DYNAMIC RESPONSE OF THE PARALLEL FLOW HEAT EXCHANGER WITH FINITE WALL CAPACITANCE

by

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This paper shows how the transient response of parallel heat exchanger with finite wall capacitance may be calculated by analytical method. Making usual idealizations for analysis of dynamic behavior of heat exchanger, the model is based on three local energy balance equations which are solved by using the Laplace transform method for step change of the primary fluid inlet temperature. The solution are found in the case of constant initial conditions and expressed in explicit analytical form in terms of the number of transfer units, heat capacity ratios, heat transfer resistance and flow capacitance ratios. Present solutions are valid in case where fluid velocities are different or equal and finite or infinite.The solutions can be very suitable for mathematical modeling systems containing such types of heat exchangers.

Introduction

Two-fluid direct-transfer heat exchangers are used extensively in nearly every industrial process such as power plants, gas turbines, air-conditioning systems and a lot of chemical plants. Any change, intentional or accidental, in the steady-state or in starting a system, causes a perturbation in the system that can have significant consequences. In all these cases, it is important to know dynamic behavior of the heat exchanger in order to select the most suitable design, control, and operation. The traditional design based on steady-state data has become inadequate, and attention has been paid to the understanding and evaluation of the dynamic behavior of heat exchangers.

As there is no enough space for reviewing already published papers, only a list of relevant references is given in this paper. However, it is important to stress that the open literature has not provided solutions for this type of problem.

This paper presents solutions to the energy equations governing convective heat transfer between a heat exchanger core, which is initially at a constant temperature, and a steady flow of fluids entering the exchanger at constant mass velocities. The temperatures of fluids and the core temperature are initially equal but, at zero time, the primary fluid is changed for the unit step increase in the inlet temperature. The presented model is valid for finite propagation speeds and finite wall capacitance. Availability of such solutions gives the engineer and the designer much more insight into the nature of transient heat transfer in parallel heat exchangers.

Mathematical formulation

On the basis of assumptions given in the Abstract and applying energy equations to both fluids and the wall, one can obtain three simultaneous partial equations in the coordinate system as shown in Fig. 1.

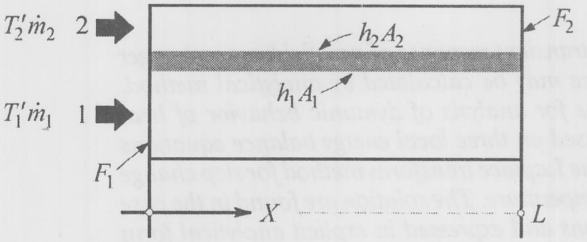

Figure 1. Schematic description of parallel heat exchanger

The space and time independent variables, X and t, range from 0 to the heat exchanger length L, and from 0 to 4, respectively. Other symbols are listed in Nomenclature.

All quantities referring to the weaker fluid (W_{min}) flowing in the X direction are denoted with subscript 1 and those referring to the stronger fluid (W_{max}), flowing in the same direction, with subscript 2 (Fig. 1).

In order to define dimensionless temperatures, it is appropriate to choose a reference temperature T and characteristic temperature difference ΔT, so that:

$$\theta_i(X,t) = \frac{T_i(X,t)-T}{\Delta T} \qquad i = 1, 2, w \tag{1}$$

Introducing dimensionless distance and dimensionless time:

$$x = \frac{X}{L}NTU; \qquad z = \frac{t}{t^*}NTU \tag{2}$$

and applying the relations given by:

$$NTU = \frac{(hA)_1(hA)_2}{(hA)_1+(hA)_2}\frac{1}{W_1}; \qquad \omega = \frac{W_1}{W_2} \qquad (0 \le \omega \le 1) \tag{3}$$

$$t^* = \frac{c_w M_w}{(hA)_1 + (hA)_2}; \quad K_1 = \frac{(hA)_1}{(hA)_1 + (hA)_2}; \quad K_2 = 1 - K_1 \tag{4}$$

$$C_i = L \frac{W_i}{c_w M_w} \frac{1}{K_i U_i}; \quad U_i = \frac{\dot{m}_i}{\rho_i F_i} \quad i = 1, 2 \tag{5}$$

the set of energy equations can be written in the following dimensionless form:

$$\frac{\partial \theta_w}{\partial z} + \theta_w = K_1 \theta_1 + K_2 \theta_2 \tag{6}$$

$$C_1 \frac{\partial \theta_1}{\partial z} + K_2 \frac{\partial \theta_1}{\partial x} = \theta_w - \theta_1 \tag{7}$$

$$C_2 \frac{\partial \theta_2}{\partial z} + \frac{K_1}{\omega} = \theta_w - \theta_2 \tag{8}$$

To define a partial differential problem completely, inlet and initial conditions have to be prescribed:

$$\left.\begin{aligned} &\theta_1(0,z) = \begin{cases} 0 & \text{for } z<0 \\ 1 & \text{for } z>0 \end{cases} \\ &\theta_2(0,z) = 0 \\ &\theta_1(x,0) = \theta_w(x,0) = \theta_2(x,0) = 0 \end{aligned}\right\} \tag{9}$$

These conditions are assuming that only fluid 1 inlet condition is perturbed. Step change in inlet temperature of fluid 1 is certainly most important from a physical point of view. However, because of the linearity of Eqs. (6), (7), and (8) and following similar procedure, as it can be seen in this paper, one can find transient behavior for perturbing other inlet and initial conditions. A general solution of this problem is developed in the following section.

General solution

Since equations (6), (7), and (8) are linear in $\theta_1(x, z)$, $\theta_w(x, z)$, and $\theta_2(x, z)$, they can be solved by using the Laplace transform. Taking a two-fold Laplace transform of the mentioned equations with respect to x and z with complex parameters s and p, respectively, and using the inlet and initial conditions (Eqs. 9), result in a simple set of algebraic equations. The solution of this algebraic system reads as follows:

$$\widetilde{\theta}_w = \frac{\dfrac{K_1 K_2}{K_2 s + C_1 p + 1}}{p + 1 - \dfrac{K_1}{K_2 s + C_1 p + 1} - \dfrac{K_2}{\dfrac{K_1}{\omega} s + C_1 p + 1}} \frac{1}{p} \tag{10}$$

$$\widetilde{\theta}_1 = \frac{\widetilde{\theta}_w}{K_2 s + C_1 p + 1} + \frac{K_2}{p(K_2 s + C_1 p + 1)} \tag{11}$$

$$\widetilde{\theta}_2 = \frac{\widetilde{\theta}_w}{\dfrac{K_1}{\omega} s + C_2 p + 1} \tag{12}$$

Using some simple mathematical transformations and the well known relation:

$$\frac{1}{(s+s_1)^{p_1+1}(s+s_2)^{p_2+1}} \sum_{j=0}^{p_1} (-1)^j \binom{p_2 + j}{p_2} \frac{1}{(s_2 - s_1)^{p_2+j+1}(s+s_1)^{p_1-j+1}} + \\ +(-1)^{p_1+1} \sum_{i=0}^{p_2} \binom{p_1 + i}{p_1} \frac{1}{(s_2 - s_1)^{p_1+i+1}} \frac{1}{(s+s_2)^{p_2-i+1}} \tag{13}$$

this algebraic system (Eqs. 10, 11, and 12) can be expressed in the following form, which is more convenient for developing the inverse Laplace transformation (using necessary condition $p_1, p_2 = 1, 2, 3, \ldots$). But, for brevity, only the final result for embed $\widetilde{\theta}_w$ will be given hereinafter.

$$\widetilde{\theta}_w(s, p) = \sum_{n=0}^{\infty} K_1^{n+1} \frac{1}{p(p+1)^{n+1}} \frac{K_2}{(K_2 s + C_1 p + 1)^{n+1}} + \\ + \sum_{n=1}^{\infty} \sum_{m=0}^{n-1} \binom{n}{m} \Bigg[K_1^{m+1} \left(\frac{\omega K_2}{a K_1} \right)^{n-m} \sum_{j=0}^{m} \binom{n-m+j-1}{n-m-1} \frac{(-1)^j}{(a K_2)^j} \cdot \\ \cdot \frac{1}{p(p+1)^{n+1}(p+b)^{n-m+j}} \frac{K_2}{(K_2 s + C_1 p + 1)^{m-j+1}} + \\ +(-1)^{m+1} \frac{\omega K_2^{n-m}}{a} \left(\frac{K_1}{a K_2} \right)^m \sum_{i=0}^{n-m-1} \binom{m+i}{m} \left(\frac{\omega}{a K_1} \right)^i \cdot \\ \cdot \frac{1}{p(p+1)^{n+1}(p+b)^{m+n+1}} \frac{\dfrac{K_1}{\omega}}{\left(\dfrac{K_1 s}{\omega} + C_2 p + 1 \right)^{n-m-i}} \Bigg] \tag{14}$$

where $a = \omega C_2/K_1 - C_1/K_2$ and $b = (\omega/K_1 - 1/K_2)/a$.

Analogous evaluations hold for the functions $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$.

From the techniques of Laplace transformation and using the Laplace transforms of special functions $F_n(x, c)$ and $H_{n,m,k}(x, c, d)$, defined in the Appendix, one can obtain the inverse Laplace transformation of equation (14), and similar equations for $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ which are not given in this paper. Using this procedure, the transient temperature distributions of both fluids and the wall for the parallel flow heat exchanger are as follow:

$$\theta_w(x,z) = \sum_{n=0}^{\infty} K_1^{n+1} F_{n+1}(x_1,1)\left[1 - \sum_{m=0}^{n} F_{m+1}(z_1,1)\right] +$$

$$+\sum_{n=1}^{\infty}\sum_{m=0}^{n-1}\binom{n}{m}\left[K_1^{m+1}\left(\frac{\omega K_2}{aK_1}\right)^{n-m}\sum_{j=0}^{m}\binom{n-m+j-1}{n-m-1}\right.$$

$$\cdot\frac{(-1)^j}{(aK_2)^j}F_{m-j+1}(x_1,1)H_{n+1,n-m+j,1}(z_1,1,1-b)+$$

$$+(-1)^{m+1}\frac{\omega K_2^{n-m}}{a}\left(\frac{K_1}{aK_2}\right)^m\sum_{i=0}^{n-m-1}\binom{m+i}{m}\cdot$$

$$\left.\cdot\left(\frac{\omega}{aK_1}\right)^i F_{n-m-i}(x_2,1)H_{n+1,m+i+1,1}(z_2,1,1-b)\right] \tag{15}$$

$$\theta_1(x,z) = F_1(x_1,1)\kappa(z_1) + \sum_{n=0}^{\infty} K_1^{n+1} F_{n+2}(x_1,1)\left[1 - \sum_{m=0}^{n} F_{m+1}(z_1,1)\right] + \sum_{n=1}^{\infty}\sum_{m=0}^{n-1}\binom{n}{m}\cdot$$

$$\cdot\left[K_1^{m+1}\left(\frac{\omega K_2}{aK_1}\right)^{n-m}\sum_{j=0}^{m+1}\binom{n-m+j-1}{n-m-1}\frac{(-1)^j}{(aK_2)^j}F_{m-j+2}(x_1,1)H_{n+1,n-m+j,1}(z_1,1,1-b)+\right.$$

$$\left.+(-1)^m\frac{\omega K_2^{n-m-1}}{a^2}\left(\frac{K_1}{aK_2}\right)^m\sum_{i=0}^{n-m-1}\binom{m+i+1}{m+1}\left(\frac{\omega}{aK_1}\right)^i F_{n-m-i}(x_2,1)H_{n+1,m+i+2,1}(z_2,1,1-b)\right] \tag{16}$$

$$\theta_2(x,z) = \frac{\omega}{a}\sum_{n=1}^{\infty}\sum_{m=0}^{n-1}\binom{n}{m}\cdot$$

$$\cdot\left[K_1^m\left(\frac{K_2}{aK_1}\right)^{n-m}\sum_{j=0}^{m}\binom{n-m+j}{n-m}\frac{(-1)^j}{(aK_2)^j}F_{m-j+1}(x_1,1)H_{n+1,n-m+j,1}(z_1,1,1-b)+\right.$$

$$\left.+(-1)^{m+1}K_2^{n-m}\left(\frac{K_1}{aK_2}\right)^m\sum_{i=0}^{n-m}\binom{m+i}{m}\left(\frac{\omega}{aK_1}\right)^i F_{n-m-i-1}(x_2,1)H_{n+1,m+i+1,1}(z_2,1,1-b)\right] \tag{17}$$

where $z_1 = z - C_1x/K_2, z_2 = z - C_2x/K_1, x_1 = x/K_2, x_2 = \omega x/K_1$.

Equations (15), (16), and (17) express analytically the transient temperature distributions for the parallel flow heat exchanger with finite wall capacitance, according to the finite propagation speed of disturbances in the exchanger.

Generally speaking, there are three different cases:

(1) $U_1 > U_2\ (C_1 < C_2)$

(2) $U_1 = U_2\ \ (C_1 = C_2)$

(3) $U_1 < U_2\ (C_1 > C_2)$

The case U_1, $U_1 \to \infty$ is a special case presented herein and discussed in detail in [10], while the cases $U_1 > 0$ and $U_2 = 0$, or $U_1 \to \infty$ and $U_2 = 0$, *etc.*, are not so interesting for practical purposes, although they can be analyzed using the given solutions.

It is important to notice that, in some special cases, validity of making assumptions must be carefully tested. More or less, it must be done always if one wishes to use certain theoretical solutions for practical purposes. For example, in the case where $U_1 > 0$, $U_2 = 0$ (secondary fluid doesn't flow) or $U_2 \approx 0$ (secondary fluid flows very slowly) and $\omega > 0$, flow passage of the secondary fluid has to be very high and, because of that, heat conduction through the fluid and in the direction perpendicular to the heat exchange surface can be significant. However, an inquiry into the practical application of this theory will not be made in this paper.

In equation (10), two special cases can be easily noticed:

A. $C_1 = C_2$ and $K_1/\omega = K_2$ or $K_1 = \omega/(1+\omega)$

B. $\omega = 0$

Case A:

The first of the mentioned conditions means that propagation speeds of both fluids are equal. The temperatures in front of disturbances must be equal to zero in accordance with initial and inlet conditions. The second condition means that the number of transfer units of both fluids is equal, *i.e.* $NTU_1 = NTU_2$. This can be easily proved by substituting expressions for ω, K_1, and K_2 (Eqs. 3 and 4) in the condition $K_2 = K_1/\omega$.

If $K_2 s + C_1 p + 1 \equiv K_1 s/\omega + C_2 p + 1$, equation (10) is transformed to:

$$\widetilde{\theta}_w = K_1 \sum_{n=0}^{\infty} \frac{1}{p(p+1)^{n+1}} \frac{K_2}{(K_2 s + C_1 p + 1)^{n+1}} \tag{18}$$

and from equations (11) and (12), the following temperatures $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ can be obtained:

$$\widetilde{\theta}_1 = \widetilde{\theta}_2 \frac{K_2}{p(K_2 s + C_1 p + 1)}$$
$$\widetilde{\theta}_2 = K_1 \sum_{n=0}^{\infty} \frac{1}{p(p+1)^{n+1}} \frac{K_2}{(K_2 s + C_1 p + 1)^{n+2}} \tag{19}$$

The inverse two-fold Laplace transform of the functions resulting from equations (18) and (19), give explicit analytical expressions for temperature distributions:

$$\theta_w(x,z) = K_1 \left[1 - \sum_{n=0}^{\infty} F_{n+1}(z_1,1) + \sum_{m=0}^{n} F_{m+1}(z_1,1) \right] \tag{20}$$

$$\theta_1(x,z) = F_1(x_1,1)\kappa(z_1) + \theta_2(x,z) \tag{21}$$

$$\theta_2(x,z) = K_1 \left[1 - F_1(x_1,1) - \sum_{n=0}^{\infty} F_{n+2}(x_1,1) + \sum_{m=0}^{n} F_{m+1}(z_1,1) \right] \tag{22}$$

Case B (applicable to any exchanger configuration):

In this case $\omega = 0$ and, therefore, $\theta_2(x, z) = 0$, which inevitably results in the reduced equation (10). After some mathematical manipulations, this equation can be transformed into:

$$\widetilde{\theta}_w = \sum_{n=0}^{\infty} \frac{1}{p(p+1)^{n+1}} \frac{K_1^{n+1} K_2}{(K_2 s + C_1 p + 1)^{n+1}} \tag{23}$$

Finally, the inverse two-fold Laplace transform of equation (23) gives:

$$\theta_w(x,z) = \sum_{n=0}^{\infty} K_1^{n+1} F_{n+1}(x_1,1) \left[1 - \sum_{m=0}^{n} F_{m+1}(z_1,1) \right] \tag{24}$$

From equation (11), using equation (23), the temperature distribution of fluid 1 can be found.

$$\theta_1(x,z) = F_1(x_1,1)\kappa(z_1) + \sum_{n=0}^{\infty} K_1^{n+1} F_{n+2}(x_1,1) \left[1 - \sum_{m=0}^{n} F_{m+1}(z_1,1) \right] \tag{25}$$

It should be emphasized that the solutions of both special cases (A and B) can be deduced from the solutions given by Romie [7], but this will not be shown in this paper.

Outlet temperatures of both fluids can be found by putting $x = NTU$.

Calculation results

The purpose of this paper was to provide an exact analytical solution by which performances of the parallel flow heat exchanger can be evaluated and compared. Many parameters are involved in the temperature distributions of both fluids and the wall and, therefore, it is not possible to present quantitative influences of all these parameters in this paper. However, there is enough space to give a particular result showing the main characteristics of the solutions.

The space temperature distributions of both fluids and the wall for various times and outlet fluid temperatures versus time will be presented herein assuming that $NTU = 1$, $\omega = 0.5$, $K_1 = 0.25$, $C_1 = 4$, and $C_2 = 0.5$. The illustrative example to this option is reported in Fig. 2. The temperature distributions of both fluids and the wall have been plotted versus dimensionless heat exchanger length for $z = 1, 3, 5$ and

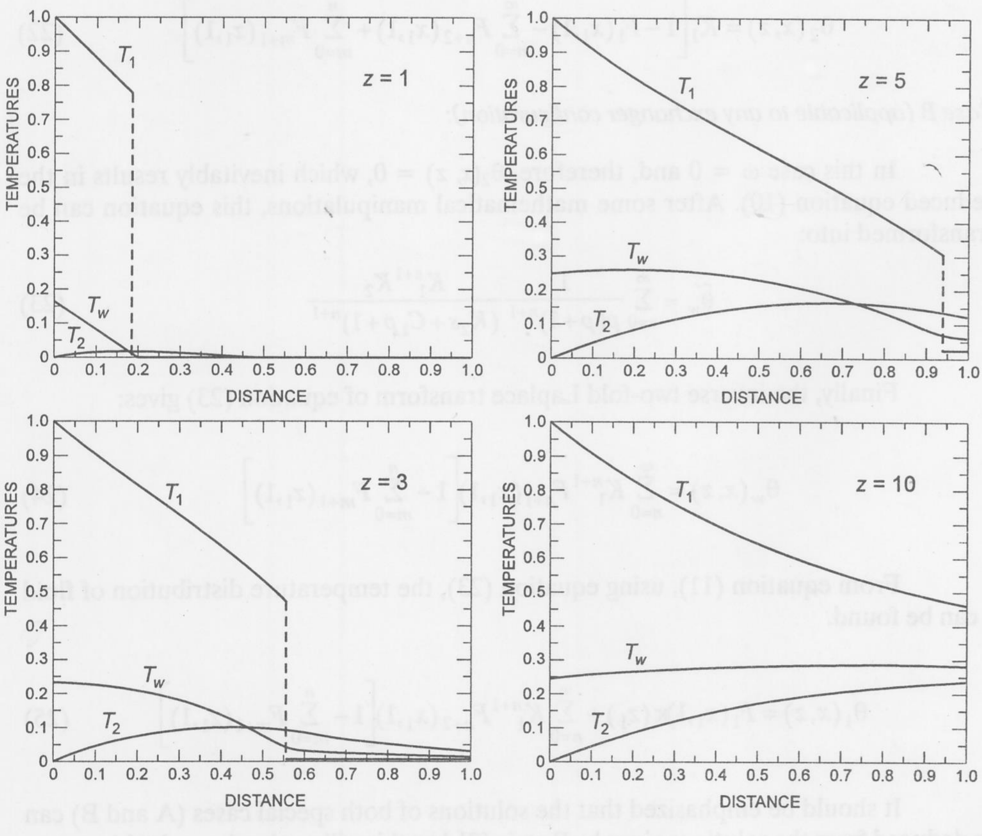

Figure 2. Temperature distribution of both fluids and the wall versus dimensionless length x for various dimensionless times z ($NTU = 1$, $\omega = 0.5$, $K_1 = 0.25$, $C_1 = 4$, and $C_2 = 0.5$)

10. It is very interesting in this example that the temperature of fluid 1 has been changed in front of the wave front (Fig. 2a, 2b, and 2c). That means that fluid 1 and the wall are heated by fluid 2 instead of vice versa. In front of the fluid 2 wave front, all temperatures are equal to zero (initial conditions). Of course, at the point of wave front 1, fluid 1 temperature is changed discontinously and has two values as follows from the step change of the inlet fluid 1. For greater values of z ($z = 10$) (Fig. 2d), distributions of both fluids and the wall are very similar to the steady-state temperature distribution and that is quite obvious.

The outlet temperatures (θ_1'' and θ_1'') of both fluids are presented in Fig. 3 for the same case. It is easy to see that fluid 1 is heated before the wave front comes to the outlet of heat exchanger becouse it gets energy from fluid 2 previously heated by fluid 1. In Fig. 3. one can find the asymtotic values for $z \rightarrow \infty$in the step response reproduction.

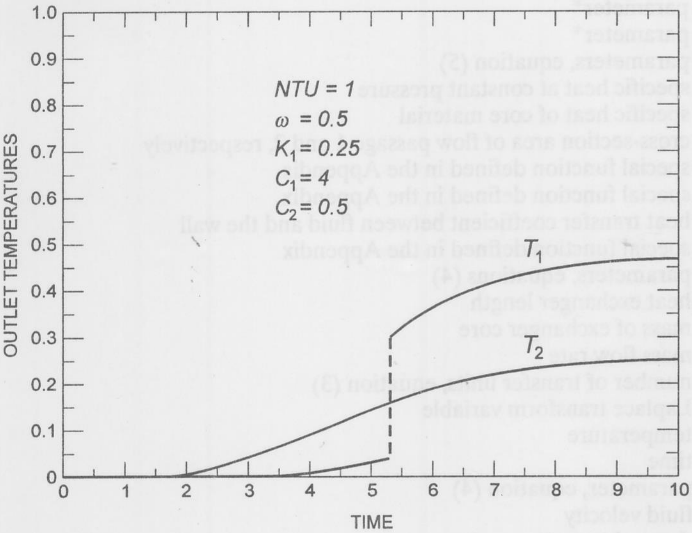

Figure 3. Outlet temperatures of both fluids versus dimensionless time z

Concluding remarks

A method providing exact analytical solutions to the transient response of parallel heat exchanger with finite wall capacitance has been presented. Solutions are valid in the case where fluid velocities are different or equal. This solution procedure provides necessary basis for the study of parameter estimation, model discrimination, and control of parallel heat exchangers.

Comparisons for the transient cases with different fluid velocities are not possible because there are no available data in literature. However, results for equal velocities can be compared with results given by Romie [5] and results for infinite velocities given by Romie [5] and Gvozdenac [8]. These tests were conducted and proved that the solutions for the above mentioned particular cases are correct.

Examination of these solutions reveals that they may be used effectively in practice for computer-aided design, control, and operation procedures.

Nomenclature

A_1, A_2 [m^2]	– total heat transfer area on side 1 and 2, respectively
a	– parameter*
b	– parameter*
C_1, C_2	– parameters, equation (5)
c_p [J/(kgK)]	– specific heat at constant pressure
c_w [J/(kgK)]	– specific heat of core material
F_1, F_2 [m]	– cross-section area of flow passage 1 and 2, respectively
F_n	– special function defined in the Appendix
$H_{n,m,k}$	– special function defined in the Appendix
h [W/(K m)]	– heat transfer coefficient between fluid and the wall
$I_{n,m}$	– special function defined in the Appendix
K_1, K_2	– parameters, equations (4)
L [m]	– heat exchanger length
M_w [kg]	– mass of exchanger core
$\dot{m}$ [kg/s]	– mass flow rate
NTU	– number of transfer units, equation (3)
p, s	– Laplace transform variable
T [K]	– temperature
t [s]	– time
t^*	– parameter, equation (4)
U [m/s]	– fluid velocity
W [W/K]	– thermal capacity rate*
X [m]	– distance from fluid 1 and 2 entrances
x, z	– dimensionless independent variables, equation (2)
θ	– dimensionless temperature, equation (1)
w	– thermal capacity rate ratio, equation (3)
κ	– unit step function

Subscript

1	– fluid 1
2	– fluid 2
w	– wall
i,j,k,n,m	– integers

Superscripts

" – at the outlet
~ – two-fold Laplace transform

*

$a = \omega\, C_2 / K_1 - C_1 / K_2$
$b = (\omega / K_1 - 1/K_2)/a$
$W = \dot{m} c_p$

Appendix

The functions $F_n(x, c), I_{n,m}(x, c, d)$ and $H_{n,m,k}(x, c, d)$ and their Laplace transforms are given as described below ($x \geq 0, -\infty < c, d < +\infty$, and $n, m, k = 1, 2, 3,...$). For $x < 0$ all these functions are equal to zero.

$$F_n(x,c) = \frac{x^{n-1}}{(n-1)!} \mathrm{e}^{-ex} \Leftrightarrow \frac{1}{(s+c)^n} \tag{A.1}$$

$$I_{n,m}(x,c,d) = \sum_{i=0}^{\infty} \binom{m+i-1}{i} d^i F_{n+m+1}(x,c) \Leftrightarrow \frac{1}{(s+c)^n (s+c-d)^m} \tag{A.2}$$

$$H_{n,m,k}(x,c,d) = \sum_{i=0}^{\infty} d^i I_{n+m+i,k}(x,c,c) \Leftrightarrow \frac{1}{s^k (s+c)^n (s+c-d)^m} \tag{A.3}$$

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