ENTHALPY EXCHANGE IRREVERSIBILITY

by

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Original scientific paper
UDK: 536.24:536.722=20
BIBLID: 0354–9836, *1* (1997), 1, 63–72

Some new cognizances initiated through the estimation of enthalpy exchange irreversibility levels that utilizes a novel concept of enthalpy exchange irreversibility norm are presented. A gas-to-gas system is analyzed.

Introduction

Efficiency analyses of any engineering system (from a complete power plant to a particular technical component, say a heat transfer device) ultimately start with basic balance relations that follow from the conservation laws. Energy and material balances in thermal engineering are founded on the First Law of thermodynamics and the mass conservation statement. If the analysis includes the Second Law of thermodynamics as well, then the efficiency of a system regarded is not just quantitated through the energetic attributes, but also through rather qualitative efficiency estimates usually expressed by the inherent thermodynamic irreversibility of the process in question.

In the field of practically all thermal engineering applications the process familiarly known as heath "transfer" and especially "heat exchange" as an overall process, are of particular importance. Both these are related to the energy degradation due to finite temperature differences, and the latter should be more correctly termed the "enthalpy exchange".

Heat exchangers are typically the devices in which enthalpy exchange process among two (or more) fluids finds its major importance. A classical task of ability to recognize, evaluate and reduce the irreversibility* for such a regular irreversible process, comes as natural to a competent engineering thermodynamist. The purpose of this paper is to present an analysis of the estimates of enthalpy exchange irreversibilities in heat exchangers. Just the contribution of finite temperature differences to the overall irreversibility is considered. Fluid friction as the source of irreversibility is not included in the present analysis.

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* We use the term "irreversibility" for the quantity $T_0\Delta S$ in the same sense as it is defined in: Van Wylen, G. J., Sonntag, R. E., Fundamentals of Classical Thermodynamics, 2nd ed., John Wiley & Sons, Inc., New York, 1973

While the analysis of the irreversibility of thermodynamic processes in general, has its long a prolific background, starting from the first significant work by Gouy [1] to the latest monographs [2, 3] dedicated exclusively to this question, the Second Law study of heat exchangers is just a topic of recent researches [4, 5, 6, 7, 8]. The discussion of some cognizances presented in this text has been impelled by a desire for development of a systematic approach to the analysis of irreversibilities in neither just a single class of heat exchangers nor some particular cases as in [4, 5]. Instead an approach that empowers a systematic comparison of various heat exchanger flow arrangements from the standpoint of enthalpy exchange irreversibility is ment. Here, just the avenue to this concept is given, while a complex second law heat exchanger analysis is a subject of a report [9] that will be published elsewhere.

The concept of enthalpy exchange irreversibility norm

Let us consider atypical open system which does not exchange work with the surrounding and for which the contributions of kinetic and potential energy are negligible. In order to establish a quantitative statement for the irreversibility of such a system (the enthalpy exchange being the sole generator of irreversibility), and this to obtain an insight into the governing arguments, one should start from its thermodynamic portrait given by the following sketch (see Fig. 1 as well):

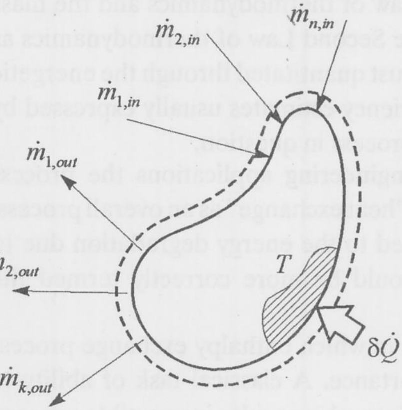


Figure 1. Typical open system

MASS BALANCE:

$$\sum_{i=1}^{n} \dot m_{i,in} - \sum_{j=1}^{k} \dot m_{j,out} = \frac{\partial m}{\partial \tau} \qquad (1)$$

ENERGY BALANCE:

$$\sum_{i=1}^{n} (\dot m \cdot h)_{i,in} - \sum_{j=1}^{k} (\dot m \cdot h)_{i,out} + \dot Q = \frac{\partial e}{\partial \tau} \qquad (2)$$

ENTROPY IMBALANCE:

$$\sum_{i=1}^{n} (\dot m \cdot s)_{i,in} - \sum_{j=1}^{k} (\dot m \cdot s)_{j,out} + \int \frac{\partial Q}{T} \le \frac{\partial s}{\partial \tau} \qquad (3)$$

For an adiabatic system in a steady state Eqs. (1–3) reduce to

$$\sum_{i=1}^{n} \dot m_{i,in} = \sum_{j=1}^{k} \dot m_{j,out} \qquad (4)$$

$$\sum_{i=1}^{n}(\dot{m}\cdot h)_{i,in}=\sum_{j=1}^{k}(\dot{m}\cdot h)_{j,out} \tag{5}$$

$$\Delta\dot{S}=\sum_{j=1}^{k}(\dot{m}\cdot s)_{j,out}-\sum_{i=1}^{n}(\dot{m}\cdot s)_{i,in}\geq 0 \tag{6}$$

The simplest situation of enthalpy-exchange among two streams can be taken as a representative for the description of a two fluid heat exchanger of arbitrary flow arrangement (Fig. 2).

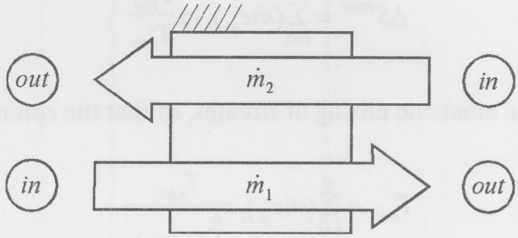


Figure 2. Two fluid heat exchanger scheme

The entropy production of such a system is simply:

$$\Delta\dot{S}=\dot{m}_1(s_{1,out}-s_{1,in})+\dot{m}_2(s_{2,out}-s_{2,in}) \tag{7}$$

If we restrict our consideration (for the sake of simplicity and obviousness of the analysis) to the enthalpy exchange of gaseous media within the perfect gas approximations, then the quantitative statement of enthalpy exchange irreversibility (EEI) is

$$N_{S,\Delta h}=\omega\ln[1-\varepsilon(1-\theta^{-1})]+\ln[1-\omega\varepsilon(1-\theta)] \tag{8}$$

where $\varepsilon = (T_{1,in} - T_{1,out})/(T_{1,in} - T_{2,in}) = \varepsilon(NTU, \omega, \text{flow arrangement})$ is the effectiveness of the enthalpy exchange among streams 1 and 2, $\omega=(\dot{m}c_p)_1/(\dot{m}c_p)_2$ is the fluids capacity rate ratio, $\theta = T_{1,in}/T_{2,in}$ – fluid inlet temperatures ratio, and $N_{S,\Delta h}=T_0\Delta\dot{S}/[T_0(\dot{m}c_p)_2]$ is the dimensionless measure of EEI.

A useful quantity termed the "enthalpy exchange irreversibility norm" (EEIN) can be defined as

$$\boldsymbol{N}_{S,\Delta h}=N_{S,\Delta h}/N_{S,\Delta h}^{\max} \tag{9}$$

where $N_{S,\Delta h}^{max}$ is the largest possible EEI. Under the given circumstances (same values of θ and ω) this will correspond to the entropy production in an adiabatic mixing of the incoming streams. Normalization as in Eq. (9) is suitable for unified comparison of various EEI's since all EEIN values lie between 0 and 1.

Some properties of EEIN

The denominator in Eq. (9) has its origin in the change of entropy

$$\Delta\dot{S}^{max} = \sum_{i=1}^{n}(\dot{m}c_p)_i \ln\frac{T_{out}}{T_{i,in}} \tag{10}$$

that correspond to the adiabatic mixing of streams, so that the common outlet temperature is

$$T_{out} = \sum_{i=1}^{n}(\dot{m}c_p)_i \frac{T_{i,in}}{\sum_{i=1}^{n}(\dot{m}c_p)_i} \tag{11}$$

as follows from the enthalpy balance, Eq. (5).

For the system involving just two streams, as in Fig. 3, these two relations yields $\Delta\dot{S}^{max}$ that can be nondimensionalized to give

$$N_{S,\Delta h}^{max} = \frac{T_0\Delta\dot{S}^{max}}{T_0(\dot{m}c_p)_2} = \omega\ln\frac{\omega\theta+1}{(\omega+1)\theta} + \ln\frac{\omega\theta+1}{\omega+1} \tag{12}$$

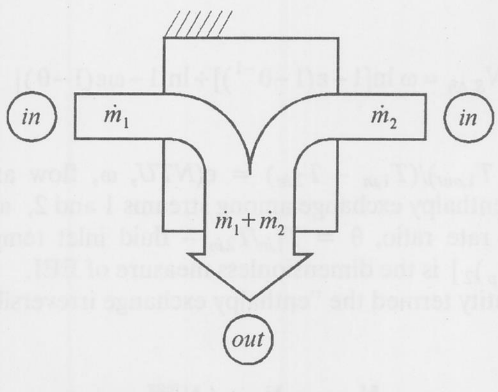


Figure 3. Two fluid adiabatic mixing scheme

This maximal dimensionless EEI depend just on the parameters at the system entrances and exists for $\omega > 0$ and $\theta \neq 1$, excluding, thus, the enthalpy exchange with phase changes and same inlet temperatures, respectively.

The behavior of EEIN, Eq. (9), when regarded as a function of ε for given ω and θ, is hence, exactly the same as that of its numerator:

$$N_{S,\Delta h} = N_{S,\Delta h}(\varepsilon, \omega, \theta) \tag{13}$$

It is easy to verify the following properties of EEI function $N_{S,\Delta h}$. As ε tends to zero, *i.e.* there is no enthalpy exchange, $N_{S,\Delta h}$ tends to zero as well. As $\varepsilon \to 1$, which correspond to $NTU \to \infty$ in some heat exchanger flow arrangements, $N_{S,\Delta h}$ tends to a certain fixed, usually minimal, value $N_{S,\Delta h}^{f}$ for given inlet parameters ω and θ. For the special value of effectiveness

$$\varepsilon = \varepsilon^* = \frac{1}{1+\omega} \tag{14}$$

EEI function $N_{S,\Delta h}$ reaches its maximum which is exactly the same $N_{S,\Delta h}^{\max}$ as given by Eq. (12). This is the case when both fluids are leaving the exchanger at the same temperature.

In general case the surface where

$$\frac{\partial N_{S,\Delta h}}{\partial \varepsilon} = 0 \quad \text{and} \quad \frac{\partial^2 N_{S,\Delta h}}{\partial \varepsilon^2} < 0 \tag{15}$$

determines $N_{S,\Delta h}^{\max}$ as in Eq. (12) and ε^* as given by Eq. (14) and separates the region with

$$\frac{\partial N_{S,\Delta h}}{\partial \varepsilon} > 0 \quad \text{for} \quad 0 \leq \varepsilon < \frac{1}{1+\omega} \tag{16}$$

from that with

$$\frac{\partial N_{S,\Delta h}}{\partial \varepsilon} < 0 \quad \text{for} \quad \frac{1}{1+\omega} < \varepsilon < 1 \tag{17}$$

The unidirectional flow of streams through the exchanger is an exeption of these properties and will be discussed in the next section, however the above analysis clearly shows the EEIN dependence on effectiveness as presented in Fig. 4.

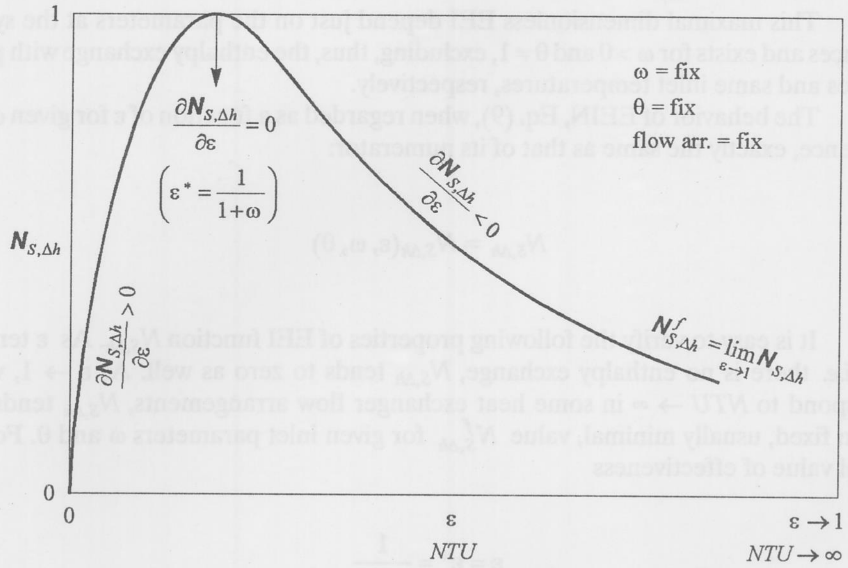


Figure 4. EEIN vs. effectiveness

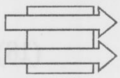
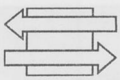
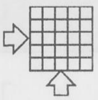
Since the exchanger effectiveness is strongly dependent on the type of flow arrangement one might immediately conclude that the way of conducting the streams through the exchanger might play a selective and far-reaching role in EEIN based study of certain flow arrangements capability. This will be especially pronounced in the region $1/(1+\omega) < \varepsilon \leq 1$. Numerous implications of this fact are discussed in [9].

Here we just note that the unbalance factor ε moves the location of EEIN maximum from $\varepsilon^* = 1/2$ for $\omega = 1$ to $\varepsilon^* = 1$ for $\omega = 0$. In the case of balanced streams the maximal values of $N_{S,\Delta h}$ is the largest, while it decreases as ε decreases until a complete disappearance for $\omega \to 0$ which is, of course, beyond the limits of the basic assumptions as well.

Excluding the senseless case $\theta = 1$ one can conclude that both $\theta \ll 1$ and $\theta \gg 1$ symmetrically increase $N_{S,\Delta h}$ for $\omega = 1$. This is a natural consequence of the EEI irrelevance with respect to the actual inlet temperature levels ($T_{1,in} > T_{2,in}$ or $T_{1,in} < T_{2,in}$).

Three particular cases

In this section we discuss the EEIN in three particular flow arrangements: co-current, counter-current and crossflow with neither fluid mixed. These are the basic heat exchanger flow arrangements. Peculiarities of enthalpy exchange conditions in these cases result in different effectiveness relationships:

Flow arrangement	$\varepsilon-\omega-NTU$	$\lim\limits_{NTU\to\infty}\varepsilon$
	$\varepsilon=\dfrac{1-e^{-NTU(1+\omega)}}{1+\omega}$	$\dfrac{1}{1+\omega}$
	$\varepsilon=\dfrac{1-e^{-NTU(1-\omega)}}{1-\omega e^{-NTU(1-\omega)}}$	1
	$\varepsilon=1-\dfrac{e^{-NTU(1+\omega)}}{\omega NTU}\sum\limits_{n=1}^{\omega} n\omega^{\frac{n}{2}}\ln(2NTU\omega)$	1

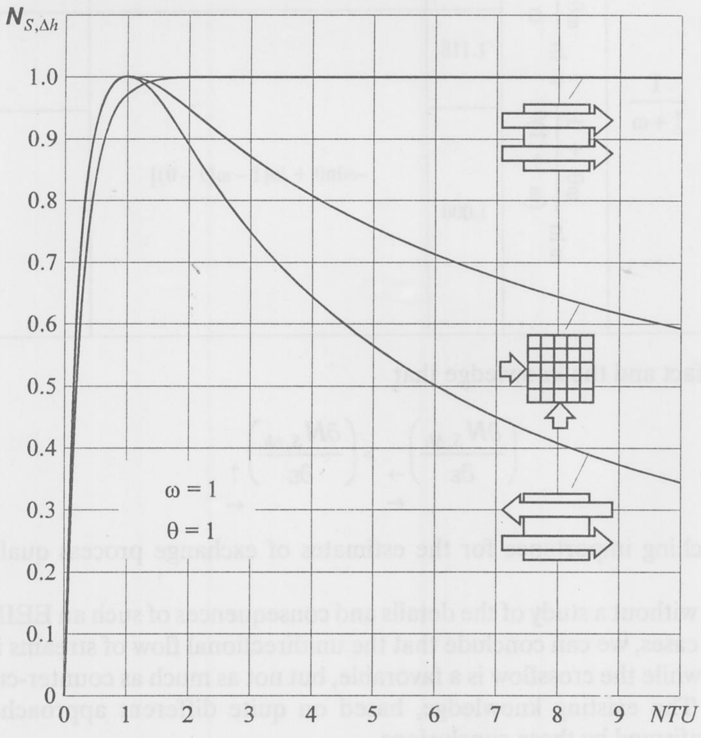
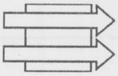
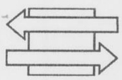
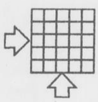


Figure 5. EEIN vs. effectiveness for particular flow arrangements

Combining these relations with Eqs. (8–9) one can evaluate EEIN for each case. Fig. 5 shows the calculated results for balanced streams ($\omega = 1$) and $\theta = 0.05$, while some striking data for these three situations are set in Table 1. The course of the EEIN curve for co-current flow of steams is immediately noticeable as the one without the region of negative slope $\partial \boldsymbol{N}_{S,\Delta h}/\partial\varepsilon < 0$. This is a consequence of

$$\lim_{NTU\to\infty} \varepsilon(NTU,\omega,\substack{\to\\ \to}) = \frac{1}{1-\omega} \tag{18}$$

Table 1.

Flow arrangemen	ε^*	$N_{S,\Delta h}^{\max}$	NTU^f	$N_{S,\Delta h}^{f}$, $\omega \neq 1$	$N_{S,\Delta h}^{f}$, $\omega = 1$
	$\frac{1}{1+\omega}$	$\omega \ln \frac{\omega\theta+1}{(\omega+1)\theta} + \ln \frac{\omega\theta+1}{\omega+1}$	$+\infty$	$\omega \ln\left[1-\frac{1-\theta^{-1}}{1+\omega}\right] + \ln\left[1-\omega\frac{1-\theta}{1+\omega}\right]$	$\ln\left[1+\frac{1}{4}(\theta+\frac{1}{\theta}-2)\right]$
			1.118	$-\omega\ln\theta + \ln[1-\omega(1-\theta)]$	0
			1.000		0

This fact and the knowledge that

$$\left(\frac{\partial \boldsymbol{N}_{S,\Delta h}}{\partial\varepsilon}\right)_{\substack{\to\\ \leftarrow}} > \left(\frac{\partial \boldsymbol{N}_{S,\Delta h}}{\partial\varepsilon}\right)_{\substack{\uparrow\\ \leftarrow}} \tag{19}$$

are of far-reaching importance for the estimates of exchange process quality in heat exchangers.

Even without a study of the details and consequences of such an EEIN behavior in these three cases, we can conclude that the unidirectional flow of streams is the most inconvenient, while the crossflow is a favorable, but not as much as counter-current flow arrangement. The existing knowledge, based on quite different approaches, is thus completely confirmed by these conclusions.

Concluding remarks

The conception of enthalpy exchange process quality analysis based on the dimensionless irreversibility norm offers a good possibility for a selective estimate of different enthalpy exchange conditions. The approach is not restricted to some particular cases and gives the opportunity for a complete analysis of heat exchangers of arbitrary flow arrangements.

Nomenclature

A [m^2]	– heat transfer area
c_p [J/kgK]	– specific heat at constant pressure
e [J]	– energy
h [J/kg]	– enthalpy per unit mass
m [kg]	– mass
$\dot{m}$ [kg/s]	– mass flow rate
NTU	– number of transfer units*
NTU^*	– number of transfer units at $\varepsilon = \varepsilon^*$
$N_{S,\Delta h}$	– dimensionless irreversibility*
$\dot{N}_{S,\Delta h}$	– enthalpy exchange irreversibility norm defined in Eq. (9), dimensionless
$\dot{Q}$ [W]	– heat flux
s [J/kgK]	– entropy per unit mass
T [K]	– temperature
T_0 [K]	– surrounding temperature
U [W/m^{2}K]	– overall heat transfer coefficient
$\Delta\dot{S}$ [W/K]	– entropy production rate
ε	– effectiveness of the exchanger*
ε^*	– effectivenes as defined by Eq. (14)
θ	– inlet temperatures ratio*
τ [s]	– time
ω	– thermal capacity rate ratio*

*

$NTU = UA/(\dot{m}c_p)_1$

$N_{S,\Delta h} = T_0\Delta\dot{S}/[T_0(\dot{m}c_p)_2]$

$\varepsilon = (T_{1,in} - T_{1,out})/(T_{1,in} - T_{2,in})$

$\theta = T_{1,in}/T_{2,in}$

$\omega = (\dot{m}c_p)_1/(\dot{m}c_p)_2$

Subscripts

in – at exchanger inlet
out – at exchanger outlet
1 – refers to fluid with $(mc_p)_{min}$
2 – refers to fluid with $(mc_p)_{max}$

Superscripts

f – refers to $NTU \to \infty$
max – maximal value
· – per unit time
* – at maximal irreversibility

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