THERMAL INSTABILITY OF COMPRESSIBLE WALTERS' (MODEL B′) FLUID IN THE PRESENCE OF HALL CURRENTS AND SUSPENDED PARTICLES

by

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Effect of Hall currents and suspended particles is considered on the hydromagnetic stability of a compressible, electrically conducting Walters' (Model B′) elastico-viscous fluid. After linearizing the relevant hydromagnetic equations, the perturbation equations are analyzed in terms of normal modes. A dispersion relation governing the effects of visco-elasticity, magnetic field, Hall currents and suspended particles is derived. It has been found that for stationary convection, the Walters' (Model B′) fluid behaves like an ordinary Newtonian fluid due to the vanishing of the visco-elastic parameter. The compressibility and magnetic field have a stabilizing effect on the system, as such their effect is to postpone the onset of thermal instability whereas Hall currents and suspended particles are found to hasten the onset of thermal instability for permissible range of values of various parameters. Also, the dispersion relation is analyzed numerically and the results shown graphically. The critical Rayleigh numbers and the wavenumbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behavior of various parameters on critical thermal Rayleigh numbers has been depicted graphically. The visco-elasticity, suspended particles and Hall currents (hence magnetic fieldintroduce oscillatory modes in the system which were non-existent in their absence.

Key words: *Walters' (model B′) fluid, Hall currents, suspended particles, thermal instability, compressibility*

Introduction

A detailed account of the theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics is given by Chandrasekhar [1] in his celebrated monograph. Chandra [2] observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Bénard-type cellular convection with the fluid descending at a cell centre was observed when the predicted gradients were imposed for

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layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, "Columnar instability". Chandra [2] added an aerosol to mark the flow pattern. Scanlon *et al.* [3] investigated some of the continuum effects of particles on Bénard convection and found that a critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The effect of suspended particles was thus found to destabilize the layer. Palaniswamy *et al.* [4] have considered the stability of shear flow of stratified fluids with fine dust and have found that the effect of fine dust is to increase the region of instability. Sharma *et al.* [5] considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics while Sharma *et al.* [6] investigated the effect of Hall currents and suspended particles on thermal instability of compressible fluids saturating a porous medium.

If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current to flow across an electric field in the presence of magnetic field is called Hall effect. The Hall current is likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. Sherman *et al.* [7] have considered the effect of Hall currents on the efficiency of a magnetohydrodynamics (MHD) generator while Gupta [8] studied the effect of Hall currents on the thermal instability of electrically conducting fluid in the presence of uniform vertical magnetic field. Sharma *et al.* [9] investigated the effect of Hall currents and finite Larmor radius on thermosolutal instability of a rotating plasma and established the destabilizing influence of Hall currents. For compressible fluids, the equations governing the system become quite complicated. Spiegel *et al.* [10] have simplified the set of equations governing the flow of compressible fluids assuming that the depth of the fluid layer is much smaller than the scale height as defined by them and the motions of infinitesimal amplitude are considered. Sharma [11] investigated the thermal instability of compressible fluids in the presence of rotation and magnetic field while Sharma *et al.* [12] studied the effect of finite Larmor radius on thermal instability of compressible rotating plasma. Thermal instability of compressible, finite Larmor radius Hall plasma has been studied by Sharma *et al.* [13] in a porous medium.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology, and petroleum industry. Bhatia *et al.* [14] studied the problem of thermal instability of a Maxwellian visco-elastic fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to its stabilizing effect on a viscous Newtonian fluid. But Maxwell's model does not describe all the characteristics of a visco-elastic fluid. Thermal instability of an Oldroydian visco-elastic fluid acted on by a uniform rotation has been studied by Sharma [15]. An experimental demonstration by Toms *et al.* [16] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd [17]. There are many visco-elastic fluids which cannot be characterized by Maxwell's constitutive relations or Oldroyd's [17] constitutive relations. Two such classes of elastico-viscous fluids are Rivlin-Ericksen [18] and Walters' (Model B′) fluids. Walters [19] has proposed a theoretical model for such elastico-viscous fluids and reported [20] that the mixture of polymethyl methacrylate and pyridine at 25 °C containing 30.5 g of polymer per liter behaves very nearly as the Walters' (Model B′) elastico-viscous fluid. Such and other polymers are used in agriculture, communication appliances and in biomedical applications. Sharma *et al.* [21] have studied the stability of two superposed Walters' (Model B′) liquids whereas thermosolutal convection problem in the presence of magnetic field for Walters' (Model B′) fluid has been investigated by Sunil et al. [22]. Recently, Sharma *et al.* [23] considered the stability of stratified Walters' (Model B′) fluid in the presence of magnetic field and rotation in porous medium.

Motivated by the fact that knowledge regarding fluid particle mixture is not commensurate with their industrial and scientific importance and the importance of flow of visco-elastic fluids in paper industry, petroleum industry, chemical technology, and geophysical fluid dynamics; we set out to study the effect of suspended particles and Hall currents on thermal instability of a compressible Walters' (Model B′) fluid. Here, it is worthwhile to mention that Hall currents are important in many geophysical and astrophysical situations in addition to the flow of laboratory plasmas. To the best of our knowledge, the problem has not been investigated so far.

Formulation of the problem

Here, as shown in fig. 1, we have considered an infinite, horizontal, compressible electrically conducting Walters' (model B′) fluid layer of thickness d, permeated with suspended particles, which is heated from below (at $z = 0$) so that temperature at the bottom layer and at the upper layer are T_0 and T_d, respectively. A uniform temperature gradient $\beta(= \mathrm{d}T/\mathrm{d}z)$ is maintained and the layer is acted upon by the gravity force $\vec{g} = (0, 0, -g)$ and uniform vertical magnetic field $\vec{H} = (0, 0, H)$.

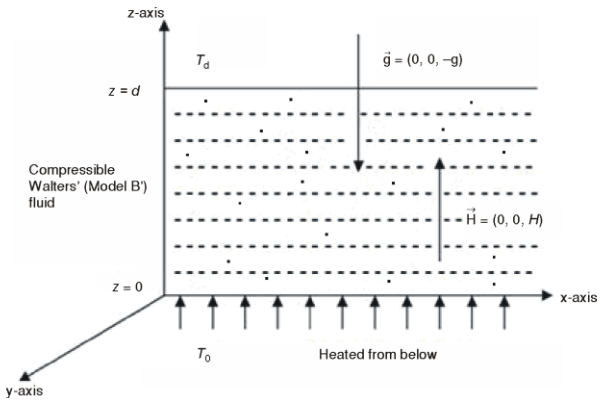


Figure 1. Geometrical configuration

The equations of motion and continuity for Walters' (model B′) visco-elastic fluid in the presence of suspended particles and Hall currents are:

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v}\right] = -\nabla p + \rho\vec{g} + \left(\mu - \mu'\frac{\partial}{\partial t}\right)\nabla^2\vec{v} + \frac{\mu_e}{4\pi}(\nabla\times\vec{H})\times\vec{H} + K_1 N(\vec{v}_d - \vec{v}) \tag{1}$$

$$\frac{\partial\rho}{\partial t} + \nabla(\rho\vec{v}) = 0 \tag{2}$$

where ρ, μ, μ', p, and $\vec{v}(u,v,w)$ denote, respectively, the density, viscosity, viscoelasticity, pressure, and velocity of the pure fluid, $\vec{v}_d(l, r, s)$, and $N(\bar{x},t)$ denote the velocity and number density of the suspended particles, μ_e is the magnetic permeability, $\bar{x} = (x, y, z)$ and $K_1 = 6\pi\mu\eta'$, η' being the particle radius, is the Stokes' drag coefficient. Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term, in the equations of motion (1), proportional to the velocity difference between particles and fluid.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Interparticle reactions are ignored because the distances between the particles are assumed to be quite large compared with their diameter. The effects due to pressure, gravity and magnetic field on the particles are small and so ignored. If *mN* is the mass of the particles per unit volume, then the equations of motion and continuity for the particles are:

$$mN\left[\frac{\partial \vec{v}_d}{\partial t}+(\vec{v}_d.\nabla)\vec{v}_d\right]=K_1N(\vec{v}-\vec{v}_d) \tag{3}$$

$$\frac{\partial N}{\partial t}+\nabla(N\vec{v}_d)=0 \tag{4}$$

Let C_f , C_{pt} , T, and q denote, respectively, the heat capacity of the pure fluid, the heat capacity of the particles, the temperature, and the "effective thermal conductivity" of the pure fluid. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives:

$$\rho C_f\left(\frac{\partial}{\partial t}+\vec{v}\nabla\right)T+mNC_{pt}\left(\frac{\partial}{\partial t}+\vec{v}_d\nabla\right)T=q\nabla^2T \tag{5}$$

The Maxwell's equations in the presence of Hall currents yield:

$$\frac{\partial \vec{H}}{\partial t}=\nabla\times(\vec{v}\times\vec{H})+\eta\nabla^2\vec{H}-\frac{1}{4\pi N'e}\nabla\times\left[(\nabla\times\vec{H})\times\vec{H}\right] \tag{6}$$

$$\nabla\vec{H}=0 \tag{7}$$

where η, N', and e denote, respectively, the resistivity, the electron number density, and the charge of an electron. The state variables pressure, density and temperature are expressed in the form [10]:

$$f(x,y,z,t)=f_m+f_0(z)+f'(x,y,z,t) \tag{8}$$

where f_m stands for constant space distribution of f, f_0 is the variation in the absence of motion and $f'(x,y,z,t)$ is the fluctuation resulting from motion. The basic state of the system with a uniform particle distribution is given by:

$$p = p(z),\ \rho = \rho(z),\ T = T(z),\ \vec{v} = (0, 0, 0),\ \vec{H} = (0, 0, H),\ \vec{v}_d = (0, 0, 0), \text{ and } N = N_0 = \text{constant}$$

where following Spiegel *et al*. [10], we have:

$$\begin{gathered} p(z)=p_m-g\int_0^z(\rho_m+\rho_0)\mathrm{d}z \\ \rho(z)=\rho_m[1-\alpha_m(T-T_m)+K_m(p-p_m)] \\ T(z)=-\beta z+T_0 \\ \alpha_m=-\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m\ (=\alpha) \\ K_m=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m \end{gathered} \tag{9}$$

where p_m and ρ_m stand for a constant space distribution of p and ρ while ρ_0 and T_0 stand for density and temperature of the fluid at the lower boundary. Following the assumptions given by Spiegel *et al*. [10] and results for compressible fluids, the flow equations are found to be the same as those of incompressible fluids except that the static temperature gradient β is replaced by its excess over the adiabatic ($\beta - g/C_p$), where C_p being specific heat of the fluid at constant pressure.

Perturbation equations

Let $\delta p, \delta\rho, \theta, \vec{v}\,(u, v, w)$, $\vec{v}_d\,(l, r, s)$, $\vec{h}\,(h_x, h_y, h_z)$, and N denote the perturbations in fluid pressure, density, temperature, fluid velocity, particle velocity, magnetic field $\vec{H}$ and particle number density N_0, respectively. Then the linearized hydromagnetic perturbation equations of the fluid-particle layer under Speigel *et al*. [10] assumptions are:

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_m}(\nabla \delta p) + \vec{g}\frac{\delta\rho}{\rho_m} + \left(\nu - \nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{v} + \frac{\mu_e}{4\pi\rho_m}(\nabla\times\vec{h})\times\vec{H} + \frac{K_1N_0}{\rho_m}(\vec{v}_d - \vec{v}) \quad (10)$$

$$\nabla\vec{v} = 0 \quad (11)$$

$$mN_0\frac{\partial \vec{v}_d}{\partial t} = K_1N_0(\vec{v} - \vec{v}_d) \quad (12)$$

$$\frac{\partial M_d}{\partial t} + \nabla\vec{v}_d = 0 \quad (13)$$

$$\nabla\vec{h} = 0 \quad (14)$$

$$\frac{\partial \vec{h}}{\partial t} = (\vec{H}\nabla)\vec{v} + \eta\nabla^2\vec{h} - \frac{1}{4\pi N'e}\nabla\times[(\nabla\times\vec{h})\times\vec{H}] \quad (15)$$

$$(1 + h_d)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)(w + h_d s) + \kappa\nabla^2\theta \quad (16)$$

where $\alpha_m = 1/T_m = \alpha$, $\nu = \mu/\rho_m$, $\nu' = \mu'/\rho_m$, $\kappa = q/\rho_m C_f$, and g/C_p, ν, ν', and κ stand for the adiabatic gradient, kinematic viscosity, kinematic viscoelasticity, and thermal diffusivity, respectively. Also,

$$M_d = \frac{N}{N_0} \quad \text{and} \quad h_d = \frac{mN_0C_{pt}}{\rho_m C_f}$$

Eliminating $\vec{v}_d$ between eqs. (10)-(12), we obtain:

$$\left(\frac{m}{K_1}\frac{\partial}{\partial t} + 1\right)\left[\frac{\partial}{\partial t}\nabla^2 w + g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - \frac{\mu_e H}{4\pi\rho_m}\frac{\partial}{\partial z}\nabla^2 h_z\right] + \frac{kN_0}{\rho_m}\frac{m}{K_1}\frac{\partial}{\partial t}\nabla^2 w = \left(\frac{m}{K_1}\frac{\partial}{\partial t} + 1\right)\left(\nu - \nu'\frac{\partial}{\partial t}\right)\nabla^4 w \quad (17)$$

$$\left(\frac{m}{K_1}\frac{\partial}{\partial t} + 1\right)\frac{\partial\varsigma}{\partial t} = \frac{\mu_e H}{4\pi\rho_m}\left(\frac{m}{K_1}\frac{\partial}{\partial t} + 1\right)\frac{\partial\xi}{\partial z} - \frac{m}{K_1}\frac{\partial}{\partial t}\left(\frac{kN_0\varsigma}{\rho_m}\right) + \left(\frac{m}{K_1}\frac{\partial}{\partial t} + 1\right)\left(\nu - \nu'\frac{\partial}{\partial t}\right)\nabla^2\varsigma \quad (18)$$

Equations (15) and (16) can be rewritten as:

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z}-\frac{H}{4\pi N'e}\frac{\partial \xi}{\partial z} \tag{19}$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)\xi = H\frac{\partial \zeta}{\partial z}+\frac{H}{4\pi N'e}\frac{\partial}{\partial z}(\nabla^2 h_z) \tag{20}$$

$$\left(\frac{m}{K_1}\frac{\partial}{\partial t}+1\right)\left[(1+h_d)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta = \beta\left(\frac{G-1}{G}\right)\left(\frac{m}{K}\frac{\partial}{\partial t}+H_d\right)w \tag{21}$$

where $\zeta = \partial v/\partial x$ is the z-component of vorticity, $\xi = (\partial h y/\partial x) - (\partial h_x/\partial y)$ is the z-component of current density and $G = (C_p/g)\beta$.

Normal mode analysis method and dispersion relation

Analyze the perturbation quantities in normal modes by seeking solutions in the form:

$$[w, h_z, \theta, \zeta, \xi] = [W(z), K(z), \Theta(z), Z(z), X(z)]\exp(ik_x x + ik_y y + nt) \tag{22}$$

where k_x and k_y are the wave numbers along x- and y-directions and the resultant wave number is given by $k = (k_x^2 + k_y^2)^{1/2}$, n is the growth rate, and W, K, Z, X, and Θ are the z-components of fluid velocity w, magnetic field h_z, vorticity ζ, current density ξ, and temperature θ, respectively, after applying normal mode analysis.

Using expression (22), eqs. (17)-(21), can be written as:

$$1+\mathrm{Pr}_1\sigma\tau\ \ (D^2-a^2)\left[(1-\sigma F)(D^2-a^2)-\sigma\right]W - f\sigma(D^2-a^2)W +$$
$$+(1+\mathrm{Pr}_1\sigma\tau)\frac{\mu_e Hd}{4\pi\rho_m \nu}(D^2-a^2)DK = (1+\mathrm{Pr}_1\sigma\tau)\frac{ga^2d^2\alpha}{\nu}\Theta \tag{23}$$

$$(1+\mathrm{Pr}_1\sigma\tau)\left[(1-\sigma F)(D^2-a^2)-\sigma\right]Z - f\sigma Z = -\frac{\mu_e Hd}{4\pi\rho_m \nu}(1+\mathrm{Pr}_1\sigma\tau)DX \tag{24}$$

$$(1+\mathrm{Pr}_1\sigma\tau)[D^2-a^2-H_d\,\mathrm{Pr}_1\sigma]\Theta + \frac{\beta d^2}{\kappa}\left(\frac{G-1}{G}\right)(H_d+\mathrm{Pr}_1\sigma\tau)W = 0 \tag{25}$$

$$[D^2-a^2-\mathrm{Pr}_2\sigma]K + \frac{Hd}{\eta}DW - \frac{Hd}{4\pi N'e\eta}DX = 0 \tag{26}$$

$$[D^2-a^2-\mathrm{Pr}_2\sigma]X + \frac{Hd}{\eta}DZ + \frac{H}{4\pi N'e\eta}D(D^2-a^2)K = 0 \tag{27}$$

where we have non-dimensionalized various parameters:

$$a = k\mathrm{d},\ \sigma = \frac{nd^2}{\nu},\ \mathrm{Pr}_1 = \frac{\nu}{\kappa},\ \mathrm{Pr}_2 = \frac{\nu}{\eta},\ F = \frac{\nu'}{d^2},\ H_\mathrm{d} = 1 + h_\mathrm{d},\ f = \frac{mN_0}{\rho_m},\ \tau = \frac{m\kappa}{K_1 d^2},$$

$$x^* = \frac{x}{d},\ y^* = \frac{y}{d},\ z^* = \frac{z}{d},\ D = \frac{d}{dz^*}$$

Let us consider the case of two free boundaries which are perfect conductors of heat. Though the case of two free boundaries is of little physical interest yet is mathematically very important as it enables us to get analytical solutions and draw some qualitative conclusions. For the case of free boundaries the boundary conditions are [1]:

$$\begin{aligned} &W = D^2W = 0,\ DZ = 0,\ \Theta = 0 \ \text{ at } \ z = 0 \ \text{ and } 1, \\ &K = 0, \qquad \text{on perfectly conducting boundaries,} \end{aligned} \tag{28}$$

and h_x, h_y, h_z are continuous. Since the components of magnetic field are continuous and the tangential components are zero outside the fluid, we have:

$$DK = 0, \quad \text{on the boundaries.} \tag{29}$$

Using these boundary conditions (28) and (29), we can see that all the even order derivatives of W must vanish for z = 0 and 1. Therefore the proper solution of W characterizing the lowest mode is:

$$W = W_0 \sin\pi z \tag{30}$$

where W_0 is a constant. After eliminating Θ, X, Z and K between eqs. (23)-(27), we obtain:

$$\boxed{\begin{aligned} \mathrm{R}_1 = \frac{G}{G-1}\Bigg\{ & (1+x)(1+i\,\mathrm{Pr}_1\,\sigma_1\pi^2\tau)[(1+x)(1-i\sigma_1 F\pi^2)+i\sigma_1] + i(1+x)f\sigma_1 + \\ & + \frac{Q_1(1+i\,\mathrm{Pr}_1\,\sigma_1\pi^2\tau)(1+x)\left(\begin{aligned}&(1+x+i\sigma_1\,\mathrm{Pr}_2)\{(1+i\,\mathrm{Pr}_1\,\sigma_1\pi^2\tau)[(1+x)(1-iF\sigma_1\pi^2)+i\sigma_1] \\ &\qquad +if\sigma_1\} + Q_1(1+i\,\mathrm{Pr}_1\,\sigma_1\pi^2\tau)\end{aligned}\right)}{\left(\begin{aligned}&[M(1+x)+(1+x+i\sigma_1\,\mathrm{Pr}_2)^2]\{(1+i\,\mathrm{Pr}_1\,\sigma_1\pi^2\tau)\cdot \\ &\cdot[(1+x)(1-iF\sigma_1\pi^2)+i\sigma_1]+if\sigma_1\} + Q_1(1+i\,\mathrm{Pr}_1\,\sigma_1\pi^2\tau)(1+x+i\sigma_1\,\mathrm{Pr}_2)]\end{aligned}\right)}\Bigg\}\cdot \\ & \cdot\frac{1+x+iH_d\,\mathrm{Pr}_1\,\sigma_1}{x(H_d+i\,\mathrm{Pr}_1\,\sigma_1\pi^2\tau)} \end{aligned}} \tag{31}$$

where

$$\mathrm{R}_1 = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4},\quad Q_1 = \frac{\mu_\mathrm{e}H^2d^2}{4\pi\rho_\mathrm{m}\nu\eta\pi^2},\quad M = \left(\frac{H}{4\pi N'e\eta}\right)^2,\quad x = \frac{a^2}{\pi^2},\quad \text{and} \quad i\sigma_1 = \frac{\sigma}{\pi^2}$$

Equation (31) is the required dispersion relation including the effects of Hall currents and compressibility on the thermal instability of Walters' (Model B′) fluid permeated with suspended particles. The dispersion relation reduces to the one derived by Sharma *et al.* [24] if the Hall current parameter is vanishing.

Case of stationary convection

Let us consider the case when instability sets in the form of stationary convection. For stationary convection, $\sigma = 0$ and the dispersion relation (31) reduces to:

$$\mathrm{R}_1 = \frac{G}{G-1}\left\{(1+x)^2 + \frac{Q_1[(1+x)^2+Q_1]}{(1+x)(M+1+x)+Q_1}\right\}\frac{1+x}{xH\ d} \tag{32}$$

which expresses the modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters Q_1, G, M, and H_d. Here, it is clear that for stationary convection the visco-elastic parameter F vanishes with σ and the Walters' (model B′) fluid behaves like an ordinary Newtonian fluid. In the absence of Hall currents, the above expression for Rayleigh number R_1 reduces to:

$$\mathrm{R}_1 = \frac{G}{G-1}[(1+x)^2 + Q_1]\frac{1+x}{xH_d}$$

which is identical with the expression for R_1 derived by Sharma *et al.* [24] wherein thermal instability of a compressible Walters' (model B′) fluid in hydromagnetics is studied in the presence of suspended particles.

Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get:

$$\overline{\mathrm{R}_\mathrm{c}} = \frac{G}{G-1}\mathrm{R}_\mathrm{c} \tag{33}$$

where R_c and $\overline{\mathrm{R}_\mathrm{c}}$ denote, respectively, the critical Rayleigh numbers in the absence and presence of compressibility. Thus, the effect of compressibility is to postpone the onset of thermal instability. The cases $G < 1$ and $G = 1$ correspond to negative and infinite value of Rayleigh number which are not relevant in the present study. Hence, compressibility has a stabilizing effect on the thermal convection.

For investigating the effects of magnetic field, suspended particles and Hall currents, we examine the natures of $\mathrm{dR}_1/\mathrm{d}Q_1$, $\mathrm{dR}_1/\mathrm{d}H_\mathrm{d}$, and $\mathrm{dR}_1/\mathrm{d}M$ analytically.

For analyzing the effect of magnetic field, expression (32) yields:

$$\frac{\mathrm{dR}_1}{\mathrm{d}Q_1} = \frac{G}{G-1}\frac{1+x}{xH_d}\frac{(1+x)(M+1+x)[(1+x)^2+2Q_1]+Q_1^2}{[(1+x)(M+1+x)+Q_1]^2} \tag{34}$$

which shows the usual stabilizing effect of magnetic field on thermal convection. This is in agreement with the result of fig. 2, where R_1 is plotted against x for various values of $Q_1 =$ $= 100$, 150, 200, 250, and 300. This stabilizing effect of magnetic field is in good agreement with earlier works of Sharma *et al.* [25] and Kumar *et al.* [26].

For analyzing the effect of suspended particles; from the expression (32), we obtain

$$\frac{\mathrm{dR}_1}{\mathrm{d}H_d} = -\frac{G}{G-1}\frac{1+x}{xH_\mathrm{d}^{\,2}}\left\{(1+x)^2 + \frac{Q_1[(1+x)^2+Q_1]}{(1+x)(M+1+x)+Q_1}\right\} \tag{35}$$

which states that suspended particles have a destabilizing effect on the system. Also fig. 3 confirms the above result numerically for the permissible range of values of various parameters. This result is in agreement with the result of Sharma *et al.* [6] in which effect of Hall currents and suspended particles is investigated on the thermal instability of compressible fluids in porous medium. The result is again identical with that of Kumar *et al.* [26] where the effect of suspended particles is studied on thermal instability of Walters' (Model B') fluid in the presence of magnetic field saturating a porous medium.

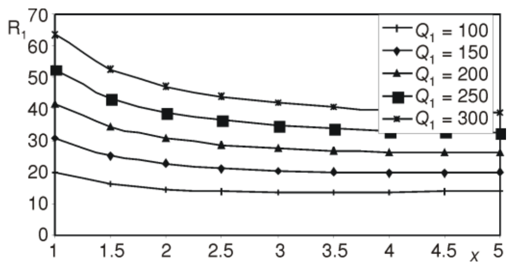


Figure 2. Variation of Rayleigh number R_1 with wavenumber x for fixed G = 10, H_d = 10, M = 10 and various values of Q_1 = 100, 150, 200, and 300

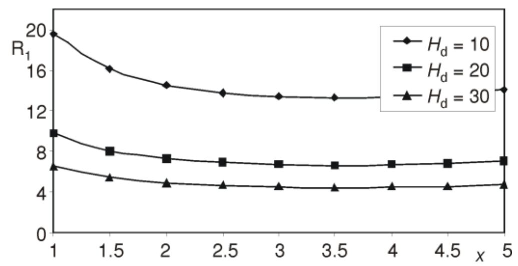


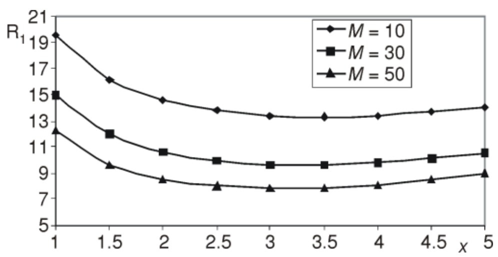
Figure 3. Variation of Rayleigh number R_1 with wavenumber x for fixed G = 10, Q_1 = 100, M = 10 and for various values of H_d = 10, 20, and 30

Expression for observing the effect of Hall currents is obtained as:

$$\frac{\mathrm{d}R_1}{\mathrm{d}M} = -\frac{G}{G-1}\frac{(1+x)^2}{xH_d}\left\{\frac{Q_1[(1+x)^2+Q_1]}{[(1+x)(M+1+x)+Q_1]^2}\right\} \tag{36}$$

which reflects the destabilizing influence of Hall currents on thermal instability of Walters' (Model B') fluid. Also in fig. 4, R_1 decreases with the increase in M which confirms the above result numerically. This result is identical with that of Sunil *et al.* [27] in which Hall effect on thermal instability of Walters' fluid has been investigated.

Figure 4. Variation of Rayleigh number $R_1$with wavenumber x for fixed G = 10, Q_1 = 100, H_d = 10 and for various values of M = 10, 30, and 50



Let us now find out critical Rayleigh number R_c and the associated critical wavenumber x_c for various values of the parameters Q_1, M, and H_d. As a function of x, R_1 given by eq. (32) attains its extremal value when:

$$(1+x)^4(2x-1)(1+x+M)^2+(1+x)^2(3x-2)Q_1^2+2(1+x)^3(4x-1)(1+x+M)Q_1 - \\ -(1-x)^2(1+x+M)Q_1^2-(1+x)^4(1+2x+M)Q_1=0 \tag{37}$$

However, this equation is not very useful for determining the critical Rayleigh numbers for assigned values of G, Q_1, M, and H_d. It is more convenient to evaluate R_c as a function of x in accordance with eq. (32) for various values of Q_1, H_d, and M as depicted in figs. 2-4 and locate the minimum numerically. The critical numbers listed in tab. 1 and illustrated in figs. 5-7 are obtained in this fashion.

Table 1. The critical Rayleigh numbers and the wavenumbers of the associated disturbances for the onset of instability as stationary convection for various values of Q_1, H_d, and M

G = 10	H_d = 10	M = 10	G = 10	Q_1 = 100	M = 10	G = 10	Q_1 = 100	H_d = 10
Q_1	x_c	R_c	H_d	x_c	R_c	M	x_c	R_c
100	3.5	13.288	10	3.5	13.288	10	3.5	13.288
200	4.5	26.032	20	3.5	6.644	30	3.5	9.623
300	5.0	38.739	30	3.5	4.429	50	3.0	7.809
500	5.0	64.755	50	3.5	2.658	100	2.5	5.683
1000	5.0	130.834						

Figure 5 shows that critical Rayleigh number R_c increases with the increase in magnetic field parameter Q_1 depicting the usual stabilizing effect of magnetic field.

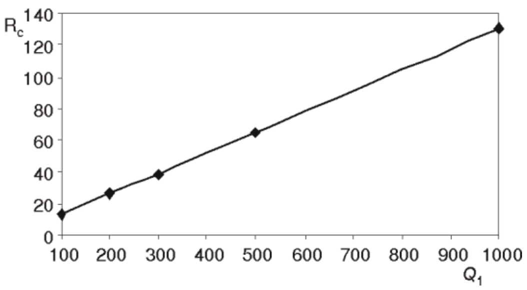


Figure 5. Variation of critical Rayleigh number R_c with magnetic field parameter Q_1 for fixed G = 10, H_d = 10, and M = 10

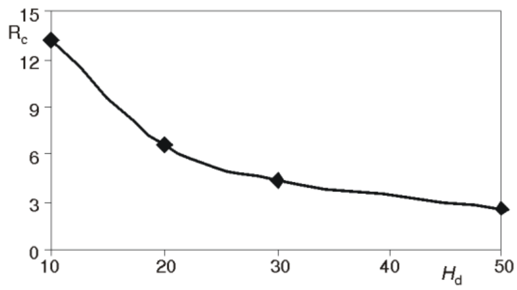


Figure 6. Variation of critical Rayleigh number R_c with suspended particle factor H_d for fixed G = 10, Q_1 = 100, and M = 10

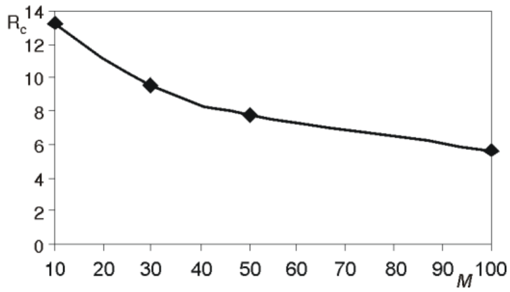


Figure 7. Variation of critical Rayleigh number R_c with Hall current parameter M for fixed G = 10, Q_1 = 100, H_d = 10

Figures 6 and 7 show the decrease in critical Rayleigh number R_c with the increase in suspended particles parameter H_d and Hall current parameter M, respectively. Thus figs. 6 and 7 confirm the destabilizing influence of suspended particles and Hall currents, respectively.

Stability of the system and oscillatory modes

To determine the possibility of oscillatory modes we multiply eq. (23) by W^*, the complex conjugate of W and using eqs. (24)-(27) together with the boundary conditions (28) and (29), we obtain:

$$(1+\Pr_1\sigma\tau)(1-F\sigma)I_1+(1+f+\Pr_1\sigma\tau)\sigma I_2+\frac{\mu_e\eta}{4\pi\rho_m\nu}(1+\Pr_1\sigma\tau)\left[I_3+\Pr_2\sigma^* I_4\right]+$$
$$\frac{\mu_e\eta d}{4\pi\rho_m\nu}[I_7+\Pr_2\sigma I_8]+\frac{nd^2}{\nu}\left(\frac{1+f+\Pr_1\sigma^*\tau}{1+\Pr_1\sigma^*\tau}\right)I_9-$$
$$\frac{G}{G-1}\frac{g\alpha\kappa a^2}{\nu\beta}\frac{(1+\Pr_1\sigma\tau)(1+\Pr_1\sigma^*\tau)}{H_d+\Pr_1\sigma^*\tau}(I_5+H_d\Pr_1\sigma^* I_6)=0 \tag{38}$$

where

$$I_1=\int_0^1(|D^2W|^2+2a^2|DW|^2+a^4|W|^2)\,dz$$
$$I_2=\int_0^1(|DW|^2+a^2|W|^2)\,dz$$
$$I_3=\int_0^1(|D^2K|^2+2a^2|DK|^2+a^4|K|^2)\,dz$$
$$I_4=\int_0^1(|DK|^2+a^2|K|^2)\,dz,\quad I_5=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)\,dz$$
$$I_6=\int_0^1(|\Theta|^2)\,dz,\quad I_7=\int_0^1(|DX|^2+a^2|X|^2)\,dz$$
$$I_8=\int_0^1(|X|^2)\,dz,\quad I_9=\int_0^1(|Z|^2)\,dz$$

where integrals I_1, I_2 … I_9 are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating real and imaginary parts of eq. (38), we get:

$$(1+\Pr_1\tau\sigma_r-F\sigma_r-F\Pr_1\tau\sigma_r^2+F\Pr_1\tau\sigma_i^2)I_1+[\sigma_r(1+f+\Pr_1\tau\sigma_r)-\Pr_1\tau\sigma_i^2]I_2+$$
$$+\frac{\mu_e\eta}{4\pi\rho_m\nu}\ (1+\Pr_1\tau\sigma_r)I_3+[\Pr_2\sigma_r(1+\Pr_1\tau\sigma_r)+\Pr_1\Pr_2\tau\sigma_i^2]I_4\ +$$
$$+\frac{\mu_e\eta d}{4\pi\rho_m\nu}[I_7+\Pr_2\sigma_r I_8]+\frac{nd^2}{\nu}\frac{[(1+f+\Pr_1\tau\sigma_r)(1+\Pr_1\tau\sigma_r)+\Pr_1^2\tau^2\sigma_i^2]}{(1+\Pr_1\tau\sigma_r)^2+\Pr_1^2\tau^2\sigma_i^2}I_9-$$
$$-\frac{g\alpha\kappa a^2}{\nu\beta}\frac{G}{G-1}\left\{\frac{(1+\Pr_1\tau\sigma_r)^2+\Pr_1^2\tau^2\sigma_i^2}{(H_d+\Pr_1\tau\sigma_r)^2+\Pr_1^2\tau^2\sigma_i^2}\right\}\cdot$$
$$\cdot[(H_d+\Pr_1\tau\sigma_r)(I_5+H_d\Pr_1\sigma_r I_6)+\Pr_1^2\tau H_d\sigma_i^2 I_6]=0 \tag{39}$$

$$
\begin{gathered}
i\sigma_i\{[\mathrm{Pr}_1\,\tau - F\,\mathrm{Pr}_1\,\tau\sigma_r - F(1+\mathrm{Pr}_1\,\tau\sigma_r)]I_1 + \\
+(1+f+2\,\mathrm{Pr}_1\,\tau\sigma_r)I_2 + \frac{\mu_e\eta}{4\pi\rho_m\nu}(\mathrm{Pr}_1\,\tau I_3 - \mathrm{Pr}_2\,I_4) + \\
+\frac{\mu_\mathrm{e}\eta d}{4\pi\rho_\mathrm{m}\nu}\mathrm{Pr}_2\,I_8 + \frac{nd^2}{\nu}\frac{f\,\mathrm{Pr}_1\,\tau}{1+\mathrm{Pr}_1\,\tau\sigma_\mathrm{r}{}^2 + \mathrm{Pr}_1{}^2\,\tau^2\sigma_i{}^2}I_9 + \\
+\frac{g\alpha\kappa a^2}{\nu\beta}\frac{G}{G-1}\frac{(1+\mathrm{Pr}_1\,\tau\sigma_\mathrm{r})^2 + \mathrm{Pr}_1^2\,\tau^2\sigma_i^2}{(H_\mathrm{d}+\mathrm{Pr}_1\,\tau\sigma_r)^2 + \mathrm{Pr}_1^2\,\tau^2\sigma_i^2}(-\mathrm{Pr}_1\,\tau I_5 + \mathrm{Pr}_1\,H_\mathrm{d}^2 I_6)\} = 0
\end{gathered}
\tag{40}
$$

It follows from eq. (39) that σ_r may be positive or negative which means that the system may be stable or unstable. Also, from eq. (40) σ_i may be zero or non-zero, meaning thereby that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of visco-elasticity, magnetic field (hence Hall currents) and suspended particles. In the absence of magnetic field (hence Hall currents), suspended particles and viscoelasticity of the fluid; all the terms in eq. (40) are positive meaning thereby that σ_i is zero. Therefore in the absence of these effects only non-oscillatory modes will prevail. *i. e.* the principle of exchange of stabilities will hold good. This result is in agreement with the result derived by Sunil *et al.* [27] where the effect of Hall currents has been investigated on Walters' (Model B′) fluid and Sharma *et al.* [24] wherein effect of compressibility and suspended particles is studied on thermal instability of Walters' fluid in hydromagnetics.

Conclusions

In the present paper, the combined effect of Hall currents, magnetic field and suspended particles on the stability of a compressible Walters' (Model B′) elastico-viscous fluid heated from below is considered. The effect of various parameters such as magnetic field, compressibility, Hall currents, and suspended particles has been investigated analytically as well as numerically. The main results from the analysis of the paper are:

- for the case of stationary convection, Walters' (Model B′) fluid behaves like an ordinary Newtonian fluid due to the vanishing of the visco-elastic parameter,
- the expressions for $\mathrm{dR}_1/\mathrm{d}Q_1$, $\mathrm{dR}_1/\mathrm{d}H_\mathrm{d}$, and $\mathrm{dR}_1/\mathrm{d}M$ are examined analytically and it has been found that the magnetic field has a stabilizing effect on the system whereas suspended particles and Hall currents have a destabilizing influence on the system. Figures (2)-(4) support the analytic results graphically. The reasons for stabilizing effect of magnetic field and destabilizing effect of suspended particles and Hall currents are accounted by Chandrasekhar [1] and Scanlon *et al*. [3], respectively. These are valid for second-order fluids as well,
- the effect of compressibility is to postpone the onset of instability, as is clear from eq. (33),
- the critical thermal Rayleigh numbers and the associated wavenumbers are found for stationary convection for various parameters involved and it has been found that it increases with the increase in magnetic field parameter and decreases with the increase in suspended particle factor and Hall current parameter thereby confirming the stabilizing role of magnetic field and destabilizing role of suspended particles and Hall currents, and

- the oscillatory modes are introduced due to the presence of visco-elasticity, Hall currents, and suspended particles. In the absence of these effects, the principle of exchange of stabilities is found to hold good.

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Nomenclature

C_f	–	heat capacity of the fluid, [$Jkg^{-1}K^{-1}$]
C_p	–	specific heat of the fluid at constant pressure, [$Jkg^{-1}K^{-1}$]
C_{pt}	–	heat capacity of particles, [$Jkg^{-1}K^{-1}$]
D	–	(= d/dz)
d	–	depth of the fluid layer, [m]
e	–	charge of an electron, [C]
F	–	kinematic visco-elasticity, [–]
$\vec{g}$	–	acceleration due to gravity, (= 0, 0, –g), [ms^{-2}]
$\vec{H}$	–	magnetic field vector having components, (= 0,0, H) [G]
$\vec{h}$	–	perturbation in magnetic field $\vec{H}$ (0,0, H), (= h_x, h_y, h_z), [G]
i	–	(= $-1^{1/2}$), a complex number
K	–	z-component of magnetic field after applying normal mode analysis
K_1	–	Stokes' drag coefficient (= $6\pi\mu\eta'$), [kgs^{-1}]
k	–	wave number of the disturbance, (= $k_x^2 + k_y^2)^{1/2}$, [m^{-1}]
k_x, k_y	–	wavenumbers in x and y directions, respectively, [m^{-1}]
M	–	dimensionless Hall current parameter, [–]
N	–	perturbation in suspended particle number density, [m^{-3}]
N_0	–	particle number density, [m^{-3}]
N'	–	electron number density, [m^{-3}]
n	–	growth rate of the disturbance, [s^{-1}]
Pr_1	–	thermal Prandtl number, [–]
Pr_2	–	magnetic Prandtl number, [–]
p	–	fluid pressure, [Pa]
Q_1	–	Chandrasekhar number, [–]
q	–	effective thermal conductivity of the pure fluid, [$Wm^{-1}K^{-1}$]
R_1	–	Rayleigh number, [–]
R_c	–	critical Rayleigh number, [–]
T	–	temperature, [K]
$\vec{v}$	–	fluid velocity vector having components (u, v, w), [ms^{-1}]
$\vec{v}_d$	–	velocity of suspended particles (= l, r, s), [ms^{-1}]
X	–	z-component of current density after applying normal mode analysis
x, y, z	–	x, y, z-directions
x	–	wavenumber, [m^{-1}]
x_c	–	critical wavenumber, [m^{-1}]
Z	–	z-component of vorticity after applying normal mode analysis

Greek symbols

α	–	thermal coefficient of expansion, [K^{-1}]
β	–	temperature gradient (= $\lvert dT/dz \rvert$), [Km^{-1}]
∂	–	curly operator, [–]
∇	–	del operator, [-]
δ	–	perturbation in the respective physical quantity, [–]
ζ	–	z-component of vorticity
η'	–	particle radius, [m]
η	–	resistivity, [m^2s^{-1}]
Θ	–	temperature after applying normal mode analysis
θ	–	perturbation in temperature, [K]
κ	–	thermal diffusivity, [m^2s^{-1}]
μ	–	viscosity of the fluid, [$kgm^{-1}s^{-1}$]
μ'	–	visco-elasticity of the fluid, [$kgm^{-1}s^{-1}$]
μ_e	–	magnetic permeability, [Hm^{-1}]
ξ	–	z-component of current density
ν	–	kinematic viscosity, [m^2s^{-1}]
ν'	–	kinematic visco-elasticity, [m^2s^{-1}]
ρ	–	density of the fluid, [kgm^{-3}]

References

[1] Chandrasekhar, S., Hydrodynamic and Hydromagnetic Stability, Dover Publication, New York, USA, 1981
[2] Chandra, K., Instability of Fluids Heated from Below, *Proc. Roy. Soc., London., A164* (1938), 917, pp. 231-242

[3] Scanlon, J. W., Segel, L. A., Some Effects of Suspended Particles on the Onset of Bénard Convection, *Phys. Fluids*, *16* (1973), 10, pp. 1573-1578
[4] Palaniswamy, V. A., Purushotham, C. M., Stability of Shear Flow of Stratified Fluids with Fine Dust, *Phys.Fluids, 24* (1981), 7, pp. 1224-1229
[5] Sharma, R. C., Prakash, K., Dube, S. N., Effect of Suspended Particles on the Onset of Bénard Convection in Hydromagnetics, *Acta Physica Hungarica, 40* (1976), 1, pp. 3-10
[6] Sharma, R. C., Gupta, U., Thermal Instability of Compressible Fluids with Hall Currents and Suspended Particles in Porous Medium, *Int. J. Eng. Sci., 31* (1993), 7, pp. 1053-1060
[7] Sherman, A., Sutton, G. W., Magnetohydrodynamics, Northwestern University Press, Evanston, Ill., USA, 1962
[8] Gupta, A. S., Hall Effects on Thermal Instability, *Rev. Roum. Math. Pures Appl., 12* (1967), pp. 665-677
[9] Sharma, K. C., Gupta, U., Hall Effects on Thermosolutal Instability of a Rotating Plasma, *Il Nuovo Cimento, 12D* (1990), 12, pp. 1603-1610
[10] Spiegal, E. A., Veronis, G., On the Boussinesq Approximation for a Compressible Fluid, *Astrophys. J., 131* (1960), pp. 442-447
[11] Sharma, R. C., Thermal Instability in Compressible Fluids in the Presence of Rotation and Magnetic Field, *J. Math. Anal. Appl., 60* (1977), 1, pp. 227-235
[12] Sharma, K. C., Gupta, U., Finite Larmor Radius Effect on Thermal Instability of a Compressible Rotating Plasma, *Astrophysics and Space Science*, *167* (1990), 2, pp. 235-243
[13] Sharma, R. C., Sunil, S., Thermal Instability of Compressible Finite-Larmor-Radius Hall Plasma in Porous Medium, *Journal of Plasma Physics, 55* (1996), 1, pp. 35-45
[14] Bhatia, P. K., Steiner, J. M., Convective Instability in a Rotating Visco-Elastic Fluid Layer, *Z. Angew. Math. Mech., 52* (1972), 6, pp. 321-327
[15] Sharma, R. C., Effect of Rotation on Thermal Instability of a Visco-elastic Fluid, *Acta Physica Hungrica, 40* (1976), 1, pp. 11-17
[16] Toms, B. A., Strawbridge, D. J., Elastic and Viscous Properties of Dilute Solutions of Polymethyl Methacrylate in Organic Liquids, *Trans. Faraday Soc., 49* (1953), pp. 1225-1232
[17] Oldroyd, J. G., Non-Newtonian Effects in Steady Motion of Some Idealized Elastico-Viscous Liquids, *Proc. Roy. Soc. (London), A245* (1958), 1241, pp. 278-297
[18] Rivlin, R. S., Ericksen, J. L., Stress Deformation Relations for Isotropic Materials, *J. Rat. Mech. Anal., 4* (1955), pp. 323-329
[19] Walters, K., The Motion of an Elastico-Viscous Liquid Contained between Coaxial Cylinders, *Quart. J. Mech. Appl. Math., 13* (1960), 4, pp. 444-461
[20] Walters, K., Non-Newtonian Effects in Some Elastico-Viscous Liquids Whose Behaviour at Some Rates of Shear is Characterized by a General Linear Equation of State, *Quart. J. Mech. Appl. Math., 15* (1962), 1, pp. 63-76
[21] Sharma, R. C., Kumar, P., Study of the Stability of Two Superposed Walters' (Model B′) Visco-Elastic Liquids, *Czechoslovak Journal of Physics, 47* (1997), 2, pp. 197-204
[22] Sunil, S., Sharma, R. C., Chand, S., Thermosolutal Convection in Walters' (Model B′) Fluid in Porous Medium in Hydromagnetics, *Studia Geotechnica et Mechanica, 22* (2000), 3-4, pp. 3-14
[23] Sharma, V., Sunil, S., Gupta, U., Stability of Stratified Elastico-Viscous Walters' (Model B′) Fluid in the Presence of Horizontal Magnetic Field and Rotation in Porous Medium, *Arch. Mech., 58* (2006), 2 pp. 187-197
[24] Sharma, R. C., Aggarwal, A. K., Effect of Compressibility and Suspended Particles on Thermal Convection in a Walters' Elastico-Viscous Fluid in Hydromagnetics, *Int. J. of Applied Mechanics and Engineering, 11* (2006), 2, pp. 391-399
[25] Sharma, R. C., Sunil, S., Chand, S., Hall Effect on Thermal Instability of Rivlin-Ericksen Fluid, *Indian J. of Pure Appl. Math., 31* (2000), 1, pp. 49-59
[26] Kumar, P., Singh, G. J., Lal, R., Thermal Instability of Walters B′ Visco-Elastic Fluid Permeated with Suspended Particles in Hydromagnetics in Porous Medium, *Thermal Science, 8* (2004), 1, pp. 51-61
[27] Sunil, S., Sharma, R. C., Chand, S., Hall Effect on Thermal Instability of Walters' (Model B′) Fluid, *Indian Journal of Theoretical Physics, 48* (2000), 1, pp. 81-92

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