

# AN OPTIMIZED LASER-DOPPLER ANEMOMETER FOR WALL BOUNDARY LAYER MEASUREMENTS

by

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*For velocity measurements in turbulent wall boundary layers with laser-Doppler anemometry (LDA), a probe volume diameter of 30  $\mu\text{m}$  or less is needed in flow rigs of typical laboratory scale to resolve steep gradients in mean velocity and turbulence quantities. It is also advantageous if signals from very small particles can be detected to enhance the data rate. This paper describes optical solutions to optimize the LDA sensitivity and to achieve such small measuring volumes. The feasibility of a suitably designed optical system is demonstrated in a two-dimensional boundary layer flow.*

## INTRODUCTION

For near-wall velocity measurements in a wall boundary layer a high spatial resolution is necessary because of the steep gradient of mean velocity. If a laser-Doppler anemometer is used as the measuring instrument a probe volume diameter not exceeding about 30  $\mu\text{m}$  is needed to reach a non-dimensional distance  $y^+ \simeq 2$  from the wall in a flow rig of typical laboratory size. To enhance the data rate in the slowly moving fluid immediately adjacent to the wall, it is also desirable to detect signals even from very small particles. This paper demonstrates the advantages of using shorter wave length laser radiation both to increase the signal-to-noise ratio (SNR) and to minimize the probe volume dimensions of the laser-Doppler anemometer.

In the following section a short review of the principles of laser-Doppler anemometer is presented. Thereafter, the improvement of the Doppler signal detectability through the use of ultraviolet light is analyzed. The benefits of ultraviolet radiation for reducing the probe volume diameter are then discussed, and demonstrated by reference to boundary layer velocity measurements in a water flow. Conclusions from the work are given in the final section of the paper.

## PRINCIPLES OF LASER-DOPPLER ANEMOMETRY

A particle passing through a laser beam can be considered as a moving receiver and emitter. Due to the Doppler effect, the moving particle does not see the angular

frequency  $\omega$  of the output laser's electric field vector  $\mathbf{E} = \mathbf{E}_0 \exp(i(\mathbf{K} \cdot \mathbf{x} - \omega t))$ , but rather an angular frequency  $\omega'$  that is Doppler-shifted according to

$$\omega' = \omega - \mathbf{K} \cdot \mathbf{v} \quad (1)$$

where  $\mathbf{K}$  is the wave vector of the laser beam and  $\mathbf{v}$  the particle velocity relative to the laser. Using the unit vector in the direction of the laser beam,

$$\hat{\mathbf{k}} = \frac{\mathbf{K}}{|\mathbf{K}|} = \frac{\lambda}{2\pi} \mathbf{K} \quad (2)$$

eq. (1) can be rewritten in terms of the laser frequency  $\nu$  and wavelength  $\lambda$  as:

$$\nu' = \nu - \frac{1}{\lambda} \hat{\mathbf{k}} \cdot \mathbf{v} \quad (3)$$

A detector placed in the direction  $\hat{\mathbf{I}}$  again does not receive the light scattered by the particle at the frequency  $\nu'$ , but at a frequency Doppler shifted a second time

$$\nu'' = \nu' - \frac{1}{\lambda} \hat{\mathbf{I}} \cdot \mathbf{v}' \quad (4)$$

The velocity  $\mathbf{v}'$  is equal to  $\mathbf{v}$  except for a change in sign, since the particle's velocity now has to be taken relative to the detector:

$$\mathbf{v}' = -\mathbf{v} \quad (5)$$

Putting eqs. (3), (4) and (5) together leads to the light frequency received by the detector

$$\nu'' = \nu + \frac{1}{\lambda} (\hat{\mathbf{I}} - \hat{\mathbf{k}}) \cdot \mathbf{v} \quad (6)$$

The frequency difference  $\nu'' - \nu$  is, therefore, directly proportional to the particle velocity  $\mathbf{v}$ . For two reasons, however, it cannot be measured directly. Firstly, it is of an order  $10^6$  smaller than the laser light frequency  $\nu$ . Secondly, photodetectors, such as photomultipliers or photodiodes, do not react to the amplitude of the electric field vector  $\mathbf{E}$ , but rather to the electromagnetic wave's intensity given by the Poynting vector  $\mathbf{S}$ :

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \quad (7)$$

where  $\mathbf{E}$  is the electric and  $\mathbf{H}$  the magnetic field vector of the monochromatic electromagnetic wave. Since  $\mathbf{E}$  and  $\mathbf{H}$  are in phase, the light intensity is modulated with the angular frequency  $2\omega$  which is too high to be detected. Photodetectors respond, therefore, only to the time averaged intensity [4] and cannot respond to the frequency difference in eq. (6).

In order to measure the frequency difference due to the Doppler shifts, the scattered light signal is mixed, *i. e.* brought to interference, with a second light signal. This is most commonly done by letting a second laser beam pass through the control

volume, as shown in Fig. 1. The photodetector then receives two scattered light signals, one with the frequency

$$\nu_A = \nu + \frac{1}{\lambda} (\hat{\mathbf{I}} - \hat{\mathbf{k}}_A) \cdot \mathbf{v} \quad (8)$$

and the other with the frequency

$$\nu_B = \nu + \frac{1}{\lambda} (\hat{\mathbf{I}} - \hat{\mathbf{k}}_B) \cdot \mathbf{v} \quad (9)$$

The mixing of these two scattered light signals leads to an intensity modulated signal, where the modulation frequency is given by the Doppler frequency  $\nu_D$ :

$$\nu_D = \frac{1}{\lambda} (\hat{\mathbf{k}}_B - \hat{\mathbf{k}}_A) \cdot \mathbf{v} \quad (10)$$

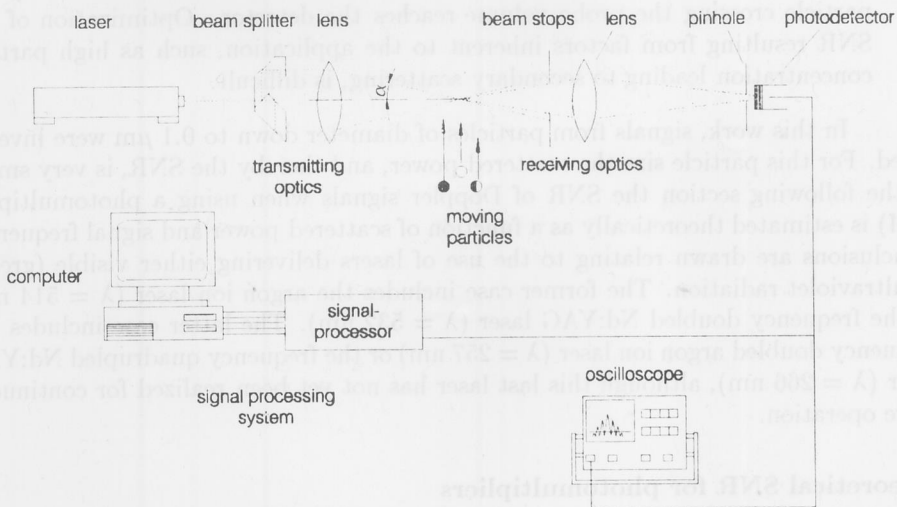


Figure 1. Schematic representation of a two-beam LDA

An important characteristic of the Doppler frequency  $\nu_D$  is its independence of the detector position  $\hat{\mathbf{I}}$ . From eq. (10) it is also clear that only the velocity component parallel to the vector  $\hat{\mathbf{k}}_B - \hat{\mathbf{k}}_A$  has an influence on the value of  $\nu_D$ . The vector  $\hat{\mathbf{k}}_B - \hat{\mathbf{k}}_A$  is perpendicular to the bisector of the angle between  $\hat{\mathbf{k}}_B$  and  $\hat{\mathbf{k}}_A$  and has the magnitude  $2 \sin \alpha$ . Therefore, the measured Doppler frequency is a function of the beam intersection half angle  $\alpha$ , the velocity component  $v_{\perp}$ , and the wavelength  $\lambda$  according to

$$\nu_D = \frac{v_{\perp} 2 \sin \alpha}{\lambda} = \frac{v_{\perp}}{\Delta x} \quad (11)$$

where  $\Delta x$  is the interference fringe spacing.

For further details on the technique of laser-Doppler anemometry the reader is referred to [1].

## DETECTABILITY OF PARTICLES

In order to measure velocity distributions with LDA, the signal-to-noise ratio (SNR) of Doppler signals must be larger than the smallest SNR necessary for the signal processing electronics to extract frequency with sufficient accuracy. The SNR of a Doppler signal can be impaired for several reasons, usually in combination.

1. *Low scattered power* results either from a low laser intensity in the probe volume or because the scattering particles are very small.
2. *High Doppler signal frequencies*, eq. (11), are a consequence of a high velocity  $v_{\perp}$  or a large beam intersection half angle  $\alpha$ .
3. *Stray light*: The SNR is reduced when light other than that scattered by the particle crossing the probe volume reaches the detector. Optimization of the SNR resulting from factors inherent to the application, such as high particle concentration leading to secondary scattering, is difficult.

In this work, signals from particles of diameter down to  $0.1 \mu\text{m}$  were investigated. For this particle size the scattered power, and thereby the SNR, is very small. In the following section the SNR of Doppler signals when using a photomultiplier (PM) is estimated theoretically as a function of scattered power and signal frequency. Conclusions are drawn relating to the use of lasers delivering either visible (green) or ultraviolet radiation. The former case includes the argon ion laser ( $\lambda = 514 \text{ nm}$ ) or the frequency doubled Nd:YAG laser ( $\lambda = 532 \text{ nm}$ ). The latter case includes the frequency doubled argon ion laser ( $\lambda = 257 \text{ nm}$ ) or the frequency quadrupled Nd:YAG laser ( $\lambda = 266 \text{ nm}$ ), although this last laser has not yet been realized for continuous wave operation.

### Theoretical SNR for photomultipliers

The SNR of a photomultiplier (PM) can be estimated by dividing the mean signal current  $\bar{i}_s$  by the square root of the sum of mean square shot noise  $\bar{i}_n^2$ , mean square dark count noise  $\bar{i}_d^2$ , and mean square thermal noise  $\bar{i}_T^2$ .

$$\text{SNR} = \frac{\bar{i}_s}{\sqrt{\bar{i}_n^2 + \bar{i}_d^2 + \bar{i}_T^2}} \quad (12)$$

The mean signal current  $\bar{i}_s$  produced by a Doppler burst is

$$\bar{i}_s = \frac{g_{PM}}{\sqrt{2}} V \bar{i}_c \quad (13)$$

$g_{PM}$ : gain of PM  
 $V$ : signal visibility.

The factor  $1/\sqrt{2}$  accounts for the fact that the Doppler signal is an oscillating signal. The mean photocathode current  $\bar{i}_c$  is given by

$$\bar{i}_c = \frac{P_s}{h\nu} \eta_Q e \quad (14)$$

$P_s$ : power scattered on to photocathode  
 $h$ : Planck's constant  
 $\nu$ : laser light frequency  
 $\eta_Q$ : quantum efficiency of photocathode  
 $e$ : electron charge

The term  $P_s \eta_Q / (h\nu)$  represents the photon detection rate, *i. e.* the number of photons detected per unit time. The mean square shot noise  $\bar{i}_n^2$  of a PM is given by

$$\bar{i}_n^2 = g_{PM}^2 \cdot 2e \cdot \bar{i}_c \Delta f \quad (15)$$

$\Delta f$  is the bandwidth of the electronics which is at least as large as the Doppler frequency  $\nu_D$  of the maximum velocity to be measured. Similarly, the mean dark count noise  $\bar{i}_d^2$  is given by

$$\bar{i}_d^2 = g_{PM}^2 \cdot 2e \cdot \dot{n}_d \Delta f \quad (16)$$

$\dot{n}_d$ : dark counts per second

The mean square thermal noise is given by

$$\bar{i}_T^2 = \frac{4k_B T}{R_L} \Delta f \quad (17)$$

where  $k_B$  is Boltzmann's constant,  $T$  the temperature, and  $R_L$  the load resistance. Putting eqs. (13) through (17) together, the SNR of a PM can be given as

$$\text{SNR} = \frac{g_{PM}}{\sqrt{2}} \frac{V P_s \eta_Q e}{h\nu} \left[ g_{PM}^2 2e \left( \frac{P_s \eta_Q e}{h\nu} + \dot{n}_d \right) \Delta f + \frac{4k_B T}{R_L} \Delta f \right]^{-1/2} \quad (18)$$

In eq. (18) the values of all quantities except  $P_s, \eta_Q, \nu$  and  $\Delta f$  are the same for green and UV light; the corresponding differences will be discussed in the following section. A final SNR estimate requires that the SNR due to quantum fluctuations in the scattering process is also taken into account.

### SNR comparison green-UV for 0.1 $\mu\text{m}$ particles

The main advantage of using UV light is the increase in the scattered power  $P_s$ . Figure 2 shows a logarithmic plot of scattered green and UV light versus particle size assuming equal light intensities in the probe volume. At 257 nm wavelength, the scattered power is about 16 times larger than at 514 nm up to  $d_p \simeq 0.5 \mu\text{m}$ .

While the assumption of equal light intensities is convenient to start with, it is unrealistic when comparing light scattering and detection of UV and green light in laser-Doppler anemometry, since the following influences have to be taken into account.

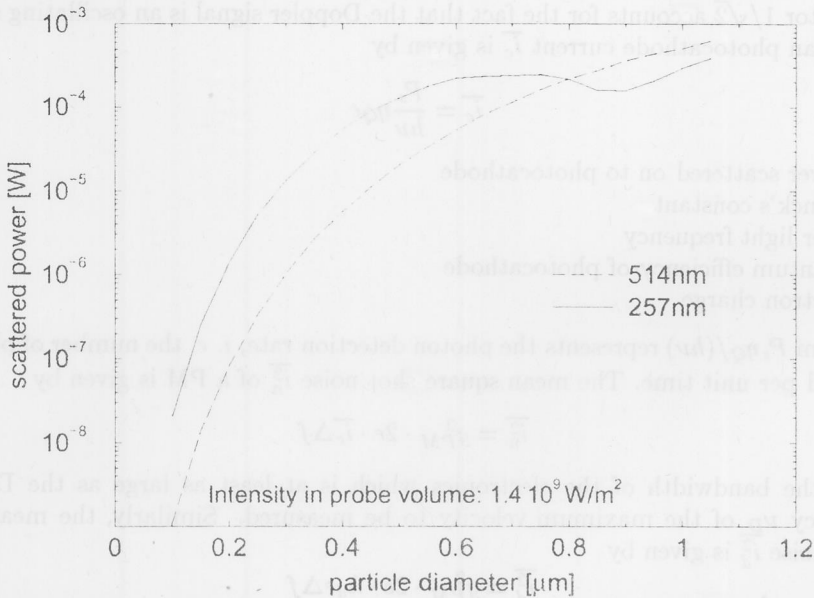


Figure 2. Scattered power versus particle size for the wavelengths 257 nm and 514 nm as calculated with Mie theory for identical optical parameters

1. Continuous wave (cw) lasers generally deliver less power when emitting ultraviolet rather than visible radiation. The UV laser source used for this work was an argon ion laser with a built-in frequency doubling crystal to convert radiation in the green line to UV. It produced about 10 times less output power at 257 nm than at 514 nm at equal plasma tube currents.
2. The interference fringe spacing  $\Delta x$  in the probe volume is given by

$$\Delta x = \frac{\lambda}{2 \sin \alpha} \quad (19)$$

Halving the wavelength thus halves the fringe spacing. Hence, the diameter of the probe volume can also be halved without reducing the number of fringes. At a particular laser output power  $P_0$ , the laser intensity in the probe volume  $I_0$  can, therefore, be increased by a factor four for a given number of interference fringes, when converting from green to UV light.

3. The quantum efficiency  $\eta_Q$  of photomultipliers optimized for UV light is generally 5 to 10% higher than that of the best photomultipliers for green light, according to the photocathode material.
4. The number of photons arriving at the detector per signal cycle is given by

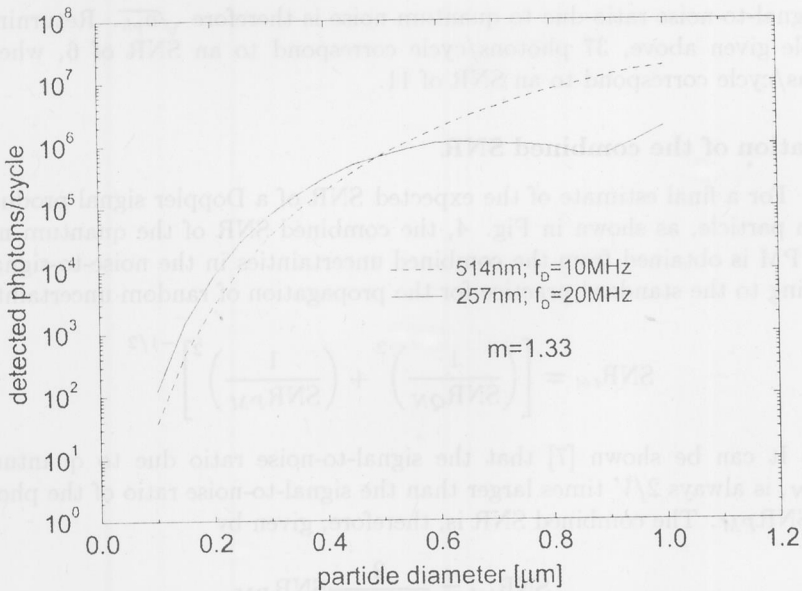
$$n_{ph} = \frac{P_s \eta_Q}{h\nu \nu_D} = \frac{P_s \lambda \eta_Q}{hc \nu_D} \quad (20)$$

Introducing the Doppler signal frequency  $\nu_D$  given by eq. (11), eq. (20) predicts the following dependence of  $n_{ph}$  on  $P_s$  and  $\lambda$ :

$$n_{ph} \propto \lambda^2 P_s \tag{21}$$

At equal scattered powers  $P_s$ , using laser light at a wavelength of 257 nm instead of 514 nm thus reduces the number of scattered photons by a factor four. The consequent reduction of the probe volume size, therefore, does not increase the number of photons/cycle. However, the SNR is increased, since  $SNR \propto (\Delta f)^{-1/2}$ , see eq. (18), for the PM and the connected electronics.

These four points are taken into account in Fig. 3 where the number of detected photons per signal cycle is plotted versus particle size.



**Figure 3.** Detected number of photons per Doppler signal cycle versus particle size at the wavelengths 257 nm and 514 nm (output power at 257 nm ten times less than at 514 nm)

Despite the reduction in the photon rate with the wavelength and the ten times lower power of the UV laser relative to the green laser, more photons are detected per signal cycle with UV light than with green light, for particles in the size range 0.1 to 0.5  $\mu\text{m}$ . In the example calculated for Fig. 3, there are 121 detected photons/cycle at 257 nm and only 37 at 514 nm for a 0.1  $\mu\text{m}$  particle.

### SNR due to quantum noise

Low numbers of photons per cycle lead to quantum noise, as the scattering process is statistical, *i. e.* the number of photons/cycle plotted in Fig. 3 is the expectation value. The quantum fluctuations can be quantified, since the standard deviation of the number of photons/cycle is approximately given by

$$\sigma_{ph} = \sqrt{n_{ph}} \quad (22)$$

The relative error, *i. e.* the noise-to-signal ratio, is given by

$$\frac{\sqrt{n_{ph}}}{n_{ph}} = \frac{1}{\sqrt{n_{ph}}} \quad (23)$$

The signal-to-noise ratio due to quantum noise is therefore  $\sqrt{n_{ph}}$ . Returning to the example given above, 37 photons/cycle correspond to an SNR of 6, whereas 121 photons/cycle correspond to an SNR of 11.

### Derivation of the combined SNR

For a final estimate of the expected SNR of a Doppler signal produced by a  $0.1 \mu\text{m}$  particle, as shown in Fig. 4, the combined SNR of the quantum noise and of the PM is obtained from the combined uncertainties in the noise-to-signal ratios, according to the standard practice for the propagation of random uncertainties:

$$\text{SNR}_{tot} = \left[ \left( \frac{1}{\text{SNR}_{QN}} \right)^2 + \left( \frac{1}{\text{SNR}_{PM}} \right)^2 \right]^{-1/2} \quad (24)$$

It can be shown [7] that the signal-to-noise ratio due to quantum noise,  $\text{SNR}_{QN}$ , is always  $2/V$  times larger than the signal-to-noise ratio of the photomultiplier,  $\text{SNR}_{PM}$ . The combined SNR is, therefore, given by

$$\text{SNR}_{tot} = \frac{2}{\sqrt{V^2 + 4}} \text{SNR}_{PM} \quad (25)$$

When  $\text{SNR}_{tot}$  is expressed in decibel (dB),  $20 \log_{10}(2/\sqrt{V^2 + 4})$  has to be added to the signal-to-noise ratio of the PM,  $\text{SNR}_{PM}$ , to obtain the total  $\text{SNR}_{tot}$ .

### Combined SNR comparison green-UV for $0.1 \mu\text{m}$ particles

With eq. (25) a comparison of the SNR (in dB) versus signal frequency for green and UV light is possible. This is shown in Fig. 4 for a  $0.1 \mu\text{m}$  latex particle in water, again assuming that the UV laser is a factor 10 times weaker than the green laser. This plot shows that the total SNR using UV light is about 9 to 12 dB higher than using green light, although the Doppler frequency is doubled when the wavelength is halved. As an example, the SNR using UV light at 20 MHz equals the SNR using green light at about 2 MHz. It is thus possible to measure ten times



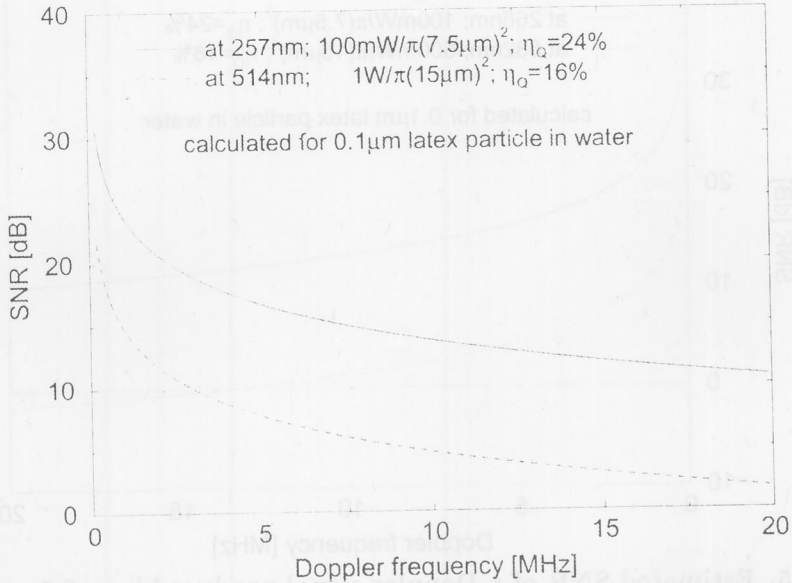


Figure 4. Estimated SNR of a Doppler signal produced by a 0.1  $\mu\text{m}$  latex particle in water versus signal frequency

higher frequencies using UV light or, according to eq. (11), five times higher velocities, without a loss in SNR.

Even greater advantages in using UV laser light can be expected by replacing the argon ion laser with a frequency doubled Nd:YAG laser which emits laser light at a wavelength of 532 nm. The conversion efficiency from 1064 nm to 532 nm is about 50 – 60%, resulting in an output power of about 500 mW. The Nd:YAG laser can also be twice frequency doubled producing a wavelength of 266 nm. The conversion efficiency from green to UV can be as high as 22% [5], which results in an output power of about 100 mW. The estimated SNR at 266 nm and 532 nm for a 0.1  $\mu\text{m}$  latex particle in water is shown in Fig. 5. The plot shows that whereas the SNR at 266 nm and 100 mW is approximately the same as for 257 nm, the SNR at 532 nm and 500 mW quickly drops to very low levels, since only about half as much output power is available at this wavelength compared to 514 nm.

### PROBE VOLUME SIZE

In the measurement of velocity in near-wall boundary layers at high Reynolds numbers with LDA particularly small probe volumes are advantageous. The minimum probe volume diameter at a given wavelength is, however, limited for two reasons:

1. A minimum number of interference fringes, typically at least 15 fringes, is necessary to ensure sufficiently accurate Doppler frequency measurements.

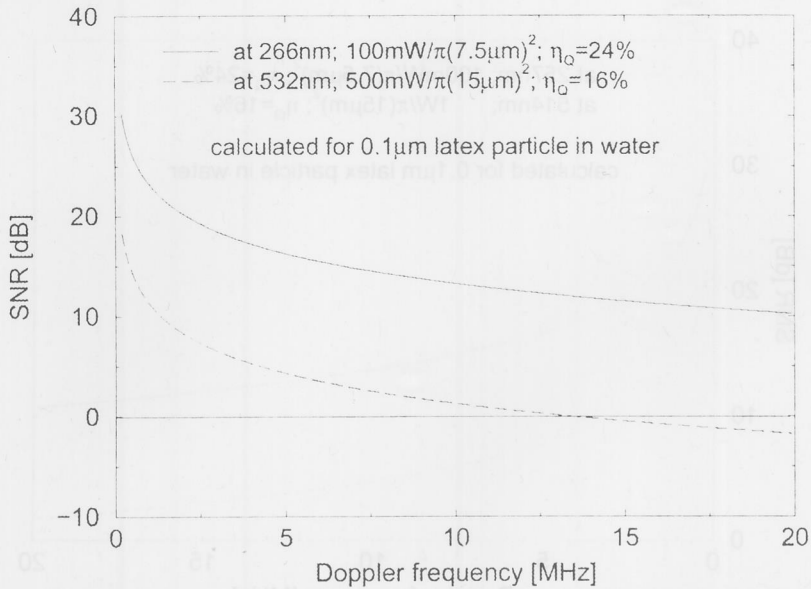


Figure 5. Estimated SNR of a Doppler signal produced by a  $0.1 \mu\text{m}$  latex particle in water versus signal frequency

2. The laser beams cannot be focused to the diffraction limit in LDA applications. The more strongly the beams are focused, the more rapidly they will expand after the focal point, thereby distorting the plane wave front necessary for constant fringe spacing throughout the probe volume and a consistent frequency measurement.

Within these constraints, reducing the wavelength by a factor of two from green to UV allows halving the probe volume diameter without a loss in frequency measurement accuracy.

### Near wall measurements

Near-wall LDA measurements are important for characterizing boundary layer flows and, in particular, for validating direct numerical simulations (DNS) of the turbulent quantities of a flow close to the wall. These quantities had been assumed to be Reynolds number independent when normalized with inner variables but direct numerical simulations at higher Reynolds numbers, which have become available in recent years, have revealed a Reynolds number dependence close to the wall for the time mean velocity, the rms values of turbulence velocity fluctuations, the Reynolds shear stresses, *etc.*

In measuring these quantities with LDA, the influence of the spatial integration due to the finite dimensions of the probe volume must be taken into consideration.

Because of the spatial distribution of the mean velocity and the moments of the turbulent velocity fluctuations, the measurements do not correspond to the values at the centre of the probe volume but rather to time and volume integrated information. Fischer *et al.* [3] derived the following correction equations for the normalized mean velocity and the mean square turbulence velocity fluctuations (Reynolds normal stress).

$$\overline{U_{cv}^+} \approx \overline{U^+}(y_c^+) + \frac{1}{8} \left( \frac{d_y^+}{2} \right)^2 \left( \frac{\partial^2 \overline{U^+}}{\partial y^{+2}} \right)_c \tag{26}$$

$$\overline{u_{cv}^{+2}} \approx \overline{u^{+2}}(x_c) + \frac{1}{8} \left( \frac{d_y^+}{2} \right)^2 \left[ 2 \left( \frac{\partial \overline{U^+}}{\partial y^+} \right)_c^2 + \left( \frac{\partial^2 \overline{u^{+2}}}{\partial y^{+2}} \right)_c \right] \tag{27}$$

Here the subscript *c* refers to a quantity measured at the centre of the probe volume and *cv* to a quantity averaged over the probe volume. The mean and fluctuating velocities,  $\overline{U^+}$  and  $\overline{u^+}$  respectively, are normalized by the shear velocity  $U_\tau$ . The coordinate  $y^+$  and the probe volume diameter  $d_y^+$  are normalized by the length scale  $\nu/U_\tau$ .

$$U_\tau = \sqrt{\tau_w/\rho}$$

$\nu$  = fluid kinematic viscosity

$\tau_w$  = wall shear stress

$\rho$  = fluid density

It is important in the present context that both correction terms are proportional to the square of the (normalized) probe volume diameter  $d_y^+$ .

According to an estimate in [2,3], the lower limit for the validity of the law of the wall [6], *i. e.* the Reynolds number independence of turbulence quantities, would be  $Re \simeq 35,000$ . To confirm this estimate, measurements at Reynolds numbers up to  $Re \simeq 40,000$  would be necessary. Since measurements at Reynolds numbers above 21,000 could not be carried out in the available flow channel (height 10.2 mm, width 18 mm) with the large probe volume diameter of a laser-Doppler anemometer with a HeNe laser, a laser-Doppler anemometer with a Nd:YAG laser ( $\lambda = 532$  nm) was set up to provide an effective probe volume diameter of only 30  $\mu\text{m}$  compared to 80  $\mu\text{m}$  of the HeNe laser based system. The smaller probe volume allowed measurements to be performed closer to the wall, as shown in Table 1, and hence permitted also the verification of the correction eqs. (26) and (27).

From Fig. 6, the measured wall limiting values of the turbulence level  $u'/\overline{U}$  (where  $u' = (\overline{u^2})^{1/2}$ ) is practically the same for both probe volume diameters. Reducing the LDA probe volume diameter with green and particularly UV radiation is a relatively low cost alternative to increasing the Reynolds number by building a larger turbulent flow field apparatus.

Table 1. Computation of wall-next measuring positions for the two different LDA setups and various Reynolds numbers, [2]

Laser	$Re_c$	$d_y$	$d_y^+$	$y_{min}^+$
HeNe (15mW)	3500	80 $\mu$ m	1.7	0.85
HeNe (15mW)	10000	80 $\mu$ m	4.3	2.15
HeNe (15mW)	21000	80 $\mu$ m	8.0	4.00
Nd:YAG (100mW)	10000	30 $\mu$ m	1.7	0.85
Nd:YAG (100mW)	21000	30 $\mu$ m	3.3	1.65

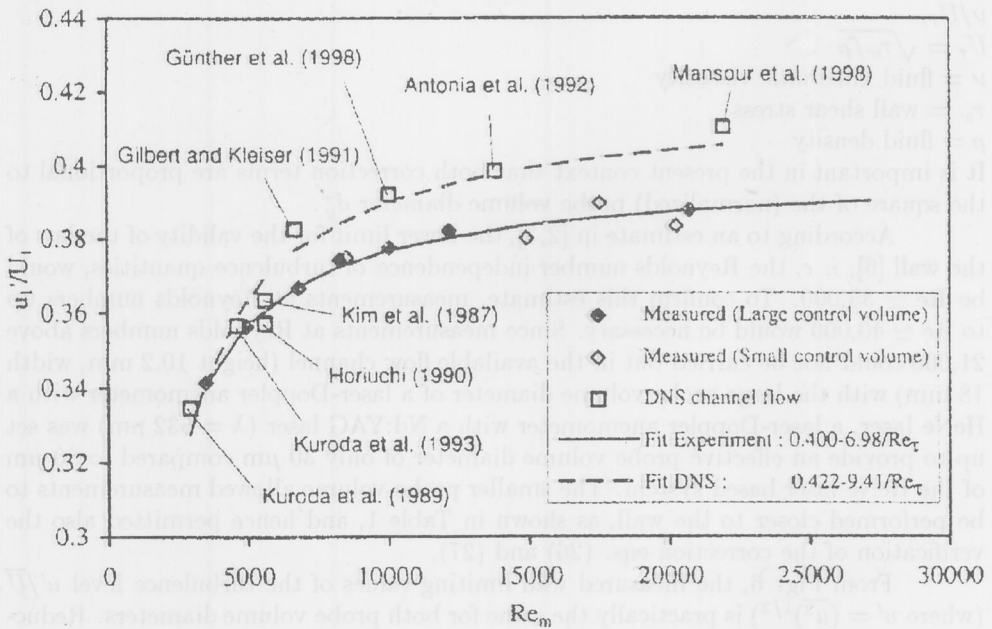


Figure 6. Plot of determined wall limiting values of the turbulence level  $u'/\bar{U}$ . Comparison with DNS-data, [3]

## CONCLUSIONS

This paper has demonstrated the advantages of using shorter wave length laser radiation both to increase the signal-to-noise ratio (SNR) and to minimize the probe volume dimensions of the laser-Doppler anemometer. Improvements are obtainable by reducing the wave length within the visible range of the electromagnetic spectrum, *e. g.* by changing from a HeNe laser (wavelength 633 nm) to an argon ion laser (wavelength 514 nm or 488 nm). More significant improvements are achievable by changing from visible to ultraviolet radiation. In the latter case, competing effects of reduced laser power and increased photomultiplier sensitivity in the ultraviolet spectral region compared with the visible range must be recognized.

These improvements are advantageous for wall boundary layer measurements with laser Doppler anemometry where a high spatial resolution is necessary because of the steep gradient of mean velocity. Through the increased detectability of small particles, it is also possible to enhance the data rate in the slowly moving fluid immediately adjacent to the wall.

## ACKNOWLEDGEMENTS

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