Liu, K.-C., *et al.*: Analysis of Non-Fourier Thermal Behaviour for …
THERMAL SCIENCE, Year 2011, Vol. 15, Suppl. 1, pp. S61-S67

S61

ANALYSIS OF NON-FOURIER THERMAL BEHAVIOUR FOR MULTI-LAYER SKIN MODEL

by

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This paper studies the effect of micro-structural interaction on bioheat transfer in skin, which was stratified into epidermis, dermis, and subcutaneous. A modified non-Fourier equation of bio-heat transfer was developed based on the second-order Taylor expansion of dual-phase-lag model and can be simplified as the bio-heat transfer equations derived from Pennes' model, thermal wave model, and the linearized form of dual-phase-lag model. It is a fourth order partial differential equation, and the boundary conditions at the interface between two adjacent layers become complicated. There are mathematical difficulties in dealing with such a problem. A hybrid numerical scheme is extended to solve the present problem. The numerical results are in a good agreement with the contents of open literature. It evidences the rationality and reliability of the present results.

Key words: *bioheat transfer, dual-phase-lag mode, laplace transform, modified discretization.*

Introduction

The Pennes bioheat transfer model is commonly used to simulate thermal behavior in biological bodies due to simplicity and validity. The Pennes bio-heat equation describes the thermal behavior based on the classical Fourier's law which depicts an infinitely fast propagation of thermal signal. In reality, the living tissues are highly non-homogenous and need a relaxation time to accumulate enough energy to transfer to the nearest element. As a result, to solve the paradox occurred in the Pennes model, the thermal wave model of bioheat transfer was proposed for the investigation of physical mechanisms and the behaviors in thermal wave propagation in living tissues [1].

The properties of hyperbolic diffusion in bio-heat transfer have attracted the relevant researchers' attention. Liu [2] and Özen [3], respectively, studied the thermal wave propagation bioheat transfer in homogenous and multi-layer tissues. Shih *et al*. [4] explored the impact of thermal wave characteristics on thermal dose distribution during thermal

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therapy. However, in order to consider the effect of micro-structural interactions in the fast transient process of heat transport, a phase lag for temperature gradient absent in the thermal wave model was introduced [5]. The corresponding model is called the dual-phase-lag (DPL) model. Antaki [6] used the DPL model to interpret heat conduction in processed meat that was interpreted with the thermal wave model. Antaki [6] estimated the phase lag time of heat flux to be 14-16 s and the phase lag time of temperature gradient to be 0.043-0.056 s for processed meat. Xu *et al.* [7] presented a system discussion on the application of the DPL model in the biothermomechanical behavior of skin tissue. The more rational prediction of temperature distribution is always needed in the development of hyperthermia.

For more realistic predictions, this work would employ the DPL model of bioheat transfer to analyze the thermal behavior in skin, which was stratified into epidermis, dermis, and subcutaneous. The DPL equation of bio-heat transfer was always developed with the first-order Taylor series expansion of DPL model. For a more general form, this paper develops a modified DPL equation of bio-heat transfer based on the second-order Taylor expansion. For convenience of statement, this paper calls the former first-order DPL equation and another second-order DPL equation. The second-order DPL equation is a fourth order partial differential equation. Due to the difference in physiological and thermal properties, the boundary conditions at the interface between two adjacent layers become complicated. There are mathematical difficulties in dealing with such a problem. The hybrid numerical scheme [8] based on the Laplace transform and the modified discretization technique is extended to solve the present problem. The deviations of the results from the bioheat transfer equations based on Pennes' model, thermal wave model and dual-phase-lag model are presented and discussed.

Mathematical formulation

In order to solve the paradox occurred in the classical heat flux model and to consider the effect of micro-structural interactions, the DPL model was suggested [5] with:

$$\bar{q}(t + \tau_q) = -k\nabla T(t + \tau_T) \tag{1}$$

where T is the temperature, k – the heat conductivity, q – the heat flux, and t – the time. τ_q and τ_T can be interpreted as phase lags arising from "thermal inertia" and "micro-structural interaction", respectively [6]. τ_q means the phase lag of the heat flux and τ_T means the phase lag of the temperature gradient. The heat flux precedes the temperature gradient for $\tau_q < \tau_T$. The temperature gradient precedes the heat flux for $\tau_q > \tau_T$. The eq. (1) reduces to the thermal wave model by setting $\tau_T = 0$ and reduces to Fourier's heat equation by setting $\tau_q = \tau_T$.

Equation (1) is, usually, developed in the first-order Taylor series expansion. As the literature [7] did, this paper would develop eq. (1) with the second-order Taylor series expansion for a more general form. Thus, it is rewritten as:

$$\left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2} \frac{\partial^2}{\partial t^2}\right)\bar{q} = -\left(1 + \tau_T \frac{\partial}{\partial t} + \frac{\tau_T^2}{2} \frac{\partial^2}{\partial t^2}\right)k\nabla T \tag{2}$$

In a local energy balance, the energy conservation equation of bioheat transfer is described as:

$$-\nabla \cdot \bar{q} + W_b C_b (T_b - T) + q_m + q_r = \rho C \frac{\partial T}{\partial t} \tag{3}$$

where t is time. ρ, c, and T denote density, specific heat, and temperature of tissue. c_b and w_b are, respectively, the specific heat and perfusion rate of blood. q_m is the metabolic heat generation and q_r is the heat source for spatial heating. T_b is the arterial temperature.

Substituting eq. (2) into the energy conservation eq. (3) leads to the second-order DPL equation of bioheat transfer with constant physiological parameters as the following:

$$\left(1+\tau_T\frac{\partial}{\partial t}+\frac{\tau_T^2}{2}\frac{\partial^2}{\partial t^2}\right)k\nabla^2 T=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho c\frac{\partial T}{\partial t}-w_b c_b(T_b-T)-q_m-q_r\right] \quad (4)$$

As $\tau_T = 0$, eq. (4) undergoes to the hyperbolic heat transfer equation with negating the second-order terms of τ_q. Deleting the second-order terms of τ_q and τ_T leads to the first-order DPL equation of bio-heat transfer. On the other hand, eq. (4) would reduce to the Pennes equation for $\tau_q = \tau_T = 0$.

In reality, skin was stratified into epidermis, dermis, and subcutaneous, as shown in fig. 1. The three layers have different physiological and thermal properties. A situation of heating on skin surface was illustrated in the literature [7] as: At $t = 0$, the skin surface is suddenly taken into contact with a hot source of constant temperature 100 °C; after contacting for 15 s, the hot source is removed and skin is cooled by the coolant of 0 °C for 30 s. This paper focuses on predicting skin temperature distributions for this heating situation, and the present problem becomes one-dimensional bio-

Figure 1. Geometry and coordinates

heat transfer problem. Heat is assumed to be incident on the skin, so the spatial heating is equal to zero. The 1-D form of eq. (4) with constant thermal parameters for q_m = constant and $q_r = 0$ is written as:
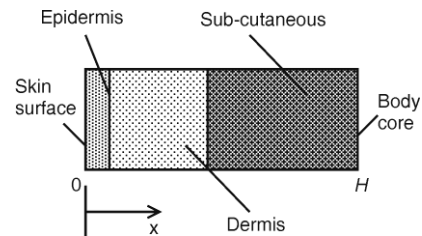
$$\left(1+\tau_T\frac{\partial}{\partial t}+\frac{\tau_T^2}{2}\frac{\partial^2}{\partial t^2}\right)k\frac{\partial^2 T}{\partial x^2}=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho c\frac{\partial T}{\partial t}-w_b c_b(T_b-T)\right]-q_m \quad (5)$$

The studied problem has the initial conditions:

$$T(x,0)=T_b \text{ and } \frac{\partial T(x,0)}{\partial t}=\frac{\partial^2 T(x,0)}{\partial t^2}=0 \quad (6)$$

and the boundary conditions

$$T(0,t)=100-100u(t-15) \qquad 45\geq t>0 \quad (7)$$

$$T(H,t)=T_b \qquad t>0 \quad (8)$$

The boundary conditions at the interfaces of two adjacent layers are obtained from the assumption that temperature and heat flux are continuous.

S64

Liu, K.-C., *et al.*: Analysis of Non-Fourier Thermal Behaviour for …
THERMAL SCIENCE, Year 2011, Vol. 15, Suppl. 1, pp. S61-S67

Numerical analysis

First, the Laplace transform method is employed to map the transient problem into steady one. Equation (5) and the boundary conditions can be written in the transform domain for the initial conditions as:

$$\frac{d^2 \tilde{T}_j}{dx^2} - \lambda_j^2 \tilde{T}_j = -g_j \quad \text{for } j = 1, 2, 3 \tag{9}$$

$$\tilde{T} = \frac{100(1 - e^{-15s})}{s} \quad \text{for } x = 0 \tag{10a}$$

$$\tilde{T} = T_b \frac{1}{s} \quad \text{for } x = H \tag{10b}$$

and at the interfaces of two adjacent layers

$$\tilde{T}_j = \tilde{T}_{j+1} \quad \text{for } j = 1, 2 \tag{10c}$$

$$\frac{d\tilde{T}_j}{dx} = \frac{K_{j+1}}{K_j} \frac{d\tilde{T}_{j+1}}{dx} \quad \text{for } j = 1, 2 \tag{10d}$$

Where

$$\lambda_j^2 = \frac{1}{K_j}[\rho_j c_j s + w_b c_b] \quad \text{for } j = 1, 2, 3 \tag{11}$$

$$K_j = \frac{k_j \left(1 + \tau_{T,j} s + \frac{\tau_{T,j}^2 s^2}{2}\right)}{1 + \tau_{q,j} s + \frac{\tau_{q,j}^2 s^2}{2}} \quad \text{for } j = 1, 2, 3 \tag{12}$$

$$g_j = \frac{q_m}{k_j \left(1 + \tau_{T,j} s + \frac{\tau_{T,j}^2 s^2}{2}\right) s} \quad \text{for } j = 1, 2, 3 \tag{13}$$

The function T is written as $\tilde{T}$ in the Laplace domain. s is the Laplace transform parameter with respect to t. The subscript j denotes the layer number.

To solve the present problem, a modified discretization scheme is proposed. Please refer to the literature [8] for the details of the present numerical scheme.

Results and discussion

Table 1 shows the values of relevant parameters used to perform all present computations in the present paper. For comparison and discussion, the arterial blood temperature and the perfuse rate are specified as $T_b = 37$ °C and $w_b = 0$, respectively. Some parameter values adjusted possibly and noted in each figure.

Liu, K.-C., *et al.*: Analysis of Non-Fourier Thermal Behaviour for …
THERMAL SCIENCE, Year 2011, Vol. 15, Suppl. 1, pp. S61-S67

S65

Table 1. Values of relevant parameters for calculations [7]

Parameters	Blood	Epidermis	Dermis	Sub-cutaneous
Specific heat [J/kgK]	3770.0	3600.0	3300.0	2700.0
Density [kg/m^3]	1060.0	1190.0	1116.0	971.0
Thermal conductivity [W/mK]	–	0.235	0.445	0.185
Metabolic heat generation [W/m^3]	–	368.1	368.1	368.3
Thickness [m]	–	0.0001	0.0015	0.0044

Figure 2 shows the variations of temperature predicted with various equations at the location $x = 0.1$ mm. The Pennes equation depicts an infinitely fast propagation of thermal signal, so it predicts that the thermal response is simultaneous with the surface heating. The temperature of the location $x = 0.1$ mm drops down at the end of heating immediately. The result from the Pennes equation is in agreement with that in the literature [7]. In the thermal wave equation, the effect of τ_q makes the thermal response delay. The transient temperatures occasionally exhibit the behavior of oscillation for the reflection-transmission effect at the interfaces. Tzou [5] stated that the DPL model can be regarded as the classical Fourier's law for $\tau_q = \tau_T$. It is observed that the results from the Pennes equation, the first-order DPL equation, and the second-order DPL equation are coincident for $\tau_q = \tau_T = 10$ s. However, they are different in the literature [7].

The temperature distributions in skin are presented in fig. 3 for $t = 45$ s. The behavior of reflection and transmission forms multi sharp discontinuities in the temperature distribution from the thermal wave equation. The major difficulty in the numerical solution of the hyperbolic heat transfer problem is numerical oscillations near the sharp discontinuities. The situation of multi wave-fronts and the composite interfaces of dissimilar materials increase the difficulty for solving such problems. It can be seen that the present numerical results do not exhibit severe numerical oscillations in the vicinity of the jump discontinuity. As $t = 45$ s, the thermal wave has not reached the boundary $x = H$. The domain $0.65 < x/H < 1$ is not disturbed. But,
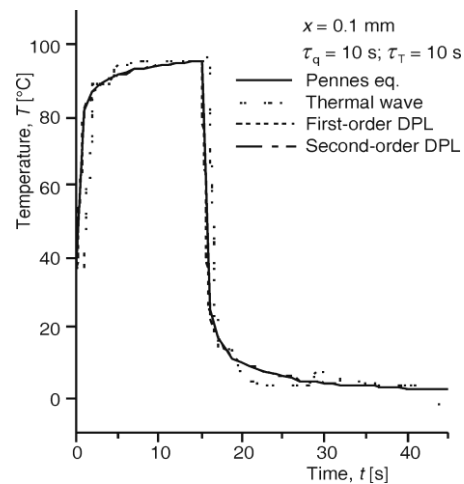

Figure 2. Variations of temperature predicted with various equations at the location $x = 0.1$ mm

Figure 3. Temperature distributions at $t = 45$ s

S66

Liu, K.-C., *et al.*: Analysis of Non-Fourier Thermal Behaviour for …
THERMAL SCIENCE, Year 2011, Vol. 15, Suppl. 1, pp. S61-S67

the skin surface temperature is below 0 $^{\circ}$C for the superposition of transmitted wave and reflected wave. It is also found that the temperature distributions are coincident for the Pennes equation, the first-order DPL equation, and the second-order DPL equation. This result implies the present results do not violate the corollary in the literatures [5]. In other words, the difference between the first-order DPL equation and the second-order DPL equation can not be shown as $\tau_q = \tau_T$.
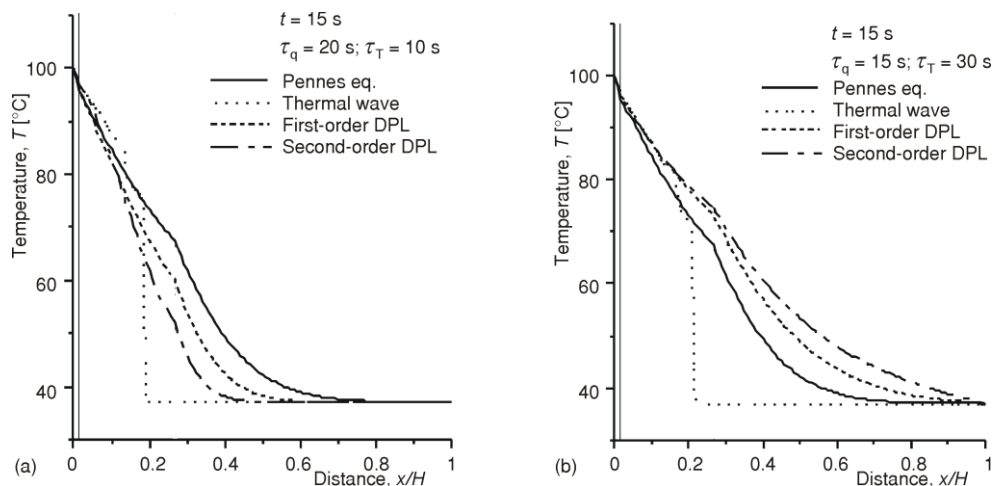

Figure 4. Temperature distributions at $t = 15$ s with: (a) $\tau_q = 20$ s, $\tau_T = 10$ s and (b) $\tau_q = 15$ s, $\tau_T = 30$ s

In order to depict the effects of the relaxation times, τ_q and τ_T, the predicted results of temperature distribution in skin are presented in fig. 4 for $\tau_q = 20$ s; $\tau_T = 10$ s and $\tau_q = 15$ s; $\tau_T = 30$ s at $t = 15$ s. The result from the Pennes equation also agrees with the literature [7]. The propagation speed of thermal wave has an inverse ratio to the value of τ_q. Therefore, the penetration depth of the wave-front for $\tau_q = 15$ s is larger than that for $\tau_q = 20$ s. The effect of τ_T destroys the structure of thermal wave, so the sharp wave-front does not appear in the results for the first-order DPL equation and the second-order DPL equation. The second-order DPL equation is more sensitive to the value of the relaxation times due to the second-order terms of τ_q and τ_T. The phenomenon of over diffusion would happen as $\tau_q/\tau_T < 1$[5]. Thus, the temperatures predicted with the first-order DPL equation and the second-order DPL equation for $\tau_q = 15$ s; $\tau_T = 30$ s are higher than that from the Pennes equation.

Acknowledgment

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Liu, K.-C., *et al.*: Analysis of Non-Fourier Thermal Behaviour for …
THERMAL SCIENCE, Year 2011, Vol. 15, Suppl. 1, pp. S61-S67

S67

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