small values of Sc and lower for larger values of Sc. Also, increases in Sc cause reduction in the fluid velocity. These behaviors are clearly shown in figs. 11 and 12.

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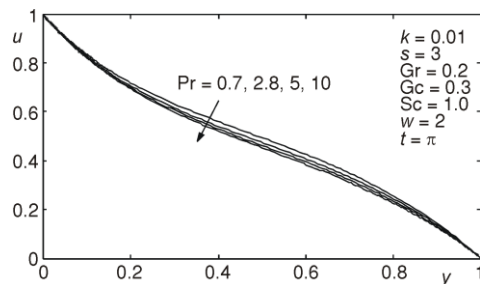


Figure 9. Velocity distribution for various values of Pr

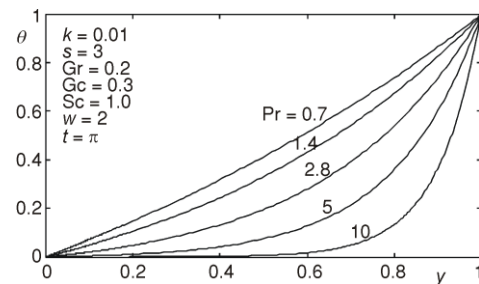


Figure 10. Temperature distribution for various values of Pr



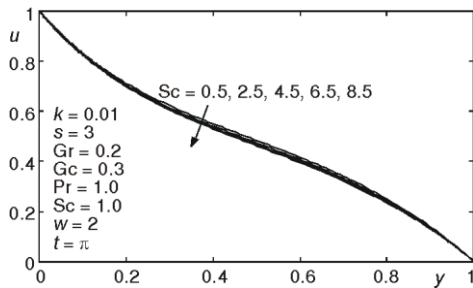


Figure 11. Velocity distribution for various values of Sc

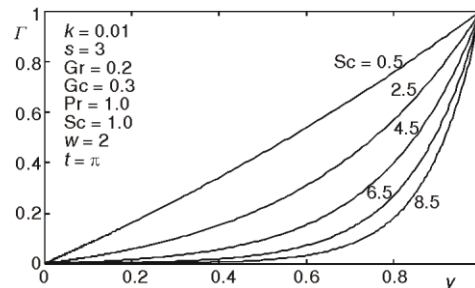


Figure 12. Concentration distribution for various values of Sc

### Conclusion

Theoretical analysis has been carried out for momentum, heat and mass transfer characteristics of an incompressible flow of electrically conducting non-Newtonian fluid between two infinite parallel walls under the action of a transfer magnetic field taking into account the induced magnetic field. The method of solution can be applied for small perturbation approximation. Numerical results are presented to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on the physical parameters. It was found that the effect of the viscoelasticity of the fluid is to decrease both the flow and induced magnetic field. Also, it was found that when the Grashof number increased, the concentration buoyancy effects were enhanced and thus, the fluid velocity increased. However, the presence of Prandtl number effects caused reduction in the fluid temperature which resulted in decrease in the fluid velocity. Also, when the Schmidt number was increased, the concentration level was decreased resulting in a decreased fluid velocity. On the other hand, it was found that the induced magnetic field profiles decreased as either the Grashof number or the modified Grashof number increased near the stationary plate, while reverse behaviour is observed near the moving plate. In addition, An increase in the value of Schmidt number resulted in decreases in both velocity and concentration.

### Nomenclature

$C$  – concentration, [ $\text{kmolm}^{-3}$ ]  
 $D$  – mass diffusivity, [ $\text{m}^2\text{s}^{-1}$ ]  
 $\vec{E}$  – electric field vector  
 $Gc$  – modified Grashof number, [–]  
 $Gr$  – Grashof number, [–]  
 $g$  – gravitational acceleration, [ $\text{ms}^{-2}$ ]  
 $\vec{H}$  – magnetic induction vector  
 $H_0$  – applied magnetic field  
 $\vec{J}$  – current density vector  
 $k$  – visco-elastic parameter  
 $k_0$  – elastic parameter  
 $M$  – magnetic field parameter  
 $Pr$  – Prandtl number, [–]  
 $R_m$  – magnetic Reynolds number, [–]  
 $T$  – temperature of the fluid, [K]

$T_0$  – surface temperature, [K]  
 $t$  – time, [s]  
 $U$  – surface velocity, [ $\text{ms}^{-1}$ ]  
 $u, v$  – velocity components along x- and y-axes, respectively, [ $\text{ms}^{-1}$ ]  
 $x, y$  – Cartesian co-ordinates along x- and y-axes, respectively, [m]

#### Greek symbols

$\alpha$  – fluid thermal diffusivity, [ $\text{m}^2\text{s}^{-1}$ ]  
 $\beta_c$  – concentration expansion coefficient, [ $\text{m}^3\text{kmol}^{-1}$ ]  
 $\beta_T$  – temperature expansion coefficient, [ $\text{K}^{-1}$ ]  
 $\Gamma$  – dimensionless concentration  
 $\varepsilon$  – scalar constant ( $<1$ )

$\mu$	– dynamic viscosity of the fluid, [Nsm <sup>-2</sup> ]	$\sigma$	– electrical conductivity, [sm <sup>-1</sup> ]
$\nu$	– kinematic viscosity, [m <sup>2</sup> s <sup>-1</sup> ]	$\theta$	– dimensionless temperature
$\rho$	– density of the fluid, [kgm <sup>-3</sup> ]	$\tau$	– shearing stress, [Nm <sup>-2</sup> ]

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