Introductory paper

ANALYTICAL METHODS FOR THERMAL SCIENCE – AN ELEMENTARY INTRODUCTION

by

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Most thermal problems can be modeled by nonlinear equations, fractional calculus and fractal geometry, and can be effectively solved by various analytical methods and numerical methods. Analytical technology is a promising tool to outlining various features of thermal problems.

Thermal science appears everywhere, and it becomes one of the hottest topics in research especially due to global warming and a strong appeal to reducing carbon dioxide emissions. It becomes an efficient candidate to control global warming and a useful tool to optimal design of thermal processes. Analytical technology will play a special role in this aspect.

We just consider a cooling fin of thin rectangular section projecting from a hot plate held at a fixed temperature. The fin loses heat by convection to the surrounding air, and we want to minimize the lose. To this end, we first write down the governing equation, which reads (see the paper by Xu)

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - k\theta^\lambda = 0, \quad \theta(0) = a \tag{1}$$

We always need an approximate solution to outline the solution property. Recently some effective methods were appeared in open literature, for example, the homotopy perturbation method and the variational iteration method [1-7].

According to the homotopy perturbation method [6, 7], we need to establish a homotopy equation in the form

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - b^2\theta + p(b^2\theta - k\theta^\lambda) = 0 \tag{2}$$

where b is an unknown parameter to be further determined.

When $p = 0$, we have a linearized equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - b^2\theta = 0, \quad \theta(0) = a \tag{3}$$

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Its solution is $\theta = ae^{-bx}$, where the parameter, b, plays an important role in optimal design. The homotopy perturbation method admits a solution in the form

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_4 + \cdots \tag{4}$$

and an approximate solution can be obtained when p approaches to unity, that is

$$\theta = \theta_0 + \theta_1 + \theta_2 + \theta_4 + \cdots \tag{5}$$

The detailed solution procedure can be referred to ref. [5].

We can also construct variational iteration algorithms in the following forms [1]:

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x (s-x)\left[\frac{\mathrm{d}^2\theta_n(s)}{\mathrm{d}s^2} - k\theta_n^\lambda(s)\right]\mathrm{d}s \tag{6}$$

or

$$\theta_{n+1}(x) = \theta_0(x) - k\int_0^x (s-x)\theta_n^\lambda(s)\,\mathrm{d}s \tag{7}$$

initial solution can be chosen as $\theta_0 = a$ or $q_0 = a\mathrm{e}^{-bx}$, the solution procedure is elucidated in details in ref. [1].

The problem can be also solved by an ancient Chinese mathematics (see the paper by Xu) and other methods, please refer to the review articles [5, 6] to learn more analytical methods for thermal problems.

This special issue conveys a strong, reliable, efficient, and promising development of various analytical methods for thermal problems, for example, the heat-balance integral method (see the paper by Hristov), the Picard's iterative method (see the paper by Witula, Hetmaniok and Slota), the lattice Boltzmann method (see the paper by Wang, Yau, Lin and Kuo), the homotopy perturbation method (see the paper by Z. Z. Ganji, D. D. Ganji and H. D. Ganji), the parameterized perturbation method (see the paper by Jalaal *et al.*), and similarity solution (see the paper by Makinde).

Fractal approach to thermal problem was also outlined in this special issue by Huang and Xu. Using the Sierpinski fractal, Huang and Xu found that the thermal contact resistance can be best explained by fractional dimensions. It was reported that the thermal property of a fractal net reaches a maximum when the fractal dimensions are close to the golden ratio, $(5^{1/2} + 1)/2$, as shown in wool fibres [8].

For discontinuous media, the governing equations can be best approximated by fractional equations. Assume that the cooling fin consists of porous material, eq. (1) can be modified as

$$\frac{\mathrm{D}^\alpha\theta}{\mathrm{D}x^\alpha} - k\theta^\lambda = 0, \quad \theta(0) = a \tag{8}$$

where α is not an integer number. The analytical methods for fractional calculus were summarized in refs. [1, 2, 9]. The main problem of application of fractional calculus is different definitions of fractional derivative. To overcome the problem, this special issue suggests a novel fractal derivative (see the paper by Ji-Huan He).

Included in this special issue is a collection of original refereed research papers by well-established researchers in the field of thermal science and applied mathematics. I hope that these papers will prove to be a timely and valuable reference for researchers in this area.

Finally, I, on behave of the guest editor, would like to express my appreciation to all reviewers who took the time to review articles in a very short time.

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