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MESHLESS LOCAL RBF-DQ FOR 2-D HEAT CONDUCTION: A COMPARATIVE STUDY

by

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Meshless local radial basis function-based differential quadrature method is applied to 2-D conduction problem. Numerical results are compared with those gained by homotopy perturbation method. Outcomes are presented through graphs which prove the accuracy of homotopy perturbation method and its applicability in heat transfer problems.

Key words**:** *mesh-free method, radial basis function, homotopy perturbation method, heat conduction*

Introduction

Traditional numerical techniques such as finite difference, finite volume, and finite element methods (FDM, FVM, and FEM) are routinely used to solve complex problems. It is well-known that these methods are strongly dependent on mesh properties. Mesh-free methods (also called meshless methods) have generated considerable interest recently due to the need to overcome the high cost of mesh generation associated with human labor. In this study a 2-D conduction problem in a square cavity which is heated none uniformly is investigated numerically using the meshless local radial basis function-based differential (RBF-DQ) [1-3]. Numerical outcomes are compared with the homotopy perturbation method (HPM) results [4-10] to investigate the accuracy of this analytic method and its application in mechanical engineering especially in heat transfer problems.

Local RBF-DQ method formulation

A radial basis function is a continuous spline which depends on the separation distances of a subset of scattered points. The most commonly used RBF is:

$$\text{Multiquadrics (MQ):} \quad \varphi(r) = \sqrt{r^2 + c^2}, \ c > 0 \tag{1}$$

Local RBF-DQ Due to its popularity is also called Local MQ-DQ method. In eq. (1), c is shape parameter (a user defined parameter) and $r = \|x - x_j\|_2$. For scattered points the approximation of a function $f(x)$ can be written as a linear combination of N RBFs:

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$$f(x) \cong \sum_{i=1}^{N} \lambda_j \varphi(\left\| x - x_j \right\|_2) + \psi(x) \tag{2}$$

where N is the number of knots x, $x = (x_1, x_2, \ldots, x_d)$, d – the dimension of the problem, λ's are coefficients to be determined, φ – the RBF and $\psi(x)$ is a specific polynomial [3].

The DQ method is a numerical discretization technique for approximation of derivatives of a function $f(x)$ by which the mth order derivative with respect to x at a point x_i can be approximated as:

$$f_x^{(m)}(x_i) = \sum_{j=1}^{N} w_{ij}^m f(x_j), \ i = 1, 2, \ldots, N \tag{3}$$

where fi, $f(x_i)$ and $w_{ij}^{(m)}$ are the domain points in supporting region (fig. 1(a)), function values and weighting coefficients. To conduct numerical experiments, the irregular knot distribution (fig. 1(b)) in the square domain is fixed. The function in the domain can be approximated by MQ RBFs as:

$$f(x, y) = \sum_{j=1, j \neq i}^{N} \lambda_j g_j(x, y) + \lambda_i \tag{4}$$

where

$$g_j(x, y) = \sqrt{(x - x_j)^2 + (y - y_j)^2 + c_j^2} - \sqrt{(x - x_i)^2 + (y - y_i)^2 + c_i^2} \tag{5}$$

● Reference Knot
✛ Boundary Knots
⬤ Non-supporting Knots
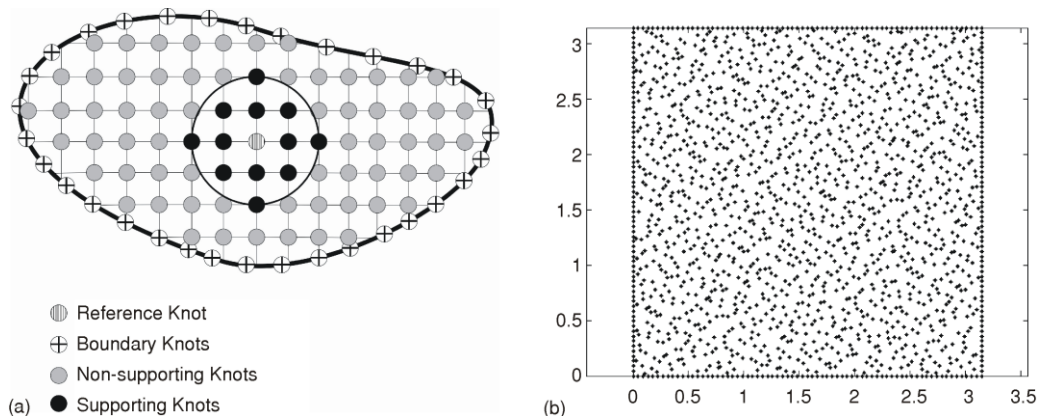(a) ● Supporting Knots
(b)

Figure 1. (a) Supporting knots around a centered knot, (b) Irregular knot distribution

From the eq. (8) and the essence of RBFs we have:

$$0 = \sum_{k=1}^{N} w_{ik}^{(m)} \tag{6}$$

$$\frac{\partial^m g_j(x_i, y_i)}{\partial x^m} = \sum_{k=1}^{N} w_{ik}^{(m)} g_j(x_k, y_k), \ j = 1, 2, \ldots, N, \quad \text{but} \quad j \neq i, \tag{7}$$

For the given i, equation system (6-7) has N unknowns with N equations. In the matrix form, the weighting coefficient matrix of the x-derivative can then be determined by:

$$[G][W^n]^T = \{G_x\} \tag{8}$$

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where $[N^n]^T$ is the transpose of the weighting coefficient matrix $[W^n]$, and:

$$[W^n] = \begin{bmatrix} w_{1,1}^{(n)} & \cdots & w_{1,N}^{(n)} \\ \vdots & \ddots & \vdots \\ w_{N,1}^{(n)} & \cdots & w_{N,N}^{(n)} \end{bmatrix} \quad [G] = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ g_N(x_1, y_1) & \cdots & g_N(x_N, y_N) \end{bmatrix}$$

$$[G_x] = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ g_x^n(N,1) & \cdots & g_x^n(N,N) \end{bmatrix}$$

(9)

It is worth while mentioning that normalization of supporting region removes the difficulty of assigning different values of c at different knots.

Mathematical modeling and solution procedure

Assume a rectangular plate with three heated boundaries as it is shown in fig. 2. The steady 2-D conduction equation and the boundary conditions for ranges of $0 < x < \pi$ and $0 < y < \pi$ can be presented in equation form as follows:

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$
$$T(0,y) = 0 \ , \ T(\pi,y) = \sinh\pi\cos y$$
$$T(x,0) = \sinh x \ , \ T(x,\pi) = -\sinh x$$

(10)

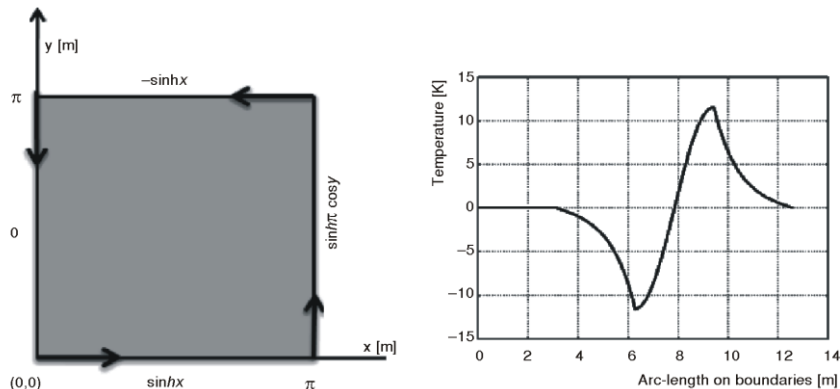where T is the temperature difference from the ambient.

Figure 2. Geometry and boundary conditions

Application of local MQ-DQ method

The steady-state results are obtained from unsteady conduction equation discritized by RBF-DQ as:

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$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \left(\sum_{k=1}^{n_i} c_{i,k}^{2x} T_i^k - \sum_{k=1}^{n_i} c_{i,k}^{2y} T_i^k \right) \tag{11}$$

where T_i represents the function value at knot i, T_i^k represents the function value at the kth supporting knot for knot i. $c_{i,k}^{2x}$ and $c_{i,k}^{2y}$ represent the computed weighting coefficients in the DQ approximation for the second order derivatives in the x and y-direction, respectively.

Application of HPM

Using HPM as discussed in detail [5] we have:
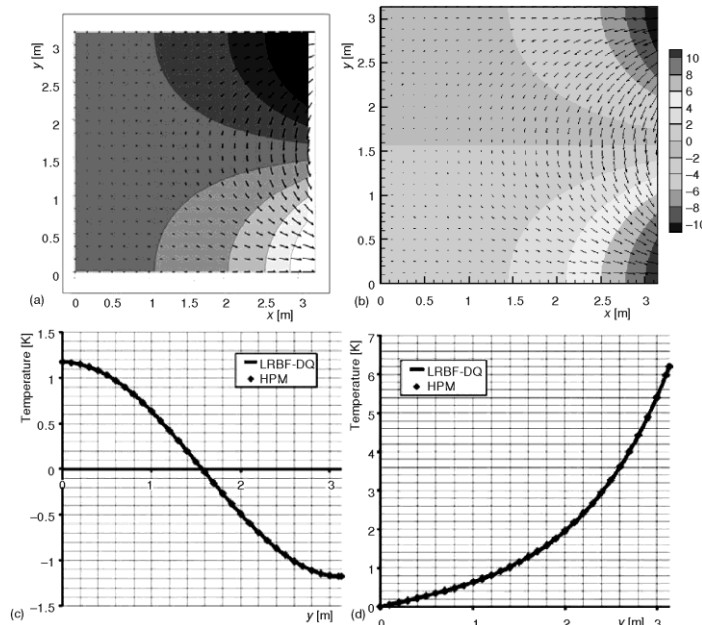
$$v = v_0 + pv_1 + p^2 v_2 + \cdots \tag{12}$$

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \tag{13}$$

with the appropriate initial condition, the v-terms are obtained as follows:

$$\upsilon_0(x, y) = x \cos y \ , \ \upsilon_1(x, y) = \frac{1}{6} x^3 \cos y \ , \upsilon_2(x, y) = \frac{1}{120} x^5 \cos y \ , \ \ldots \tag{14}$$

where $\upsilon_0(x, y) = x \cos y$ satisfies three boundary conditions considering $\sinh x \approx x$. Therefore:

$$T(x, y) = \cos y (x + \frac{1}{6} x^3 + \frac{1}{120} x^5 + \frac{1}{5040} x^7 + \ldots) \tag{15}$$

Figure 3. Comparison of temperature contour and gradients vectors obtained by (a) HPM, (b) local RBF-DQ and temperature value on (c) *x* = 1 and , (d) *y* = 1

The comparison between temperature distribution and gradients gained by HPM and RBF-DQ for the boundary conditions are shown with contour and vector plots in fig. 3.

Conclusions

In this paper, the mesh-free Local RBF-DQ and homotopy perturbation methods are applied to obtain the solution of a two-dimensional heat transfer conduction equation. Outcomes show a very good agreement between RBF-DQ and HPM for distributions and gradients of temperature. A comparative study has been performed between the two numeric and analytic techni-

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ques to investigate the simplicity, reliability, and effectiveness of HPM in solving practical PDE in heat transfer.

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