NUMERICAL STUDY ON CONVECTIVE DRYING OF REAL PARTICULATE MATERIALS IN A VIBRO-FLUIDIZED BED

by

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A mathematical model describing unsteady simultaneous one-dimensional heat and mass transfer between gas phase and wet particulate material dried in a vibro-fluidized bed is presented in this paper. Heat and mass transfer between gas phase and particle surface are calculated using well known expressions for the case of packed bed, while mixing of the particles is taken into account by means of the diffusion term in the differential equations, using an effective particle diffusion coefficient. Internal moisture transport is calculated on the basis of modified Luikov drying theory. A control volume numerical method is used for discretization of the coupled partial differential equations system. Results obtained on the basis of numerical simulation are compared successfully with corresponding experimental data using wet poppy seeds as the drying material.

Introduction

Digital computers and developed numerical mathematics are widely used for the purpose of solving heat and mass balance equations for chemical engineering processes. Contemporary research on various processes is mostly directed to modeling and numerical simulation. Results obtained on the basis of the reliable mathematical model and numerical analysis enable prediction of the relevant process parameters required for design of particular device and optimization of the process itself. A good model (numerical simulation) can successfully substitute lengthy and expensive experimental research.

Although drying processes are included in the computational environment of many commercial process simulators, the mathematical models used are simple and inappropriate for more detailed simulation. On the other hand, the expressions for thermophysical properties and transport phenomena constants are limited to commercially popular real materials.

During past years a lot of experimental and theoretical investigations directed to the convective drying processes of grained and particulate real (colloidal capillary-porous materials) in packed bed, fluidized bed and vibro-fluidized bed have been carried out in the Laboratory for Thermal Engineering and Energy. Some of the developed and successfully verified mathematical models describing drying process in packed and fluidized bed have been already reported (Stakić and Sijerčić, 1997, Stakić, 1997). The common idea of all the models is that drying process is calculated for the elementary bed of immobile particles. Particle mixing, occurring in the case of mobile beds (fluidized and vibro-fluidized), is taken into account through the diffusion term in the differential equations, using reported particle diffusion coefficients. The new approach in defining the basic drying equation, based on the modified Luikov drying theory, has enabled development of all mentioned models, being permanently innovated, improved and checked for new materials in order to obtain higher precision of calculated data.

Developed model and some of the obtained results on numerical simulation of particulate materials (poppy seed) drying in vibro-fluidized bed are shown in this paper. The new approach in drying equation definition, although discussed in some previous publications (Stakić, 1997), has been also explained.

Mathematical model

Convective drying process represents an unsteady problem of combined heat and mass transfer inside the bed of particulate solids forced by the fluid (drying agent) flow. Differential equation expressing the conservation of the general dependent variable ϕ for this problem can be written as follows:

$$\frac{\partial}{\partial \tau}(\rho\phi) + \mathrm{div}(\rho\vec{U}\phi) = \mathrm{div}(D_\phi \mathrm{grad}\phi) + S_\phi \tag{1}$$

while for the case of unsteady one-dimensional problem equation (1) has the following form:

$$\frac{\partial}{\partial \tau}(\rho\phi) + \frac{\partial}{\partial x}(\rho U\phi) = \frac{\partial}{\partial x}\left(D_\phi \frac{\partial \phi}{\partial x}\right) + S_\phi \tag{2}$$

The terms in both differential equations on the left side are the unsteady term and the convection term, while on the right side are diffusion term and the source term. All kind of the transport laws must be incorporated inside the source term, representing generation and dissipation of the variable ϕ. Forms of the variables D_ϕ and S_ϕ depend on the physical meaning of the variable ϕ. If the convective drying process is discussed, moisture content and temperature of the material being dried, as well as humidity and temperature of the drying agent used, are particular cases of the general dependent variable.

While developing the mathematical model describing unsteady simultaneous one-dimensional heat and mass transfer between gas phase and material during convective drying process in vibro-fluidized bed, following basic assumptions were used:

- each phase (gas phase, solid phase) is considered to be a quasi-homogeneous phase having effective transport coefficients;
- one-dimensional model is used (changes of parameters in vertical direction are discussed only, and other directions were neglected);
- at single moment of time, all particles have the same dimension, shape and density;
- diffusion (mixing) of particles is taken into account through the effective diffusion coefficient;
- heat and mass is transferred between solid phase and gas phase at the surface of the particles maintaining the state of thermodynamic equilibrium;
- mass transfer (moisture gradient) inside dried material is calculated using an original method based on the modified Luikov drying theory;
- heat transfer (temperature gradient) inside dried material is neglected due to small particle dimensions.

On the basis of the mentioned assumptions the following system of partial differential equations of heat and mass balance for the case of convective drying in vibro-fluidized bed can be defined:

– Conservation of gas humidity and enthalpy:

$$\rho_g V_g \left(\frac{\partial g}{\partial \tau} + U_g \frac{\partial g}{\partial x} \right) = S_M \tag{3}$$

$$\rho_g V_g C_g \left(\frac{\partial T_g}{\partial \tau} + U_g \frac{\partial T_g}{\partial x} \right) =$$

$$= h_{pg} a_p V (T_p - T_g) + h_w a_w V (T_w - T_g) + C_v (T_p - T_g) S_M \tag{4}$$

where $S_M = \rho_g k_{pg} a_p V_p (g_s - g)$ defines mass source, while $U_g = U_0 / \varepsilon$ is real gas velocity.

– Conservation of material moisture content and enthalpy:

$$\rho_{pd} V_p \left(\frac{\partial u}{\partial \tau} + U_p \frac{\partial u}{\partial x} \right) = \rho_{pd} V_p \frac{\partial}{\partial x} \left(D_{eff} \frac{\partial u}{\partial x} \right) - S_M \tag{5}$$

$$\rho_{pd} V_p C_p \left(\frac{\partial T_p}{\partial \tau} + U_p \frac{\partial T_p}{\partial x} \right) =$$

$$= V_p \frac{\partial}{\partial x} \left(\lambda_{eff} \frac{\partial T_p}{\partial x} \right) + h_{pg} a_p V (T_g - T_p) + h_w a_w V (T_w - T_p) - r S_M \tag{6}$$

– Continuity equation:

$$\frac{\partial \rho_g}{\partial \tau} + \frac{\partial}{\partial x}(\rho_g U_0) = S_M^* \tag{7}$$

Additional assumption is that $U_p = 0$, because there is no directed movement of the particles along the vertical axis inside the bed.

Analyzing the system of coupled partial differential equations (3–7) it can be seen that it is not completed, and that it has to be supplemented with the equation defining the rate of the drying process itself (drying equation). Nevertheless, an accurate definition of the drying rate for a certain convective drying process is not an easy assignment.

On the basis of an original approach defined by Stefanović and Nešić (1986), developed in order to determine the drying process of an arbitrary bed for any real material, transport phenomena inside and outside the solids are discussed separately because of the fact that during convective drying of any real material, two types of resistance exist due to:

- moisture transport from material interior to its surface,
- moisture convection from material surface into the drying agent.

At the same time, it is assumed that moisture transported from the material interior to its surface is transferred into the surrounding gas, permanently keeping particle surface and gas boundary layer close to that surface at the state of thermodynamic equilibrium.

Internal moisture transport, being a result of several different moisture transport mechanisms (Milojević and Stefanović, 1982), is very complex problem impossible to define analytically. On the basis of the simplified differential equation from the Luikov drying theory, drying rate for the case of moisture transport inside dried material can be generally expressed as:

$$-\frac{du}{d\tau} = K_i(u - u_{eq,s}) \tag{8}$$

where the internal moisture transport coefficient (K_i) can be defined on the basis of the material characteristics separately from the drying regime (drying agent velocity and temperature), and depends on material type, size and shape of the solids, material temperature, and depth of evaporation front. Analysis of the results obtained by the experimental drying process investigations for packed bed of different materials (Milojević and Stefanović, 1982) has shown that the internal moisture transport coefficient can be expressed in a form of an empirical function. It depends on the current mean moisture content and temperature of the material, the initial material moisture content and temperature, and condition of material surface:

$$K_i = A_k \left(\frac{u}{u_0}\right)^{a_1} \left(\frac{t_m}{t_{m0}}\right)^{a_2} \tag{9}$$

The coefficients in eq. (9) can be obtained for every real material using an original technique. This method is based on the experimental investigations of drying kinetics for an elementary packed bed of a given material (Stakić, 1993), assuming the bed of particles having linear and differentially small changes of parameters along the bed height as an elementary bed.

Drying rate for the case of external moisture transport (between gas boundary layer at material surface and surrounding gas) can be expressed as:

$$\frac{du}{d\tau} = k_{pg} a_p (g - g_s) \frac{\rho_p}{\rho_g} \tag{10}$$

The mass transfer coefficients between material and gas in packed bed (k_{pg}), are well known (investigated in details and generalized in the form of suitable empirical expressions – reported by large number of the authors) mainly for the case of two groups of materials:

(1) powdery and fine-grained materials (milk powder, poppy seeds, *etc.*):

$$\begin{gathered} \mathrm{Sh} = 2 + 1.8\mathrm{Sc}^{0.33}\,\mathrm{Re}_p^{0.5}, \quad \mathrm{Re}_p \geq 120 \\ \mathrm{Sh} = 0.012\mathrm{Sc}^{0.33}\mathrm{Re}_p^{1.625}, \quad \mathrm{Re}_p < 120 \end{gathered} \tag{11}$$

(2) grained and coarse-grained materials (corn grain with or without the corn-cob, coal, *etc.*):

$$\begin{gathered} \mathrm{Sh} = 0.977\mathrm{Sc}^{0.33}\,\mathrm{Re}_p^{0.595}, \quad \mathrm{Re}_p > 300 \\ \mathrm{Sh} = 1.83\mathrm{Sc}^{0.33}\,\mathrm{Re}_p^{0.485}, \quad \mathrm{Re}_p \leq 300 \end{gathered} \tag{12}$$

Gas humidity inside boundary layer (g_s) is possible to define from the state of equilibrium for a system formed by wet material contacted with humid gas, attained when partial pressure of water vapor at the material surface equalizes with partial pressure of water vapor in the surrounding gas. Due to the very complex mechanisms for moisture to material bonds, state of equilibrium for different materials and gases at certain temperature can be defined only by means of experimental methods, based on determining the moisture content of the material being in thermal and hydrological equilibrium with the surrounding gas:

$$u_{eq} = f(\varphi), \text{ at } T = \text{const., and } p = \text{const.} \tag{13}$$

Equation (13) is known as sorption and/or desorption (moisture) isotherm (depending on the manner of achieving the equilibrium). In order to simulate the drying process numerically it is necessary to generalize the experimental data for moisture isotherms in the form of empirical expression (Stakić, 1993):

$$1-\varphi_s = e^B,\ \ B = -B_0 u_{eq,s}^{B_1 T + B_2} \tag{14}$$

where:

$$\varphi_s = \frac{p_{at}}{p_s}\frac{g_s}{g_s+(1-g_s)\dfrac{M_v}{M_g}} \tag{15}$$

All the coefficients (B_0, B_1, and B_2) can be defined from the experimentally obtained equilibrium values $u_{eq,s}$ and φ_s for several corresponding temperatures T'.

The expressions for effective particle diffusion (mixing) coefficient in vibro-fluidized bed are taken from Gupta and Mujumdar (1980):

$$D_{eff} = 2.1\cdot 10^{-5}\left[\frac{A(2\pi f)^2}{g}\right]^{2.2} \tag{16}$$

and from Pakowski *et al.* (1984):

$$D_{eff} = 14.5\omega^2 R_a^{1.27} R_0^{1.588}\,\mathrm{Re}_v^{0.787}\,L_v^{1.176} \tag{17}$$

where:

$$R_a = \frac{A\omega^2}{g},\ \ R_0 = \frac{A\omega}{U_0},\ \ \mathrm{Re}_v = \frac{d_p A\omega^2\rho_g}{\mu_g},\ \ L_v = \frac{d_p}{A} \tag{18}$$

while:

$$\lambda_{eff} = D_{eff} C_p \rho_p \tag{19}$$

Particles and bed characteristics like particle equivalent diameter (d_p), particle specific surface (a_p), material specific heat capacity (C_p), material density (ρ_p), bed bulk density (ρ_b), bed porosity (ε), must be known and accurate enough. Those parameters are of great importance for heat and mass balance equations (ρ_p, C_p, a_p), for heat and mass transfer coefficients (d_p, a_p), and for momentum conservation equations (ε), and can be found from Stakić (1993). Dependencies defining the particle diameter change due to the moisture content change can be taken from Ginsburg and Savina (1982):

$$d_p = \left[\frac{6m_d}{\rho\pi}(1+u)\right]^{1/3} \tag{20}$$

but a simplified dependence is used in the model:

$$d_p = d_{pd}\left[\frac{\rho_{pd}}{\rho_p}(1+u)\right]^{1/3} \tag{21}$$

It is very important that using the parameters and dependencies obtained on the basis of the experiments performed in packed bed, the drying process in vibro-fluidized bed can be calculated with developed computer program. The basic value of developed mathematical model and the computer program is its universality. It is easily possible, by changing the very few of the data in the input files, to calculate the drying process in vibro-fluidized as well as in packed bed.

Numerical procedure

All the dependent variables of interest in the partial differential equations (3–7) obey a generalized conservation principle. Numerical procedure based on Patancar (1980) has been used to solve the partial differential equations. Each partial differential equation is discretized by means of the control-volume method (flow field is divided using vertical grid into a finite number of control volumes). Iterative line-by-line method is used to solve obtained linearized algebraic equations, using recurrence formula during calculation of variable's values for every line, following procedure for all the lines in one direction. This method is called the Thomas algorithm or the TDMA (TriDiagonal-Matrix Algorithm).

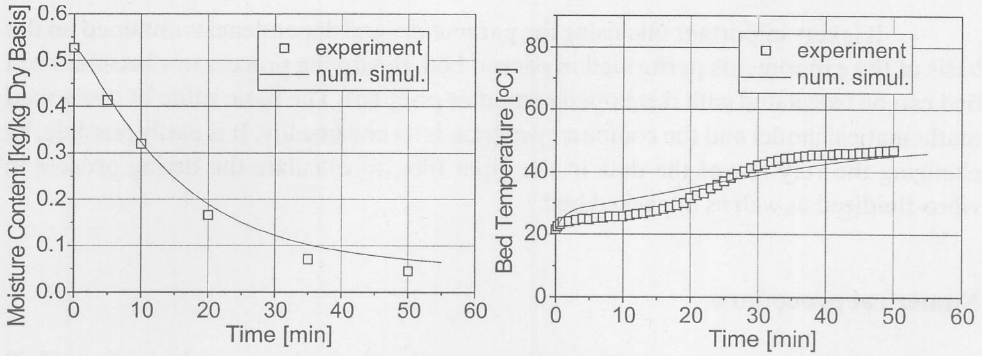
Results and discussion

Validation of described mathematical model was done on the basis of the available experimental data (Stakić, 1993). The data obtained by the numerical simulation on the basis of the model were compared with the data obtained by the experimental investigations of poppy seed drying in vibro-fluidized bed. According to the known procedure (Stakić, 1993), the coefficients in equation (9) and (14) were defined for the case of wet material used (poppy seed), obtaining the following values:

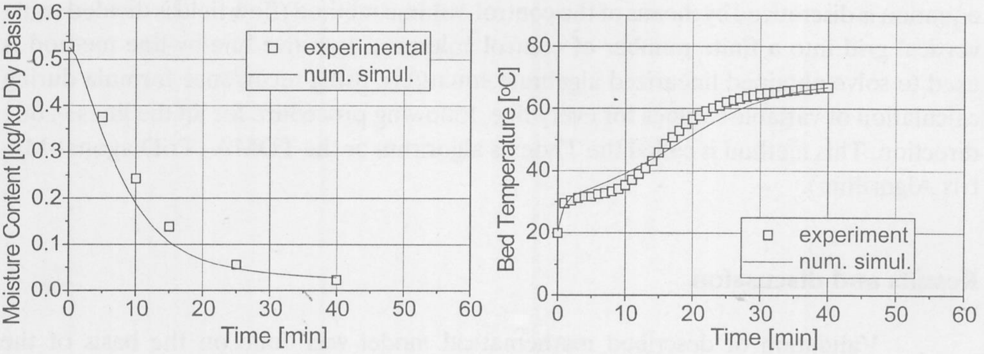
$$A_K = 0.28031 \cdot 10^{-3}, \quad a_1 = 0.9877, \quad a_2 = 3.1926 \tag{22}$$

$$B_0 = 14.82158, \quad B_1 = -3.02701, \quad B_2 = 1.948165 \tag{23}$$

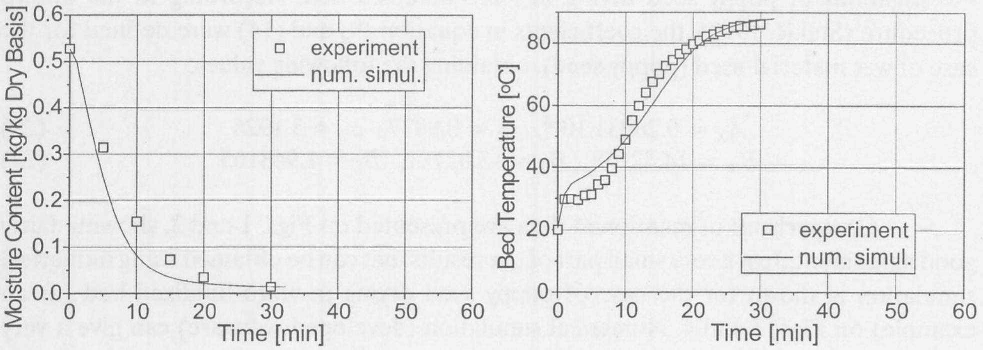
Comparisons of mentioned data are presented on Figs. 1 and 2, showing fairly good agreement. Just a very small part of the results that can be obtained using numerical simulation is shown for the case of poppy seed drying in vibro-fluidized bed (as an example) on Figs. 3 and 4. Numerical simulation (developed software) can give a very interesting and useful analysis of the drying process, showing the influence of the certain parameters on to the process, which can be clearly seen from the shown examples. This is in fact the main reason for making, developing and using mathematical modeling that is able to describe the behavior of the relevant parameters characterizing the different processes.



(a) $t_{a0} = 50.0$ °C, $x_0 = 0.011$ kg/kg, $U_{a0} = 0.185$ m/s, $t_{p0} = 21.5$ °C, $H_0 = 0.015$ m



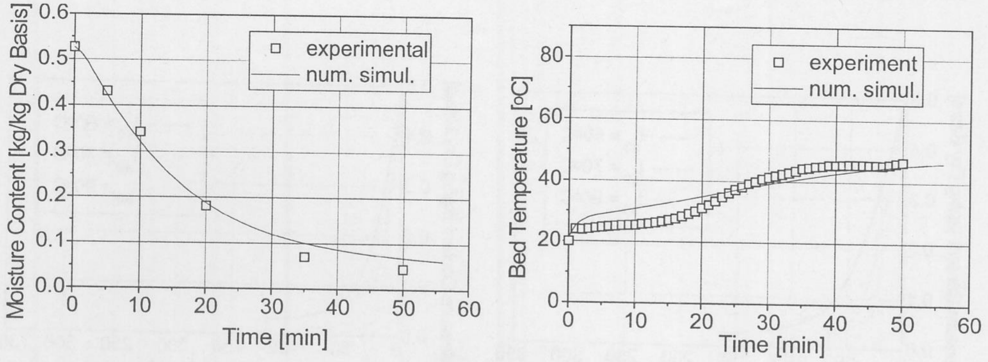
(b) $t_{a0} = 70.0$ °C, $x_0 = 0.012$ kg/kg, $U_{a0} = 0.197$ m/s, $t_{p0} = 20.5$ °C, $H_0 = 0.015$ m



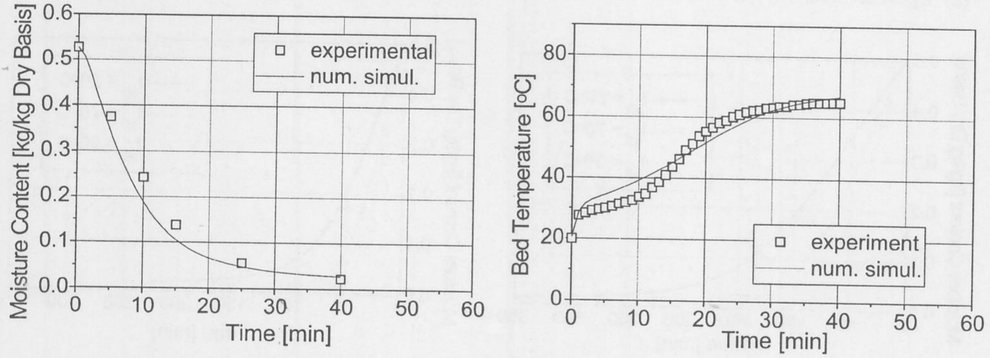
(c) $t_{a0} = 89.9$ °C, $x_0 = 0.012$ kg/kg, $U_{a0} = 0.210$ m/s, $t_{p0} = 20.5$ °C, $H_0 = 0.015$ m

Figure 1. Comparison of simulated and experimental drying kinetics for poppy seed drying in vibro-fluidized bed

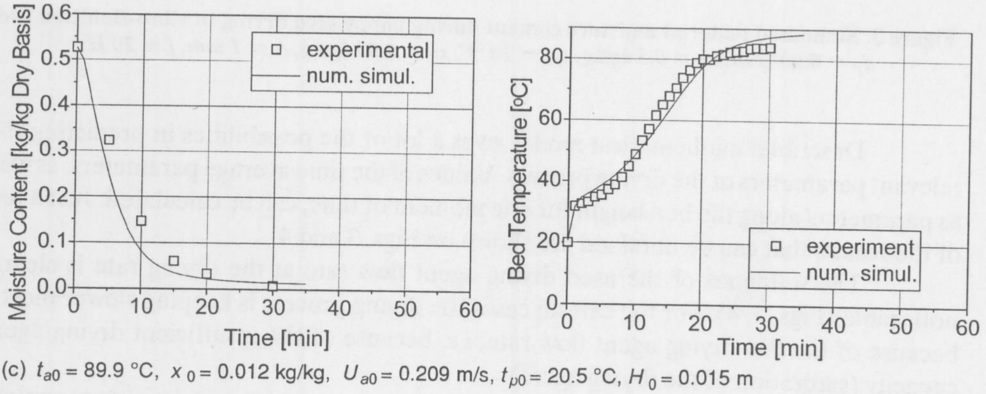
$d_p = 0.755$ mm, $u_0 = 0.527$ kg/kg, $A = 1.4$ mm, $f = 19.65$, Hz $\Gamma = 2.18$



(a) t_{a0} = 50.1 °C, x_0 = 0.011 kg/kg, U_{a0} = 0.185 m/s, t_{p0} = 21.4 °C, H_0 = 0.015 m



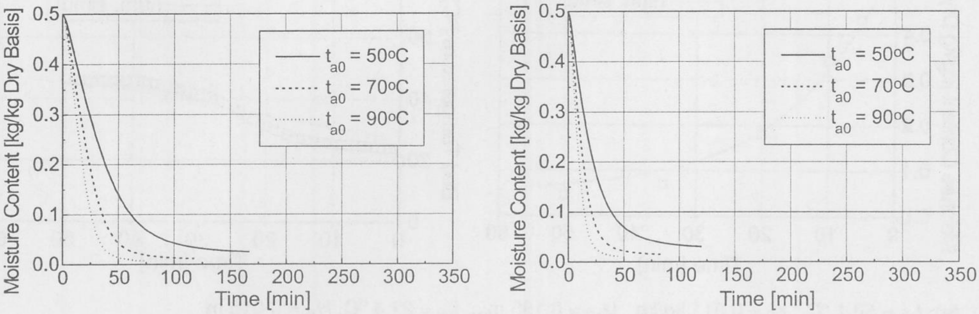
(b) t_{a0} = 70.1 °C, x_0 = 0.012 kg/kg, U_{a0} = 0.197 m/s, t_{p0} = 20.5 °C, H_0 = 0.015 m



(c) t_{a0} = 89.9 °C, x_0 = 0.012 kg/kg, U_{a0} = 0.209 m/s, t_{p0} = 20.5 °C, H_0 = 0.015 m

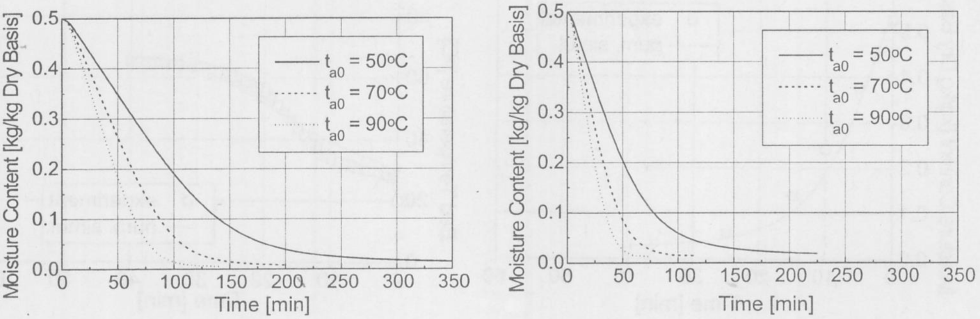
Figure 2. Comparison of simulated and experimental drying kinetics for poppy seed drying in vibro-fluidized bed

d_p = 0.755 mm, u_0 = 0.527 kg/kg, A = 4.3 mm, f = 9.65 Hz, Γ = 1.61



(a) $U_{a0} = 0.1$ m/s, $H_0 = 0.03$ m

(b) $U_{a0} = 0.2$ m/s, $H_0 = 0.03$ m



(c) $U_{a0} = 0.1$ m/s, $H_0 = 0.09$ m

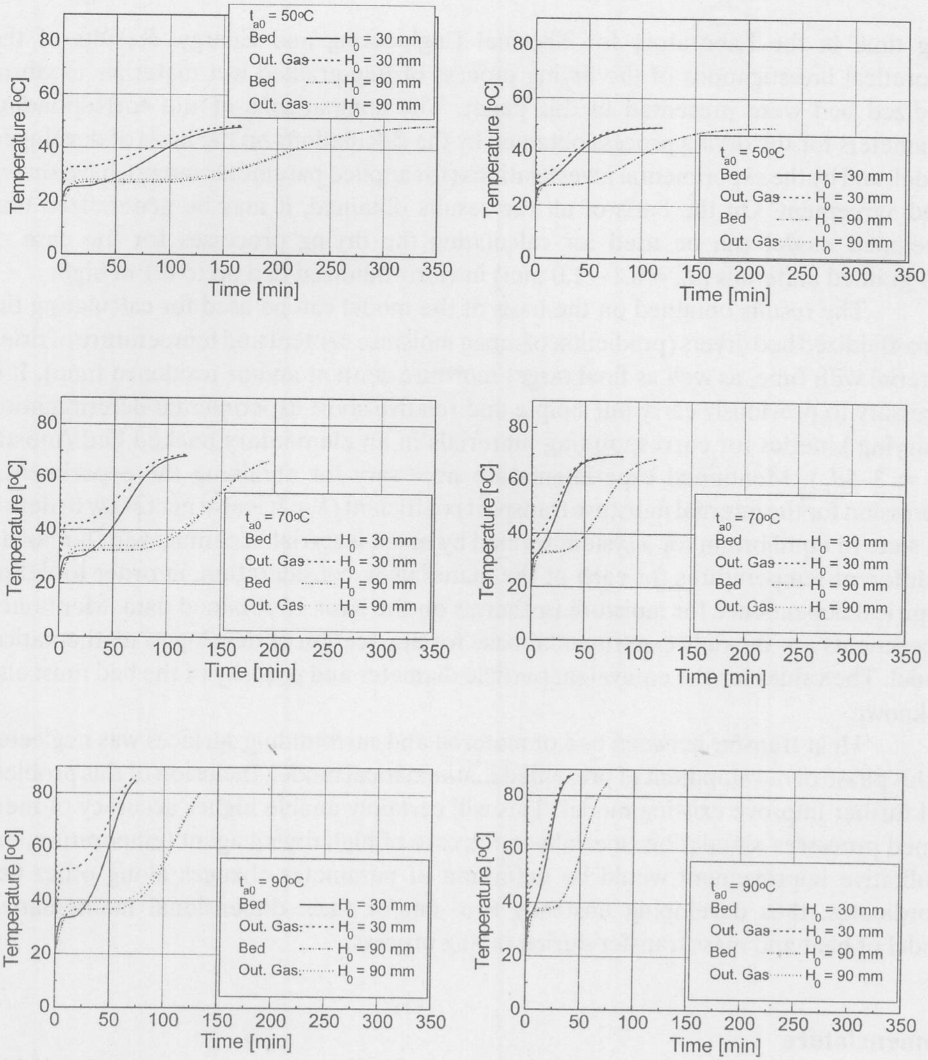
(d) $U_{a0} = 0.2$ m/s, $H_0 = 0.09$ m

Figure 3. Simulated material moisture content during poppy seed drying in vibro-fluidized bed
$d_p = 0.0057$ *m*, $u_0 = 0.5$ *kg/kg*, $t_{p0} = 20$ *°C*, $x_0 = 0.01$ *kg/kg*, $A = 1$ *mm*, $f = 20$ *Hz*

Described mathematical model gives a lot of the possibilities in predicting the relevant parameters of the drying process. Values of the time average parameters, as well as parameters along the bed height for one moment of time, can be calculated. Just a few of the results that can be obtained are shown on Figs. 3 and 4.

The influence of the used drying agent flow rate at the drying rate is clearly noticeable (Figs. 3, 4). For the certain cases the drying process is keeping slower mostly because of the low drying agent flow rate, *i.e.* because of the insufficient drying agent capacity (saturation of the drying agent).

The values of the material parameters such as temperature and moisture content are not changeable along the bed height for one moment of time, due to the mixing of the particles existing in vibro-fluidized bed.



(a) U_{a0} = 0.1 m/s (b) U_{a0} = 0.2 m/s

Figure 4. Comparison of simulated bed temperature and outlet gas (air) temperature during poppy seed drying in vibro-fluidized bed
$d_p = 0.0057$ m, $u_0 = 0.5$ kg/kg, $t_{p0} = 20$ °C, $x_0 = 0.01$ kg/kg, $A = 1$ mm, $f = 20$ Hz

Conclusions

Detailed experimental and theoretical investigations of drying the different real colloidal capillary-porous materials in vibro-fluidized bed have been carried out for a long time in the Laboratory for Thermal Engineering and Energy. Results of the theoretical investigations of the drying process of fine-grained wet materials in vibro-fluidized bed were presented in this paper. The comparisons of the corresponding parameters for the drying process obtained by the calculations on the basis of developed model, and by the experimental investigations (for applied parameter range), have shown good agreement. On the basis of all the results obtained, it may be generalized that developed model can be used for calculating the drying processes for the case of fine-grained materials ($d_p = 0.1 - 1.0$ mm) in vibro-fluidized bed up to 0.1 m high.

The results obtained on the basis of the model can be used for calculating the vibro-fluidized bed dryers (prediction of mean moisture content and temperature of dried material with time, as well as final target moisture content and/or residence time). It is necessary to previously carry out simple and relative short experimental determination of drying kinetics for corresponding materials in an elementary packed bed (mostly $H_0 = 3\text{–}5d_p$). Mentioned experiments are necessary for obtaining the experimental expression for the internal moisture transport coefficient (K_i). It is also necessary to define the state of equilibrium for a system formed by moist material in contact with humid air at different temperatures for each of the materials in consideration, in order to define empirical dependence for moisture isotherms on the basis of obtained data. Mentioned experiments are the only experimental base for application of developed mathematical model. The values for the equivalent particle diameter and porosity of the bed must also be known.

Heat transfer between bed of material and surrounding surfaces was neglected in this phase of development of presented mathematical model. Inclusion of this problem will further improve existing model. This will certainly enable higher accuracy of mentioned processes' simulation, specially in the case of high drying agent temperature, but qualitative improvement would be definition of parameter changes along other bed coordinates, thus developing unsteady two- and/or three-dimensional mathematical model of heat and mass transfer during drying process.

Nomenclature

a_p [m^2/m^3] – bed specific surface, $6(1-\varepsilon)/(\phi d_p)$
a_w [m^2/m^3] – wall specific surface, A_w/V
A [m] – amplitude of vibration
C [J/kgK] – specific heat capacity
d [m] – diameter
D [m^2/s] – mass diffusivity
f [Hz] – frequency of vibration
g [kg/kg] – gas humidity (wet basis)
h [W/m^{2}K] – heat transfer coefficient
H [m] – bed height
k, K [m/s] – mass transfer coefficient
r [J/kg] – latent heat of water phase change (evaporation)
t, T [°C, K] – temperature

u [kg/kg] – material moisture content (dry basis)
U [m/s] – velocity
V [m^3] – volume
x [kg/kg] – air humidity (dry basis)

Greek letters

Γ – intensity of vibration, $A(2\pi f)^2/g$
ε – void fraction (porosity)
λ [W/mK – thermal conductivity
ρ [kg/m^3] – density
τ [s] – time
ϕ – sphericity
φ – air relative humidity

Subscripts

a – air
at – atmosphere
b – bulk
d – dry
eff – effective
eq – equilibrium
g – gas
i – internal
m – material
p – particle
s – surface
v – vapor
w – wall
0 – initial

References

[1] Luikov, A. V., Drying Theory, Energy, Moscow, 1968, p. 471

[2] Patankar, S. V., Numerical Heat Transfer and Fluid Flow, Hemisphere P. C., New York, 1968, p. 197

[3] Gupta, R., Mujumdar, A. S, Aerodynamic and Thermal Characteristics of Vibrated Fluid Beds – A Review, *Drying 80,* (1980), 1, Hemisphere P. C., Montreal,

[4] Pakowski, Z., Mujumdar, A., Strumilo C., Theory and Application of Vibrated Beds and Vibrated Fluid Beds for Drying Processes, Advances in *Drying 3,* (1984), Hemisphere P. C., Montreal

[5] Milojević, D., Stefanović, M., Convective Drying of Thin and Deep Beds of Grain, *Chem. Eng. Comun., 13* (1982), 4–6, pp. 261–269

[6] Stefanović, M., Nešić, S., ITE Method for Determining Drying Kinetics of Solid Granular Materials, *Drying '86,* Hemisphere P. C., 1, 1986

[7] Ginzburg, A. S., Savina, I. M., Moist Transport Characteristics of Food Products, Handbook, Light and Food Industry, Moscow, 1982, p. 205,

[8] Stakić, M., Heat and Mass Transfer During Drying Process of Powdery and Fine-Grained Materials by Means of Vibro-Fluidization (in Serbian), PhD Thesis, Belgrade University, Faculty of Mechanical Engineering, Belgrade, 1993, p. 177

[9] Stakić, M., Sijerčić, M., Numerical Analysis of Coal Drying Process in a Fixed Bed, *Proceedings,* First European Congress on Chemical Engineering (ECCE-1) pp. 1145–1148, Florence, 1997

[10] Stakić, M., Mathematical Modeling and Simulation of Corn Drying in a Fixed Bed, *Proceedings*, Second International Symposium on Mathematical Modeling and Simulation in Agricultural and Bio-Industries, Budapest, 1997, pp. 119–124
[11] Stakić, M., Numerical Simulation of Heat and Mass Transfer During Particulate Solids Drying in a Fluidized Bed, *Proceedings*, Second Israel Conference for Conveying and Handling of Particulate Solids, Jerusalem, 1997, pp. 12.46–12.51
[12] Stakić , M., Numerical Simulation of Real Materials Convective Drying in Fixed Bed, *Thermal Science*, *1* (1997), 2, pp. 59–70

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