A THERMODYNAMIC CAVITATION MODEL APPLICABLE TO HIGH TEMPERATURE FLOW

by

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Cavitation is not only related with pressure, but also affected by temperature. Under high temperature, temperature depression of liquids is caused by latent heat of vaporization. The cavitation characteristics under such condition are different from those under room temperature. The paper focuses on thermodynamic cavitation based on the Rayleigh-Plesset equation and modifies the mass transfer equation with fully consideration of the thermodynamic effects and physical properties. To validate the modified model, the external and internal flow fields, such as hydrofoil NACA0015 and nozzle, are calculated, respectively. The hydrofoil NACA0015's cavitation characteristic is calculated by the modified model at different temperatures. The pressure coefficient is found in accordance with the experimental data. The nozzle cavitation under the thermodynamic condition is calculated and compared with the experiment.

Keywords: *cavitation, thermodynamics effects, NACA0015, nozzle*

Introduction

In liquid flow, cavitation generally occurs if the pressure drops below the vapor pressure. Consequently, negative pressures are relieved by means of forming gas filled or gas and vapor filled cavities [1]. Therefore, cavitation is the phenomenon of the formation, growth and rapid collapse of bubbles. It may occur through the formation of bubbles or cavities in the liquid, or it can be a result of the enlargement of cavities that are already present in the bulk liquid [2]. Cavitation research has been important in marine propellers, pump impellers, turbines and hydrofoils where cavitation is often unavoidable.

Many experiment results have shown that, the cavitation performance in high temperature and high pressure fluids for hydrofoil or hydraulic machines are better than that under room temperature. Generally speaking, the fluid physical properties are different at different temperatures. The density of water decreases with increasing temperatures, while the density of water vapor and the saturation pressure increases. Furthermore the density of water vapor and the saturation pressure change quickly at high temperatures.

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Under room temperature conditions, the amount of vapor pressure changing with the variations of temperature is smaller and can even be neglected. On the other hand, this effect cannot be ignored under high temperature and high pressure conditions.

Research on the cavitation under thermodynamic effects has generally been focused on obtaining correlations for temperature depression as a function of flow conditions and liquid properties. Particular important methods include the B-factor method [3, 4] and the entrainment theory method [5]. Deshpande [6] solves the cavity geometry which is affected by the temperature depression with CFD（computational fluid dynamics）method. The solution of the Navier-Stokes equations in combination with an algebraic turbulence model provides the details of the cavity geometry, while the temperature depression in the vicinity of the cavity interface can be obtained to solve the energy equation. This research only considers the flow field information.

In this paper, the cavitation mass transfer equation under the thermodynamic effects is modified. This modified equation mainly considers the temperature variation, pressure pulsation and physical properties. To validate the modified equation's robustness, two flow fields are solved. Cavitation around hydrofoil NACA0015 is calculated with this equation and the simulation results are compared with the experimental data. Finally, a circle nozzle cavitation is also simulated by this equation; the result is found in accordance with the experiment.

Mathematical model

In this paper, to research the cavitation phenomenon under thermodynamic conditions, some fluid physical properties are introduced, such as the saturation vapor pressure P_v, surface tension s and water and water vapor density, which are functions of the temperature. At high temperatures, the saturation vapor pressure decreases more than at the room temperature if the temperature drops by 1 °C.

The saturation vapor pressure of water P_v is given by the empirical formula as:

$$p_v = p_k \exp\left\{\left(1-\frac{T_k}{T}\right)\left[a+(b-eT)(T-d)^2\right]\right\} \tag{1}$$

where p_k = 22.13 MPa and T_k = 2647.31 K.

The surface tension is also a function of the temperature, namely:

$$s = 0.11633 - 1.09536\cdot 10^{-4}T - 1.72619\cdot 10^{-10}T^2 \tag{2}$$

In the homogeneous multiphase model, a common flow field is shared by all fluids, as well as other relevant fields such as temperature and turbulence. Thus, the governing equations of the homogeneous model for mass, momentum, thermal energy and volume conservation equation can be written as:

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial(\rho_m \bar{u}_i)}{\partial x_i} = 0 \tag{3}$$

$$\frac{\partial(\rho_m \bar{u}_i)}{\partial t} + \frac{\partial(\rho_m \bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_j} + \frac{\partial}{\partial x_j}(\bar{\tau}^*_{ij} - \rho_m \overline{u'_i u'_j}) \tag{4}$$

$$c_m\left(\frac{\partial \rho_m \overline{T}}{\partial t}+\bar{u}_j\frac{\partial \rho_m \overline{T}}{\partial x_j}\right)-\frac{\partial}{\partial x_j}\left(\lambda\frac{\partial \overline{T}}{\partial x_j}-c_m\rho_m\overline{u_i'T'}\right)-\frac{\mathrm{d}p}{\mathrm{d}t}-\overline{\Phi}=\dot{m}(c_v-c_l)\overline{T} \tag{5}$$

$$\frac{\partial(\rho_l\alpha_l)}{\partial t}+\frac{\partial(\rho_l\alpha_l\bar{u}_j)}{\partial x_j}=\rho_l\dot{m} \tag{6}$$

where
$$\bar{\tau}_{ij}^*=\begin{cases}\mu_m\left(\dfrac{\partial\bar{u}_i}{\partial x_j}+\dfrac{\partial\bar{u}_j}{\partial x_i}\right) & i\neq j\\[2ex] 2\mu_m\dfrac{\partial\bar{u}_i}{\partial x_j}-\dfrac{2}{3}\mu_m\nabla.\vec{V} & i=j\end{cases}$$

$$\overline{\Phi}=2\mu_m\left(\frac{\partial\bar{u}_j}{\partial x_j}\right)^2+\mu_m\left(\frac{\partial\bar{u}_j}{\partial x_i}+\frac{\partial\bar{u}_i}{\partial x_j}\right)^2+\lambda(\nabla.\vec{V})^2\quad i\neq j$$

$$-\rho_m\overline{u_i'u_j'}=\mu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{2}{3}\rho_m k\delta_{ij}\qquad -\rho_m\overline{u_i'T'}=\frac{\mu_t}{\sigma_T}\frac{\partial T}{\partial x_j}$$

$$\rho_m=\alpha_v\rho_v+(1-\alpha_v)\rho_l\qquad \mu_m=\alpha_v\mu_v+(1-\alpha_v)\mu_l\qquad c_m=\alpha_v c_v+(1-\alpha_v)c_l$$

In most cases, the characteristic time of bubbles (growth and collapse, *etc*.) is less than 10^{-5} s, depending on the cavity size. The time scale of turbulent fluctuation is generally far greater. Therefore, the bubble related behavior of cavities is affected by turbulence, such as inception, growth and collapse, *etc*. Singhal *et al.* [7] has assumed pressure fluctuation is a function of density and turbulence energy as additional negative pressure drops, which equals to the half of the turbulence fluctuation p'_{turb}:

$$p'_{\mathrm{turb}}=0.39\rho k \tag{7}$$

This effect is equivalent to raising the vapor pressure p_v by $p'_{\mathrm{turb}}/2$ to an equivalent vapor pressure p_v^*:

$$p_v^*=p_v+\frac{p'_{\mathrm{turb}}}{2}=p_v+0.195\rho k \tag{8}$$

The mass transfer equation for vapor-liquid mixture based on the theory of evaporation/condensation on a plane surface, and the Kinetic theory of mass transfer are used to get the source terms. The source term of eq. (9) is applied only if $p<p_v^*$; the source term of the Sharp Interfacial Dynamics Model eq. (10) is employed, or vice versa. Thus, the source term $\dot{m}$ can be expressed as:

If $$p<p_v^*:\qquad \dot{m}=C_1\frac{3a_v}{r}\frac{2s}{2-s}\left(\frac{M}{2\pi R}\right)^{1/2}\left(\frac{p_v^*}{\sqrt{T}}-\frac{p}{\sqrt{T}}\right) \tag{9}$$

If $$p>p_v^*:\quad \dot{m}=C_2\frac{3(1-a_v-a_u)}{r}\frac{2s}{2-s}\left(\frac{M}{2\pi R}\right)^{1/2}\left(\frac{p}{\sqrt{T}}-\frac{p_v^*}{\sqrt{T}}\right) \tag{10}$$

Simulation results for hydrofoil

In this paper, the 3-D hydrofoil of NACA0015 (shown in fig. 1) is simulated at 25, 50, 100 and 150 °C. The span of the hydrofoil is 0.08 m and cavitation number is 1.5.

Figure 2 shows the pressure coefficient – C_p distribution along nondimensional hydrofoil chord.

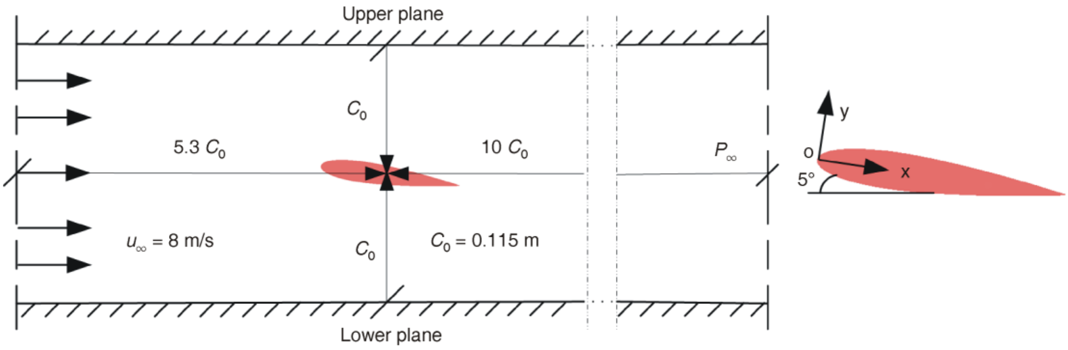

Figure 1. Model and flow conditions

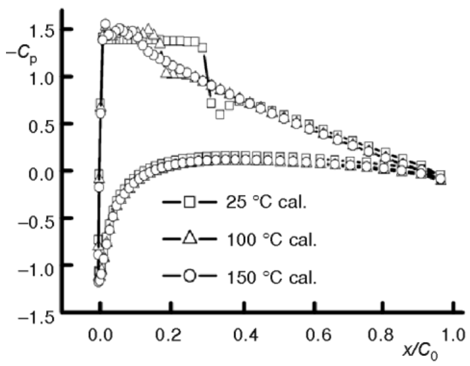

Figure 2. The pressure coefficient distribution at different temperatures

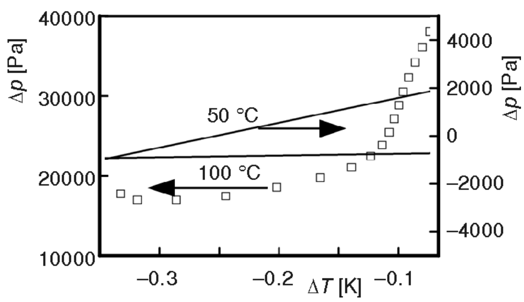

Figure 3. Pressure variations *vs.* temperature variations

The cavitation zone length is in accordance with the experimental results at 25 °C and 50 °C in the reference [8]. The cavitation zone length calculated at 100 °C is 45% shorter than the length at 25 °C and is minimum at 150 °C. The cavity zone length decreases with rising temperatures.

In fig. 3, when the temperature decreases by 0.1 K, the decrease in pressure at 100 °C is five times as large as that at 50 °C

Hence, the pressure variation is more sensitive to temperature under high temperature conditions.

Simulation results for nozzle

Cavitation occurs when the flow passes through a very small circle nozzle, which produces a high pressure decline. In the present paper, the circle nozzle geometry, as shown in fig. 4, is presented, which is the same as Abuaf's *et al.* [9] experimental circle nozzle. The calculation is given at the same pressure coefficient $C_p = (P_{in} - P_v)/(P_{in} - P_{out}) = 1$; p_v is the saturated vapor pressure.

The turbulence model is the SST *k-w* model due to the better convergence compared to other turbulence models.

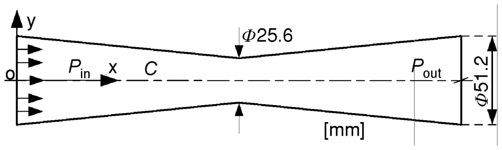

Figure 4. Circular nozzle [9]

In fig. 5, the calculated vapor fraction on the axis of the nozzle at 100 and 150 °C are plotted and compared with the references [9, 10]. A good agreement is observed between the presented prediction and the test data [9], especially when *x/C* is larger than 0.7 comparing with the published predictions [10]. In the simulation, it is also observed that the bubble occurs near the throat of the nozzle, and then is carried to the diffusion part. At 100 °C, the vapor volume fraction is larger than that at 150 °C which demonstrates that the cavitation phenomena under high temperatures and high pressures is different from under low temperatures and low pressures.

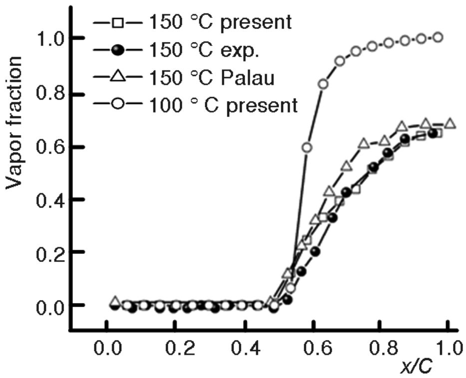

Figure 5. Cavitation vapor fraction along the axial line (*C*: the axial distance from the nozzle inlet to outlet; *x*: the axial distance from the point to inlet)

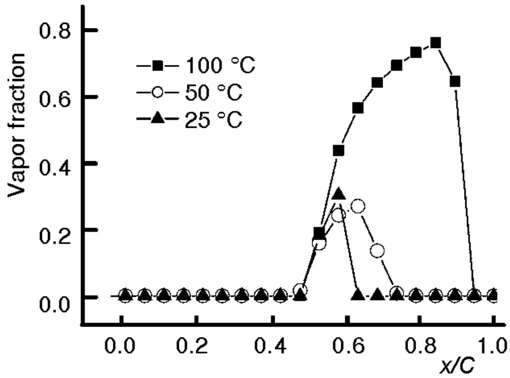

Figure 6. Cavitation vapor fraction along the axial line (*C*: the axial distance from the nozzle inlet to outlet; *x*: the axial distance from the point to inlet)

If the boundary condition is modified, such as the p_{in} = 555 kPa and p_{out} = 378 kPa at the temperature 25 °C, 50 °C, and 100 °C the vapor distribution is increasing with the temperature increasing, in fig.6. This result is different from the vapor distribution at the same pressure coefficient C_p = 1.

Conclusions

In this paper, a thermodynamic cavitation model of high temperature flows considering the surface tension, temperature and physical properties is modified based on evaporation and condensation mechanics.

The modified thermodynamic cavitation model is used to simulate the flow on NACA0015 hydrofoil at different temperatures. The length of the vapor distribution decreases with temperature increasing.

The circle nozzle is also calculated by this improved model. The cavitation model under thermodynamics effects is effective to predict the nozzle's cavitation. The prediction of the axial vapor fraction is in good agreement with the experimental data.

From these two cases, the external and internal flow, the modified cavitation model considering the thermodynamic efforts is enough to predict the cavitation phenomena at high temperatures.

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Nomenclature

a – coefficient in the eq. (1) [–]
b – coefficient in the eq. (1) [–]
C_p – pressure coefficient, $2(p-p_\infty)/\rho_l u_\infty^2$ [–]
C_1 – coefficient of evaporation, 0.39, [–]
C_2 – coefficient of condensation, 0.03, [–]
c – specific heat capacity, [kg^{-1} K^{-1}]
d – coefficient in the eq. (1) [–]
e – coefficient in the eq. (1) [–]
k – turbulent kinetic, [m^{2}s^{-2}]
M – coefficient in the eq. (9) and (10), [–]
$\dot{m}$ – heat transfer induced by interphase mass transfer, $(c_v - c_l)\overline{T}$ [m^{-3}s^{-1}]
$\dot{m}$ – interphase mass transfer rate, [kgm^{-3}s^{-1}]
p – local static pressure, [Pa]
R – coefficient in the eq. (9) and (10), [–]
r – bubble radius, [m]
T – temperature, [K]
t – time, [s]
$\overline{u}$ – average velocity, [m·s^{-1}]
u' – fluctuation velocity, [ms^{-1}]

Greek letters

α – volume fraction, [–]
α_u – non-condensable gas void fraction, $5\cdot 10^{-5}$, [–].
δ_{ij} – Kronecker symbol, [–]
μ – kinemics viscosity, [Pas]
λ – heat conductivity, [Wm^{-1}K]
ρ – fluid density, [kgm^{-3}]
Φ – enthalpy, [Jkg^{-1}]

Subscripts

i, j – 1, 2, 3
v – water vapor
l – water
m – mixture

References

[1] Batchelor, G. K., An Introduction to Fluid Dynamics, Cambridge University Press, Cambridge, United Kingdom,1967
[2] Shah, Y. T., Pandit, A. B., Moholkar, V.S., Cavitation Reaction Engineering, Kluwer Academic/Plenum Publishers, Dordrecht, The Netherlands, 1999
[3] Stahl, H. A., Stepanoff, A. J., Thermodynamic Aspects of Cavitation in Centrifugal Pumps, *ASME Journal of Basic Engineering*, *78* (1956), 8, pp. 1691-1693
[4] Moore, R. D., Ruggeri, R. S., Prediction of Thermodynamic Effects on Developed Cavitation Based on Liquid-Hydrogen and Freon-114 Data in Scaled Venturis, NASA TN D-4899, 1968
[5] Holl, J. W., Wislicenus, G. F., Scale Effects on Cavitation, *ASME Journal of Basic Engineering*, *83* (1961), pp. 385-398

[6] Deshpande, M., Feng, J., Merkle, C. L., Numerical Modeling of the Thermodynamic Effects of Cavitation, *Journal of Fluid Engineering*, *119* (1997), 2, pp. 420-427

[7] Singhal, A. K., *et al*., Mathematical Basis and Validation of the Full Cavitation Model, *Journal of Fluids Engineering*, *124* (2002), 3, pp. 617-624

[8] Liu, D. M., Liu, S. H., Wu, Y. L., Numerical Analysis of Airfoil NACA0015 Cavitation Characteristic Based on the Thermodynamic Effects, *Modern Physics Letters B, 24* (2010), 13 pp. 1499-1502

[9] Abuaf, N., *et al*., A Study of Nonequilibrium Flashing of Water in a Converging-Diverging Nozzle, NUREG/CR 1864, Vol. 2, Office of Nuclear Regulatory Research, 1981

[10] Palau-Salvador, G., González-Altozano, P., Arviza-Valverde, J., Numerical Modeling of Cavitating Flows for Simple Geometries Using FLUENT V6.1, *Spanish Journal of Agricultural Research 5* (2007), 4, pp. 460-469

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