APPLICATION OF LATTICE BOLTZMANN METHOD AND FIELD SYNERGY PRINCIPLE TO THE HEAT TRANSFER ANALYSIS OF CHANNEL FLOW WITH OBSTACLES INSIDE

by

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In this paper the lattice Boltzmann method and field synergy principle are applied to simulate two-dimensional incompressible steady channel flow under low Reynolds number, and analyze the local influence on velocity field and temperature field caused by inserting cylinder obstacles of different cross-section. Furthermore, field synergy principle of elliptic flow type is applied to demonstrate that the increased interruption within the fluid increases the synergistic level between the velocity field and temperature gradient field. As the intersection angle between the velocity vector and the temperature gradient vector decreases by inserting cylinder obstacles to fluid field, the results of heat transfer will improve significantly.

Key words: *lattice Boltzmann method, field synergy principle, intersection angle*

Introduction

Lattice Boltzmann method (LBM), which belongs to the explicit method, primarily solves the distribution function of the particles first, and therefore could skip the iterative computations. Furthermore, the stability of the method has nothing to do with the sizes of the lattices and the length of time step. As a result, the algorithms of the method are much simpler and steadier, which makes the method superior to its conventional counterparts. In addition to the simplicity of the equations, LBM is able to concretely demonstrate the ability of microscopic interaction as well. This method, hence, has been successfully applied to the simulation and calculation of a series of problems ranging from ordinary hydrodynamic problems [1] to the complex flowing phenomena such as two-phase flows [2] and porous flows [3].

Guo [4] proposed a novel concept concerning boundary layer flows when he studied on convective heat transfer. He pointed out that the intersection angle formed by

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the speed vector and the temperature gradient vector in the flow field should be reduced as much as possible in order to enhance heat transfer; the statement is the so-called 'Field Synergy Principle'. Tao *et al.* [5] further applied field synergy principle from parabolic problems to oblong fluid flows. They exemplified that under the conditions of pared-down heat boundary layer, more frequent fluid turbulence and increased velocity gradient around the boundaries, they were able to analyze the effect that helps enhance convective heat transfer by the degree of field synergy principle. In this paper, we employ Lattice Boltzmann Method to simulate two-dimensional incompressible steady channel flow field under low Reynolds number. By inserting the triangular prismatic obstacle with different vertex angles, the impact of the obstacle on the velocity and temperature fields is investigated, and Field Synergy Principle is used in the meantime to analyze and verify the results.

Mathematical modeling

Lattice Boltzmann method for velocity and temperature fields

In the research, the most widely-used D2Q9 Model [6] is utilized to simulate a two-dimensional flow field. The D2Q9 Model is comprised of square lattices on which there are three types of particles: a motionless particle, a horizontally- or vertically-moving particle, and a diagonally-moving (forms a 45-degree angle with the line which is horizontal or vertical) particle. The transfer velocity of the lattices is set to be $c = \delta x/\delta t = \delta y/\delta t$, where δx and δy represent the moving distances of the particles in the space. The velocity vector of the discrete lattices $\vec{e}_i$ is defined as follows:

$$\begin{gathered}\vec{e}_i = (0,0),\ i = 0,\ \vec{e}_i = (\pm c, 0), (0, \pm c), \\ i = 1,2,3,4,\ \vec{e}_i = (\pm c, \pm c), (\pm c, \pm c), i = 5,6,7,8\end{gathered} \tag{1}$$

Lattice Boltzmann method describes the particle distribution function $f(\vec{x}, \vec{V}, t)$ in the phase space formed by position and velocity; $\vec{x}$ and $\vec{V}$ are the position vector and velocity vector, respectively, and t denotes the time. The governing equation of the density distribution function is:

$$f_i(\vec{x} + \vec{e}_i\delta t, t + \delta t) - f_i(\vec{x}, t) = -\frac{1}{\tau_v}[f_i(\vec{x}, t) - f_i^{(eq)}(\vec{x}, t)] \tag{2}$$

where τ_v symbolizes the relaxation time in which the density distribution function approaches the local equilibrium state $f^{(\mathrm{eq})}$. The density distribution function of the equilibrium state is:

$$\begin{gathered}f_0^{(eq)} = \frac{4\rho}{9}\left[1 - \frac{3}{2}\frac{\vec{u}^2}{c^2}\right], \quad f_{1,2,3,4}^{(eq)} = \frac{\rho}{9}\left[1 + 3\frac{(\vec{e}_i\vec{u})}{c^2} + \frac{\rho}{2}\frac{(\vec{e}_i\vec{u}^2)}{c^2}\right], \\ f_{5,6,7,8}^{(eq)} = \frac{\rho}{36}\left[1 + 3\frac{(\vec{e}_i\vec{u})}{c^2} + \frac{9}{2}\frac{(\vec{e}_i\vec{u})^2}{c^2} - \frac{3}{2}\frac{\vec{u}^2}{c^2}\right]\end{gathered} \tag{3}$$

The macroscopic density and velocity fields can be expressed as:

$$\rho = \sum_i f_i, \quad \rho\vec{V} = \sum_i \vec{e}_i f_i \tag{4}$$

Concerning the thermal model in the study, we adopt Peng *et al.*'s [7] model. The simplified governing equation of energy distribution model is:

$$g_i(\vec{x}+\vec{e}_i\delta t, t+\delta t)-g_i(\vec{x},g)=-\frac{1}{\tau_c}[g_i(\vec{x},t)-g_i^{(eq)}(\vec{x},t)] \tag{5}$$

where τ_c represents the relaxation time as the energy distribution function approaches the local equilibrium state $g^{(eq)}$. The equilibrium state energy distribution function is:

$$g_0^{(eq)}=-\frac{2\rho\varepsilon}{3}\frac{\vec{V}^2}{c^2},\quad g_{5,6,7,8}^{(eq)}=\frac{\rho\varepsilon}{9}\left[\frac{3}{2}+\frac{3}{2}\frac{(\vec{e}_i\vec{V})}{c^2}+\frac{9}{2}\frac{(\vec{e}_i\vec{V})^2}{c^2}-\frac{3}{2}\frac{\vec{V}^2}{c^2}\right] \tag{6a}$$

$$g_{5,6,7,8}^{eq)}=\frac{\rho}{36}\left[1+3\frac{(\vec{e}_i\cdot\vec{V})}{c^2}+\frac{9}{2}\frac{(\vec{e}_i\cdot\vec{V})^2}{c^2}-\frac{3}{2}\frac{\vec{V}^2}{c^2}\right] \tag{6b}$$

Boundary conditions treatment

During the simulation performed by LBM, the density distribution function *f* and energy distribution function *g* are the ones evolved. The boundary conditions of the realistic problems, however, are about the boundary conditions of the macroscopic physical quantity; therefore, the microscopic boundary conditions of the corresponding distribution functions have to be set according to flowing macroscopic boundary conditions.

In this study, two kinds of channels are considered: one without cylindrical obstacles and one with an obstacle of triangular prism that has different vertex angles. We place the 90°-vertex angle isosceles triangular prism right in the center of the channel; the width of the channel is *H*; the distance from the centroid to the entrance is 10*H*/3, the length of the hemline is *H*/3, and the distances from the hemline to the upper and lower walls are both *H*/3. The cross-sectional area and the position of the centroid of the 50°-vertex angle isosceles triangular prism are the same as the 90° one.

Regarding the setting of the boundary conditions of flow field velocity, the uniform velocity distribution method ($U = U_\mathrm{m}$, $V = 0$) are used to deal with the entrance boundary, and the non-slip boundary conditions for the wall boundaries; the exit boundary, to which the insertion boundary method are applied, is the fixed outlet pressure.

With regard to the setting for the boundary conditions of flow field temperature, the entrance boundary ($T = T_\mathrm{m}$), wall boundaries ($T = T_\mathrm{m}$) and exit boundary ($T = T_\mathrm{out}$, $T_\mathrm{out} = T_\mathrm{w} < < T_\mathrm{m}$) are treated as isothermal boundaries. The main reason we place the triangular prism in the channels is simply to increase the turbulence in the flow field. To prevent other effects that could be unfavorable, the adiabatic boundary is employed for the temperature boundary of the triangular prism. For the setting for adiabatic boundary, the wall temperature gradient which was proposed by Shu *et al.* [8] is utilized.

In dealing with the boundaries, we adopt Zou *et al.* [9] proposal of applying the total rebounding rule to disequilibrium distribution functions, according to the correlation between the macroscopic physical quantity and the distribution functions, to yield an equation, by the boundary conditions, that satisfies the unknown numbers on the boundary lattices. As a result, the density distribution functions on the boundaries must satisfy

$$f_\alpha^{neq} = f_\beta^{neq} \tag{7}$$

and the energy distribution functions must satisfy

$$g_\alpha^{neq} - \vec{e}_\alpha^{\,2} f_\alpha^{neq} = -(g_\beta^{neq} - \vec{e}_\beta^{\,2} f_\alpha^{neq}) \tag{8}$$

among which $\vec{e}_\alpha$ and $\vec{e}_\beta$ goes in the opposite directions.

Results and discussion

The uniform square mesh (901·61) is used in the study. As the compression effect of LBM is proportional to the square of Mach number, the flow speed within the entrance must be controlled to be lower than the sound speed by 10% in order to avert considerable degree of compression. Consequently, we set the entrance speed to be an appropriate fixed value; through the adjustment to the relaxation factors, different Reynolds numbers are able to be acquired. In addition, the definition of the Nusselt number of the wall is used to determine convective heat transfer and conductive heat transfer.

From the concept of field synergy principle, the non-dimensional integral value should be as large as possible in order to augment the effect of convective heat transfer, which means that the intersection angle between the velocity vector and temperature gradient vector should be as small as possible. The physical meaning of the non-dimensional integral value is the totality of non-dimensional heat source intensity; the greater the intensity of the heat source, the better the effect of heat transfer.

Figure 1 displays the impact of the Reynolds number on the end-wall average Nusselt number in the channels which include the triangular prisms with different vertex angles. If the vertex angle of the triangular prism gets larger, the turbulence effect will be more obvious, which makes the average Nusselt number become larger as well, and the phenomenon will be more apparent with the growth of the Reynolds number. The end-wall average Nusselt number augments by 10.6% and 14.1% respectively when the triangular prisms with 50 and 90-degree vertex angles are placed into the channels under Re = 110. Figure 2 is the diagram of the influence of the Reynolds number on the non-dimensional integral value in the channels which include the triangular prisms with different vertex angles. It is found that the non-dimensional integral value of the channel including the triangular prism with 90-degree vertex angle is larger than that of the channel including the triangular

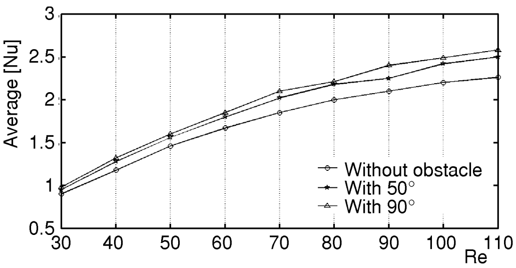

Figure 1. Impact of the Reynolds number on the end-wall varies average Nusselt number in the channels which include the triangular prisms with different vertex angles

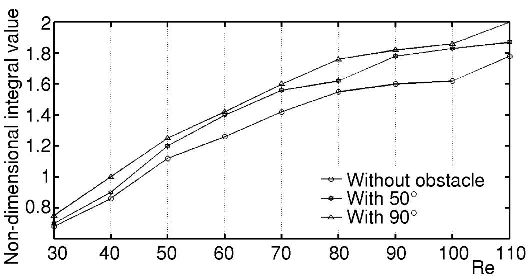

Figure 2. Influence of the Reynolds number on the non-dimensional integral value in the channels including the triangular prisms with different vertex angles

prism with 50-degree vertex angle and that of the obstacle-free channel; the phenomenon is more noticeable as the Reynolds number gets larger, which implies that the vertex angles of the obstacles greatly affect the heat transfer in the system (the greater the vertex angle, the better the effect of heat transfer). Figure 3 illustrates the impact of the Reynolds number on the average synergy angles in the channels which include the triangular prisms with different vertex angles. The placement of the triangular prisms effects better turbulence effect, and therefore promotes preferable synergy level between the velocity field and temperature gradient field. When Re = 110, the average synergy angles of the channels which include the triangular prisms with 50 and 90-degree vertex angles will make the approximately 1.4-degree and 4.01-degree differences if compared with those of the obstacle-free channel, which verifies the accuracy of the simulation results.

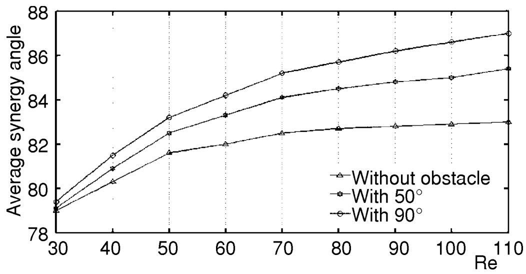

Figure 3. Influence of the Reynolds number on the average synergy angle in the channels including the triangular prisms with different vertex angles

Conclusions

In the study, the fluid flowing is simulated through the channels which include the triangular prisms with different vertex angles using Lattice Boltzmann Method. The triangular prisms play the part of the interference in the flow field; they partially alter the flowing path of the fluid. The circular back-flow that comes into existence in the rear of the triangular prisms affects the fluid that passes through, which influences the temperature gradient of the walls and partially augments the walls' Nusselt numbers. Via the Field Synergy Principle that is suitable for the backflow-accompanied oblong flowing, the effect of heat transfer in the flow field is analyzed from the perspective of the non-dimensionally integral values and average synergy angles. The analytical result proves that when the synergy level between the velocity field and temperature gradient field is higher, the effect of heat transfer will then be better, which successfully verifies the numerical results of the simulations.

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Nomenclature

$\vec{e}_i$ – velocity vector of the discrete lattices, [–]
f – particle distribution function, [–]
g – energy distribution function, [–]
$\vec{x}$ – position vector, [–]
$\vec{V}$ – velocity vector, [–]
t – time, [s]

Greek letters

ρ – density, [kgm^{-3}]
τ – relaxation time, [s]

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