MATHEMATICAL MODELING OF HEAT TRANSFER FROM IMMERSED HEATED SURFACE TO PACKED BED

by

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In this paper, the results of the investigation of the steady forced convection in cylindrical porous bed heated electrically by heater placed in bed axis, are presented. The main aim of the investigation was to provide more data on the pressure, velocity and temperature distributions for flow situations characterized by velocities lower than minimum fluidization velocity. The cylindrical porous bed consisting of glass spheres, which is heated radially and symmetrically, has been taken as physical model. The boundary conditions of the second kind (q = const.) have been realized on the surface of heater.
By analyzing the forced convection phenomenon, the dominant mechanism for heat and momentum transport, have been observed. This was the basis for establishing the mathematical model.
The peculiarity of the mathematical modeling presented in this paper is that the porous bed has been treated as pseudohomogeneous medium (i.e., quasicontinuum). According to the above assumption, the basic transport equations have been obtained by using the method of volume averaging.
The proposed mathematical model was solved numerically by using the control volume method. In order to perform this numerical procedure, the original computer program has been constructed.
The obtained results of the applied prediction method for velocity and temperature distributions give a significant verification of: the noticed transport phenomena and its mathematical modeling, the chosen unambiguity conditions and the validity of the applied numerical procedure.

Introduction

Phenomena of fluid flow, heat and mass transfer in porous bed are present in a great number of technologies in :

– chemical industry (chemical and biochemical reactors, adsorbers, filtration devices, drying processes, oil exploitation and so on);

– energetics (regenerative heat exchangers, heat pipes, heat insulation, storage of thermal energy and so on);

– environmental protection (removal of toxic materials from soil and ground water, underground storage of nuclear waste materials and so on).

In recent years a considerable amount of investigations have been performed in this field. These investigations have been directed to : the study of porous bed structures; the analysis of the occurence and simultaneous effects of transport phenomena; the estimation of the influence of geometric, flow and thermal conditions on transport phenomena and to the study of the effective transport coefficients and its effect on velocity, temperature, pressure and concentration fields.

Physical model

As a basis for mathematical modeling of transport phenomena, the physical model of a reactor (chemical or biochemical) with packed glass sphere bed is taken (Fig. 1). The bed is bounded by cylindrical glass tube and it is heated radially and symetrically by electric heater placed in bed axis.

It can be noted that the choice of such a physical model of such a reactor does not diminish the generality of the established mathematical model. This mathematical model, with small modifications, can be used for prediction of velocity and temperature

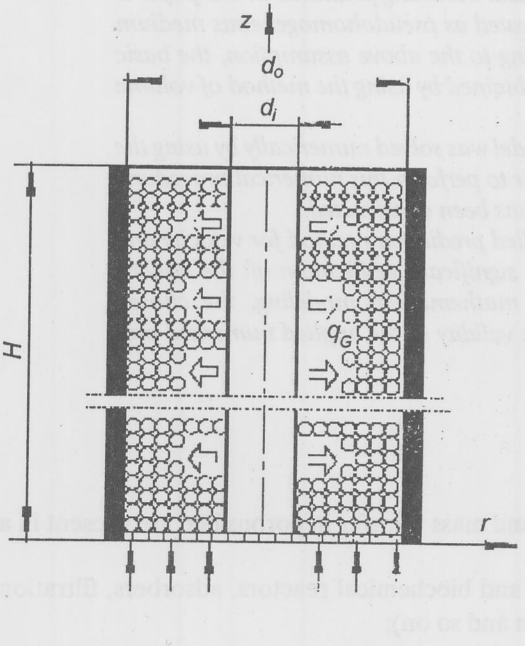

Figure 1. Physical model of reactor

fields in various apparatuses operating under similar flow, geometric and thermal conditions.

The numerical analysis results, evaluated from mathematical model, are compared with experimental data from reference [6]. In this reference, the experimental data on temperature and pressure drop measurements for forced convection of air through the above presented model of reactor, are given. The temperatures in the bed were measured at four different positions along radius and at three different axial positions (heights) for various flow rate of air and different specific heat fluxes.

Mathematical model

By analyzing the forced convection phenomenon, the dominant mechanisms for heat and momentum transport, have been observed. This was the basis for establishing the mathematical model consisting of: continuity equation, momentum equation and energy equation.

The peculiarity of the mathematical modeling presented in this paper is that the porous bed has been treated as pseudohomogeneous medium (*i. e.*, quasicontinuum). According to the above assumption, the basic transport equations have been obtained by using the method of volume averaging [5, 9].

In the analysis of the forced convection phenomenon and in its mathematical modeling the special attention is paid to non-Darcian effects: boundary, inertia, non-uniform porosity, hydrodynamic and thermal dispersion effects.

The basic assumptions included in mathematical model are as follows:

– steady fluid flow,

– solid phase and fluid are in local thermal equilibrium,

– fluid flow is hydrodynamically fully developed,

– hydrodynamic entry length is neglected [7],

– there is a bulk flow only in axial direction, $w_z = w_z(r)$, radial component of velocty and radial convection are neglected,

– pressure gradients exist only in axial direction,

– there is a variation of porosity only in radial direction, ($\varepsilon = \varepsilon(r)$, non-Darcian effect of non-uniform porosity),

– constant thermo-physical properties of fluid and solid phase,

– effective thermal conductivity K_{eff} (r, z) is function of porosity $\varepsilon(r)$, axial velocity $w_z(r)$, and temperature $T(r, z)$; non-Darcian effect of thermal dispersion is included by means of thermal dispersion conductivity K_d,

– conduction in axial direction is neglected,

– momentum equation after including all non-Darcian effects obtains the form of Darcy-Brinkman-Forcheimer equation.

Taking into account the above assumptions, the proposed mathematical model completed with the unambiguity conditions (geometric parameters of chosen system, thermophysical properties and effective transport coefficients, initial and boundary condition) obtain the following form:

continuity equation:

$$\frac{dw_z(r)}{dz}=0 \tag{1}$$

momentum equation:

$$\frac{dp}{dz}=-\frac{\mu_f}{K}w_z(r)-\rho_f\frac{F}{\sqrt{K}}w_z^2(r)+\frac{1}{r}\frac{d}{dr}\left[\mu_{eff}r\frac{dw_z(r)}{dr}\right] \tag{2}$$

where K and F are permeability and inertia coefficient, respectively, defined by Ergun-relation [1] in the form:

$$K=\frac{\varepsilon^3 d_p^2}{a(1-\varepsilon)^2} \quad \text{and} \quad F=\frac{b}{\sqrt{a}\varepsilon^{3/2}} \tag{3}$$

where d_p is sphere diameter.

Constants a and b have been empirically determined according to [1] as $a = 150$ and $b = 1.75$. The momentum equation (2) is the so-called Darcy-Brinkman-Forchheimer equation of an empirical nature. This form of momentum equation can be also obtained by using the volume averaging method on Navier-Stokes equations [2], if the left hand side of the obtained in such a way equation, under the condition of steady and hydrodynamicaly fully developed flow, can be neglected.

The only modification introduced in this paper is connected with Brinkman's term. There is a dilemma whether the momentum diffusion coefficient is equal to fluid viscosity μ_f, according to original Brinkman's equation [4], or it is equal to some effective value $\mu_{eff}=\mu_f/\varepsilon$. The value of μ_{eff} is the result of the application of the volume averaging method on Navier - Stokes equation. In this paper the transport coefficient μ_{eff} is under differential operator, due to both the nature of numerical method and the effect of non-uniform porosity. As an approximation for variable porosity, the empiric exponential function of Vortmeyer & Schuster [8] has been taken in the form:

$$\varepsilon=\varepsilon_\infty\left[1+C_1\exp\left(-N_1\frac{r_o-r}{d_p}\right)\right]\left[1+C_1\exp\left(-N_1\frac{r-r_i}{d_p}\right)\right] \tag{4}$$

where: ε_∞ – porosity in the bulk of bed; C_1, N_1 - empiric coefficients chosen according to [2, 6].

The boundary conditions for equation (2) are:

$$\text{for} \quad r=r_i, \quad 0\le z\le H, \quad w_z(r)=0 \tag{5}$$

$$\text{for} \quad r=r_o, \quad 0\le z\le H, \quad w_z(r)=0 \tag{6}$$

energy equation:

$$\rho_f c_{pf} w_z(r)\frac{\partial T(r,z)}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(K_{effr}(r)r\frac{\partial T(r,z)}{\partial r}\right) \tag{7}$$

where $K_{effr}(r)$ is effective radial thermal conductivity, which can be given in the form:

$$K_{effr}(r) = K_o(r) + K_d(r) \tag{8}$$

where:

– $K_o(r)$ is the stagnant thermal conductivity, which is a function of phase thermal conductivity and porosity.

It can be given according to [10] in the form:

$$K_o = k_f \left\{ 1 - \sqrt{1-\varepsilon} + \frac{2\sqrt{1-\varepsilon}}{1-\lambda B}\left[\frac{(1-\lambda)B}{(1-\lambda B)^2} \ln\left(\frac{1}{\lambda B}\right) - \frac{B+1}{2} - \frac{B-1}{1-\lambda B} \right] \right\} \tag{9}$$

where : $\lambda = \dfrac{k_f}{k_s}$, $B = 1.25\left(\dfrac{1-\varepsilon}{\varepsilon}\right)^{10/9}$.

– $K_d(r)$ is the transverse thermal dispersion conductivity, which according to [2], for $\mathrm{Re}_d \gg 10$, has the form:

$$K_d = D_i k_f \frac{1-\varepsilon}{\varepsilon} \mathrm{Pe}_m \frac{w_z(r)}{w_{zm}} \tag{10}$$

where – w_{zm} is the averaged velocity;

D_i – empirical constant chosen according to [2, 6],

$\mathrm{Pe}_m = w_{zm} \cdot d_p / a$ – Peclet number,

$a = k_f / (\rho_f c_{pf})$ – fluid thermal diffusivity.

Boundary conditions for equation (7) are:

$$\text{for} \quad z = 0, \quad r_i < r < r_o, \quad T = T_{ul} \tag{11}$$

$$\text{for} \quad r = r_i, \quad 0 < z < H, \quad \left(-K_{eff} \frac{\partial T}{\partial r}\right)_{r=r_i} = q_G \tag{12}$$

$$\text{for} \quad r = r_o, \quad 0 < z < H, \quad T = T_w \tag{13}$$

The mathematical model is closed by Ergun's relation giving the connection between the pressure drop in bed and the air flow rate:

$$-\frac{dp}{dz} = \frac{\mu_f}{K} w_{zm} + \rho_f \frac{F}{\sqrt{K}} w_{zm}^2 \tag{14}$$

The mathematical model established in such a way is the so-called WHC (wall heat conduction) type (according to Vortmeyer).

Numerical solution

The proposed mathematical model was solved numerically by using the control volume method [3]. In order to perform this numerical procedure, the original computer program in FORTRAN77 has been constructed. According to the assumption that thermo-physical properties of fluid are constant, firstly the equation (2) with boundary conditions (5) and (6) was solved numerically, and the obtained velocity profile $w_z(r)$ is used in numerical solution of energy equation (7).

Due to the form of source term (nonlinearity), the numerical solution of equation (2) demands the iteration procedure with the convergence condition. Solution of the discretization equations is carried out by using TDMA-procedure. The discretization step sizes can be chosen. The discretization of the convective term in equation (7) is carried out, according to the "upwind" scheme. Since the temperature field is two-dimensional, the solution of the discretization equations resulting from equations (7) has been carried out by using "line-by-line" method and TDMA-procedure.

Experimental investigation

The numerical analysis results, evaluated from mathematical model, are compared with experimental data. Detailed experimental measurement of pressure and temperature fields for different fluid flow rate and different specific heat flux verify the proposed mathematical model [6].

According to the proposed physical model a test rig was made, with the test section consisting of a cylindrical annular porous bed formed of glass spheres with 4.8 mm in diameter. The bed is heated radially and symmetrically from inner cylinder which is the body of electric heater. The boundary conditions of the second kind (q_w = const.) were achieved at the surface of the inner cylinder.

The forced air flow through the bed is in axial direction. During experimentation the volume air flow rate is varied within 5–19 m^3/h. For this flow rate interval the corresponding average velocity (Darcy's velocity) is within 0.4–12 1.4 m/s and Re-number within 90–350. The electric power of heater scaled to the length is varied within 40–200 W/m.

The temperature is measured at four radial positions (2, 6, 12, 20 mm from surface of heater) and at three axial positions along the height of bed (20, 100, 180 mm). The pressure drop is measured at five positions along the bed height and the obtained results are averaged.

The existence of nonuniform radial temperature profile dependent on flow and thermal conditions is stated.

Discussion and conclusions

Numerical solution of the momentum equation gives the radial non-uniform velocity profile. Velocity distributions, for different Re_d - numbers and for sphere diameters d_p = 4.8 mm, is shown in Fig. 2. From this figure one can conclude that the variation of Re_d-number has not a significant influence on velocity profiles . The influence of the diameter variation of the spheres forming the porous bed on the velocity profile shape for the same flow rate, is presented in Fig. 3. With increasing the sphere diameter the velocity "peak" is smaller and closer to the boundary wall at distances of about 0,1 d_p.

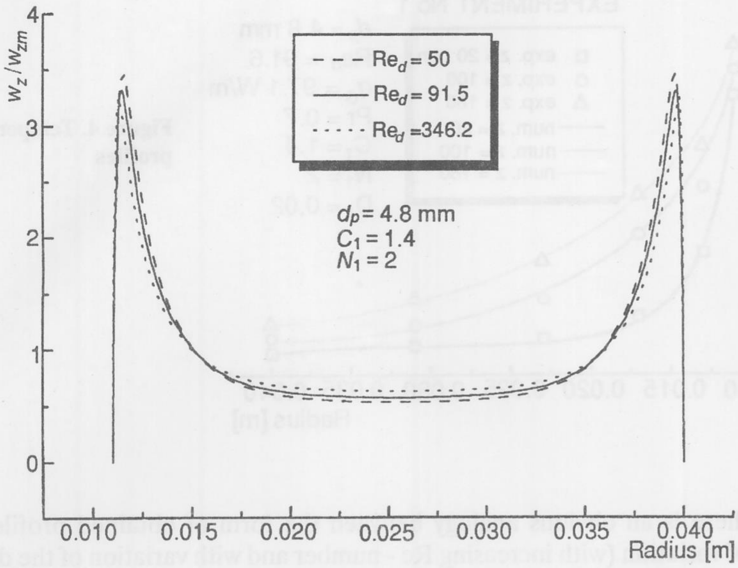

Figure 2. Velocity distributions for different Re_d-numbers

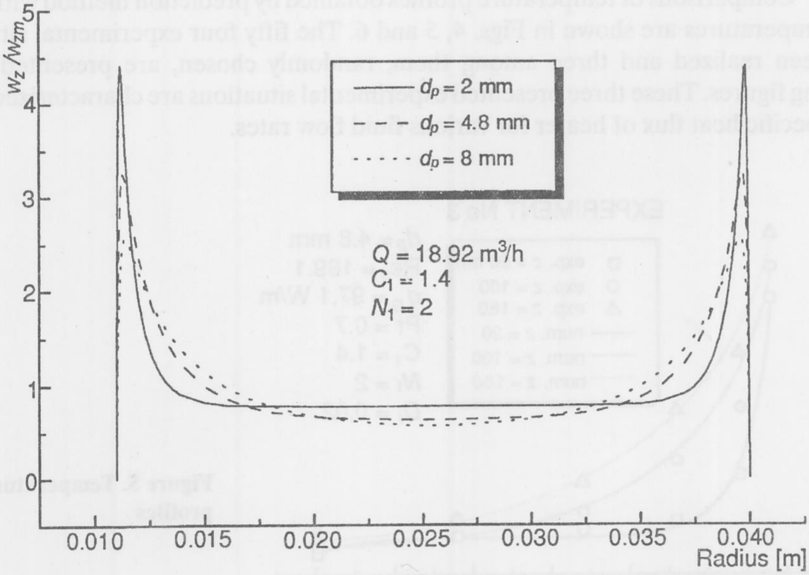

Figure 3. Velocity distributions for different diameter of spheres forming the bed

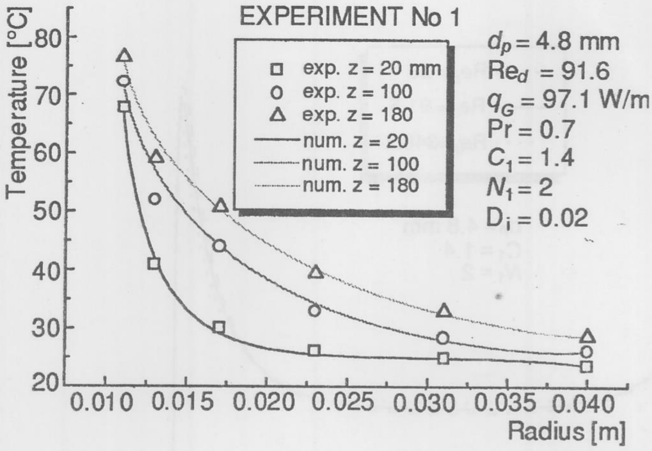

Figure 4. Temperature profiles

There is an obvious analogy between the form of obtained profiles and the tendency of variation (with increasing Re - number and with variation of the diameter of spheres forming the bed) and the results in references, where the velocity profiles were obtained by method of matched asymptotic expansions [2] or by variational method [8].

Numerical solution of energy equation gives a two-dimensional temperature distribution that is in good agreement with the experimental results cited in reference [6].

Comparisons of temperature profiles obtained by prediction method with measured temperatures are shown in Figs. 4, 5 and 6. The fifty four experimental situations have been realized and three among them, randomly chosen, are presented in the following figures. These three presented experimental situations are characterized by the same specific heat flux of heater for various fluid flow rates.

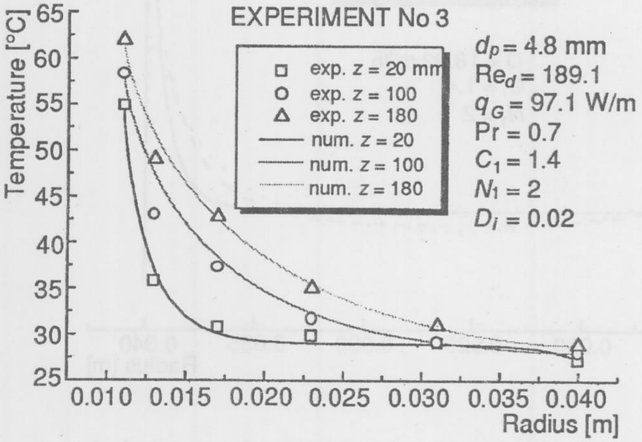

Figure 5. Temperature profiles

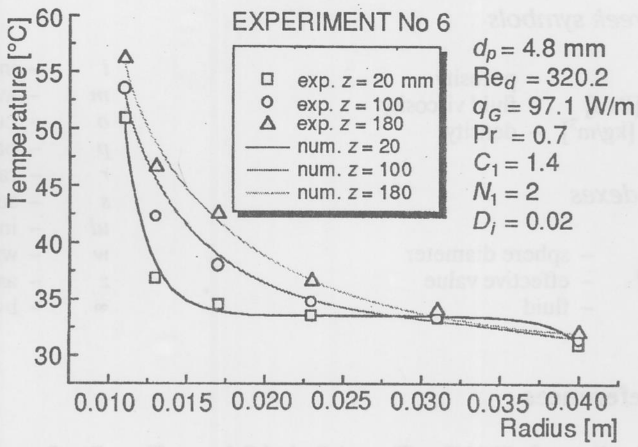

Figure 6. Temperature profiles

The obtained results show that the non-Darcian effects have a significant influence on the shape of velocity and temperature profiles especially in the vicinity of the impermeable solid convective boundary surface.

The presented mathematical model and its solving procedure, the computer program, can contribute to more correct solutions of different problems involving the forced convection in porous bed which are important for engineering practice.

Nomenclature

a [m^2/s]	–	fluid thermal diffusivity
a, b	–	constants defined by Ergun-relation
C_1, N_1	–	empiric coefficients
d [m]	–	diameter
d_p [m]	–	sphere diameter
D_i	–	empiric coefficient
F	–	inertia coefficient
H [m]	–	height of porous bed
K [m^2]	–	permeability
K_{eff} [W/(mK)]	–	effective thermal conductivity
K_o [W/(mK)]	–	stagnant thermal conductivity
K_d [W/(mK)]	–	transverse thermal dispersion conductivity
k [W/(mK)]	–	thermal conductivity
p [Pa]	–	pressure
Pe	–	Peclet number
q [W/m^2]	–	specific heat flux
r [m]	–	radial direction, radius
Re	–	Reynolds number
T [°C]	–	temperature
w_{zm} [m/s]	–	Darcy's velocity
w_z [m/s]	–	axial velocity
z	–	axial direction

Greek symbols

ε – porosity
μ [Pa·s] – fluid viscosity
ρ [kg/m^3] – density

Indexes

d – sphere diameter
eff – effective value
f – fluid
i – inner
m – averaged
o – outer
p – particle
r – radial direction
s – solid
ul – inlet,
w – wall
z – axial direction
∞ – bulk of bed

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