

A THEORETICAL AND NUMERICAL STUDY OF THERMOSOLUTAL CONVECTION: STABILITY OF A SALINITY GRADIENT SOLAR POND

by

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A theoretical and numerical study of the effect of thermodiffusion on the stability of a gradient layer is presented. It intends to clarify the mechanisms of fluid dynamics and the processes which occur in a salinity gradient solar pond. A mathematical modelling is developed to describe the thermodiffusion contribution on the solar pond where thermal, radiative, and massive fluxes are coupled in the double diffusion. More realistic boundary conditions for temperature and concentration profiles are used. Our results are compared with those obtained experimentally by authors without extracting the heat flux from the storage zone. We have considered the stability analysis of the equilibrium solution. We assumed that the perturbation of quantities such as velocity, temperature, and concentration are infinitesimal. Linearized equations satisfying appropriate prescribed boundary conditions are then obtained and expanded into polynomials form. The Galerkin method along with a symbolic algebra code (Maple) are used to solve these equations. The effect of the separation coefficient ψ is analyzed in the positive and negative case. We have also numerically compared the critical Rayleigh numbers for the onset of convection with those obtained by the linear stability analysis for $Le = 100$, $\mu_a = 0.8$, and $f = 0.5$.

Key words: *salt gradient, solar pond, solar energy, salinity, thermodiffusion, solet effect, linear stability, numerical simulation*

Introduction

In recent years, there has been increasing efforts to promote renewable energy in developing countries [1, 2]. Solar pond is one such technology, which is both appropriate and relevant in the Algerian context. It is a simple and low cost solar energy system which collects solar radiation and stores it as thermal energy in the same medium for a long period of time [3, 4]. For this reason, solar ponds present some advantages with respect to other solar energy technologies which could make it competitive, especially, for example, in those regions where salt is available for free. Many experimental and theoretical works [5-12] basically concentrate on design, application, and experimental thermal measurements in solar ponds to investigate the thermal behaviour of various types of solar ponds in different dimensions [13, 14].

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Initially, the numerical models were generally one-dimensional (1-D) and treated the problem of transient heat conduction and mass diffusion [15]. Alargo [16] developed a 1-D simulation model, which simulates the transient behaviour of the pond using a finite difference method, for a closed cycle in a salt gradient solar pond. Husain *et al.* [17] studied the estimation of radiation flux in a solar pond and proposed a simple empirical formulation.

Only in few recent works, the 2-D and 3-D character of the solar pond have been considered while studying the same problem [18, 19] with both the transfer of mass and energy and variable properties of the saline solution have been considered [20, 21]. The results have shown that, in particular, the solar radiation absorption as well as the heat losses through the pond free surface may have adverse effects on the long-term stability of the pond. Very recently, Suarez *et al.* [22] investigated numerically the effect of double diffusive convection on the thermal performance and stability of a salt gradient solar pond. They showed that the temperature in the bottom of the solar pond increased from 20° to 52°, even though the insulating layer is being eroded by double-diffusive convection which have profound impacts on the overall stability of solar pond.

With regard to the pond stability that constitutes, as stated above, a major factor in its operation and performance, it has been studied analytically by several researchers who employed, in most cases, the well-known linear perturbations theory [23-26]. Results as obtained from these studies have provided meaningful details and information regarding the onset of these instabilities, the map of marginal stability of the pond as well as the existence of several possible stable or unstable states that may be encountered in the pond depending upon the ratio of the thermal Rayleigh number with respect to the salinity Rayleigh number. However, all these studies were limited to a layer of a fluid heated from below.

The extraction of the heat from the system could also cause the damage of the non-convective zone (NCZ) (see fig. 1), and must be performed with caution. Recently, an alternative method of heat extraction from these solar ponds has been investigated analytically [27], and used for industrial process heating [28], space heating, and power generation. In all these applications, heat is extracted from the bottom of the solar pond (that is, from the lower convective zone).

To the knowledge of the authors, the only work which has introduced the thermodiffusion effect into the salt diffusion equation and thus contributing to destabilize the salinity gradient layer has been done by Angeli *et al.* [29]. However, their study was confined to a 1-D mathematical model numerically solved by a finite difference schema ruling out the heat extraction from the bottom.

Nevertheless, the thermodiffusion continues to be the only hydrodynamic transport mechanism that lacks a simple physical explanation, because it strongly varies with the nature of the components of the mixture and with their concentration. In fact this effect cannot easily be measured experimentally and predicted theoretically. The aim of the present paper is therefore to present a numerical and analytical study of the thermodiffusion contribution on the solar pond using more realistic boundary conditions such as the heat extraction characterized by the fraction f and the absorption of solar radiation characterized by an extinction coefficient μ_a .

Mathematical formulation

Solar ponds generally consist of three regions: the upper convective zone (UCZ), the NCZ, and the lower convective zone (LCZ), as shown in fig. 1. The UCZ is the topmost layer of the solar pond. It is a relatively thin layer which consists almost wholly of fresh water. The NCZ

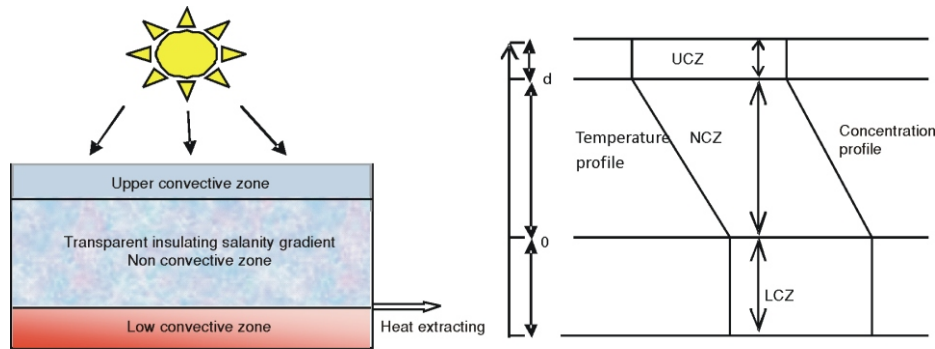


Figure 1. Schematic diagram of solar pond with typical temperature and salinity profiles

is just below the UCZ and has an increasing concentration relative to the UCZ. It also acts as a transparent insulator that lets sunlight reach the bottom where it remains entrapped, creating a storehouse of thermal energy in the form of hot brine. The LCZ is the layer in which the salt concentration is the greatest, and there is no concentration gradient in it. If the concentration gradient of the NCZ is great enough, no convective motion will occur in this region, and the energy absorbed in the bottom of the pond will be stored in the LCZ.

Let us then consider a pond in the NCZ, assumed to be a rectangular cavity of length L and depth d , filled with linearly stratified saline solution. The modelling of the gradient layer is based on the Boussinesq-Oberbeck approximation of the Navier-Stokes equations with two sources terms due to the absorption of the solar radiation in the energy equation and the thermodiffusion effect (also known as Soret effect) in the species equation. The dynamics of such system is written in their general form as:

$$\begin{aligned} \vec{\nabla} \cdot \vec{V} &= 0 \\ \rho_0 \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla}) \vec{V} &= -\vec{\nabla} P - \mu \nabla^2 \vec{V} + \rho_0 [1 - \beta_T (T - T_{ref}) - \beta_C (C - C_{ref})] g \vec{K} \\ \frac{\partial T}{\partial t} + (\vec{V} \cdot \vec{\nabla}) T &= \kappa \nabla^2 T + \frac{\dot{q}}{\rho C_p} \\ \frac{\partial C}{\partial t} + (\vec{V} \cdot \vec{\nabla}) C &= D \nabla^2 C + D_T C_0 (1 - C_0) \nabla^2 T \end{aligned} \quad (1)$$

where $\vec{V}(u, w)$ is the velocity field, P – the pressure, μ – the dynamic viscosity, κ – the thermal diffusivity, D – the Fick coefficient, and D_T – the thermodiffusion coefficient of the densest component. We assume that the product $C(1 - C)$ in the expression of the mass flux density vector of the densest component is constant and equal to $C_0(1 - C_0)$. \dot{q} is the radiative power per unit volume and C_0 is the initial concentration of the solute. The equation of state of the saline solution is given by:

$$\rho = \rho_0 [1 - \beta_T (T - T_{ref}) - \beta_C (C - C_{ref})] \quad (2)$$

where β_T and β_C are the coefficients of volumetric expansion for temperature and concentration, respectively. T is the dimensional temperature and C – the mass fraction of the densest component of the mixture. C_{ref} and T_{ref} are taken as the reference state, i. e., $C_{ref} = (C_1 + C_2)/2$ with C_1

and C_2 being the concentrations of the two horizontal walls, and $T_{\text{ref}} = [T(z=0) + T(z=d)]/2$. It may be useful to note that the two temperatures $T(z=0)$ and $T(z=d)$ are not *a priori* known; their values will be given by the steady-state profile of the temperature $T(z)$ in which we substitute $Z = 0$ and $Z = d$. The two lateral walls are supposed rigid, adiabatic, and perfectly insulated. The two horizontal walls are free implying that the vertical component of the velocity and its derivative with respect to the z variable are equal to zero. In the upper wall, the convective heat flux is produced by the zone of protection and on the lower wall a heat flux comes from the storage zone. All these conditions are given by the following equations:

$$\begin{aligned} C &= C_1, \quad \frac{\partial T}{\partial Z} = \frac{q}{\lambda} \quad \text{for } Z = 0, X \\ C &= C_2 = 0 \text{ (the pure water)}, \quad \frac{\partial T}{\partial Z} = \frac{h_d [T(z=d) - T_\infty]}{\lambda} \quad \text{for } Z = d, X \\ \frac{\partial C}{\partial X} &= \frac{\partial T}{\partial X} = 0 \quad \text{for } Z = 0, L, Z \\ w &= 0 \text{ and } \frac{\partial u}{\partial Z} = 0 \quad \text{for } Z = 0, d, X \end{aligned} \quad (3)$$

where h_d is the convective heat transfer coefficient and λ is the thermal conductivity of the water.

Study of the basic state

The steady-state solutions can be obtained by equating velocity and all time derivatives to zero in the set of eqs. (1).

Thus, we obtain:

$$\begin{aligned} \vec{P} &= \rho_0 [\beta_T (T - T_{\text{ref}}) - \beta_c (C - C_{\text{ref}})] g \vec{K} \\ \Delta T &= \frac{\dot{q}}{\lambda} \quad \text{with } \lambda = \rho C_p k \\ D \Delta C &= D_T C (1 - C_0) \Delta T \end{aligned} \quad (4)$$

The steady-state solution for the temperature profile is determined by assuming that the absorption of the solar radiation can be modeled by an extension coefficient μ_e that takes into account the transparency of the fluid (Lambert law). It can be expressed by the rate of energy generation per unit volume in the layer:

$$\dot{q}(d) = q(d) \mu_e \exp[-\mu_e (d - Z)] \quad (5)$$

The steady-state equation for the concentration is obtained by resolving the species equation in the set of eq. (4):

$$\frac{\partial^2 C}{\partial Z^2} = S_T C_0 (1 - C_0) \frac{\dot{q}}{\lambda}$$

where $S_T = D_T/D$ stands for the Soret coefficient. We then obtain:

$$\begin{aligned} \frac{\partial^2 C}{\partial Z^2} &= S_T C_0 (1 - C_0) \frac{q(d) \mu_e \exp[-\mu_e (d - Z)]}{\lambda} \\ \frac{\partial X}{\partial Z} &= S_T C_0 (1 - C_0) \frac{q(d) \mu_e \exp[-\mu_e (d - Z)]}{\lambda} = A \end{aligned}$$

$$C(Z) = S_T C_0 (1 - C_0) \frac{q(d) \mu_e \exp[\mu_e(d - Z)]}{\lambda} + AZ + B \quad (6)$$

To evaluate the integration constants A and B , we introduce the boundary conditions $C(Z = 0) = C_1$ and $C(Z = d) = C_2$. Equation (6) for the steady-state solution $C(Z)$ can be finally written as:

$$C_0(Z) = \frac{S_T C_0 (1 - C_0) q(d) \exp(-\mu_e d)}{\lambda \mu_e} \exp(\mu_e Z) + \frac{Z}{d} [\exp(\mu_e d) - 1] + 1 - \frac{C_2 - C_1}{d} Z - C_1 \quad (7)$$

At $Z = d$, $\dot{q}(Z) = q(d)$ is the heat flux due to solar radiation at the upper boundary of the gradient zone. Thus, the steady-state heat diffusion in the set of eqs. (4) with heat generation becomes:

$$\frac{\partial^2 T}{\partial Z^2} = \frac{\dot{q}}{\lambda} - \frac{q(d) \mu_e \exp[\mu_e(d - Z)]}{\lambda} + A \quad (8)$$

where A is the constant of integration. At each boundary of the gradient zone, the heat flux by conduction has to be equal to the heat transferred by convection leading to:

$$\frac{\partial T}{\partial Z}(Z = 0) = \frac{q}{\lambda} \quad \text{and} \quad \frac{\partial T}{\partial Z}(Z = d) = \frac{h_d [T(d) - T_\infty]}{\lambda} \quad (9)$$

In order to guarantee a global energy balance, the heat flux from the storage zone is set equal to the difference between the total heat absorbed in the storage zone and the total heat extracted per unit area in the same zone, *i. e.*:

$$q = q_{\text{abs}} - q_{\text{ext}} \quad (10)$$

The total heat absorbed in the storage zone can be expressed as:

$$q_{\text{abs}} = \int_0^d \dot{q}(Z) dZ = q(d) \exp(-\mu_e d) \quad (11)$$

The extracted heat flux (q_{ext}) can be represented as a fraction f of the total heat flux (q_{abs}) absorbed in the LCZ:

$$q = q_{\text{abs}} - q_{\text{ext}} = (1 - f)q_{\text{abs}} = (1 - f)q(d) \exp(-\mu_e d) \quad (12)$$

$$q(d) \exp(-\mu_e d) [1 - (1 - f)] - q(d) = -h_d [T(d) - T_\infty]$$

where

$$q(d) [1 - f \exp(-\mu_e d)] = h_d [T(d) - T_\infty] \quad (13)$$

Solving eqs. (8), (11), (12), and (13), we obtain the following temperature profile:

$$T_0(Z) = T_\infty + \frac{q(d)}{\lambda} \exp(-\mu_e d) \frac{\exp(\mu_e Z)}{\mu_e} - f(Z - d) + \frac{q(d)}{\lambda \mu_e} - \frac{1}{h_d} \frac{q(d)}{h_d} f \exp(-\mu_e d) \quad (14)$$

The case without heat extraction from the pond ($f=0$) represents the worst situation for the onset of instabilities. In this case ($f=0$), eq. (14) becomes:

$$T_0(Z) = T_\infty + \frac{q(d)}{\lambda} \exp(-\mu_e Z) - \frac{\exp(\mu_e Z)}{\mu_e} + q(d) \frac{1}{\lambda \mu_e} - \frac{1}{h_d} \quad (15)$$

Substituting $Z=0$ and $Z=d$ in eq. (14), we can easily determine the reference temperature as:

$$T_{\text{ref}} = \frac{T(Z=0) + T(Z=d)}{2} = T_\infty + \frac{q(d)}{2\lambda\mu_e} [1 - \exp(-\mu_e d)] + q(d) \frac{1}{h_d} - \frac{1}{h_d} - \frac{d}{2\lambda} f \exp(-\mu_e d) \quad (16)$$

enabling us to determine:

$$\Delta T = T(Z=0) - T(Z=d) = \frac{q(d)}{\lambda\mu_e} \exp(\mu_e d) [\exp(\mu_e d) - 1 - fd\mu_e] - 0$$

As is evident from the former equations, the temperature and concentration fields are functions of the vertical direction Z , μ_e , f , $q(d)$, T_∞ , h_d , S_T , C_1 , C_2 , and C_0 .

Dimensionless formulation of equations

The dimensionless formulation is obtained by using the following reference variables: d for the depth, d^2/κ for the time, κ/d for the velocity, $\rho_0(\kappa^2/d^2)$ for the pressure, $\lambda\Delta T/d$ for the heat flux, λ/d for the coefficient of convective heat transfer, $(C - C_{\text{ref}})/\Delta C$ for the concentration, and $(T - T_{\text{ref}})/\Delta T$ for the temperature, where $\Delta C = C_1 - C_2$ and $\Delta T = T(z=0) - T(z=d)$. Thus, the adimensional equations of conservation for mass, momentum, energy, and chemical species which take into account the absorption of solar radiation characterised by an extinction coefficient μ_a , the heat flux extracted from the storage zone characterised by the fraction f , and the thermodiffusion effect characterised by the coefficient of separation ψ , are given by:

$$\begin{aligned} \frac{\partial \bar{V}^*}{\partial t^*} &= (\bar{V}^* - \bar{V}^*) \bar{V}^* - \bar{P}^* + \text{Pr} \Delta \bar{V}^* - \text{Pr} (\text{Ra}_t T^* - \text{Ra}_s C^*) \bar{K} \\ \frac{\partial T^*}{\partial t^*} &= (\bar{V}^* - \bar{V}^*) T^* - 2T^* + \frac{\mu_a \exp(\mu_a Z)}{\exp(\mu_a) - 1} f \mu_a \\ \frac{\partial C^*}{\partial t^*} &= (\bar{V}^* - \bar{V}^*) C^* + \frac{1}{\text{Le}} (2C^* - \psi \frac{\text{Ra}_t}{\text{Ra}_s} - 2T^*) \end{aligned} \quad (17)$$

The considered problem involves therefore seven dimensionless parameters: thermal Rayleigh number $\text{Ra}_t = g\beta_T \Delta T d^3 / \kappa \nu$, solutal Rayleigh number $\text{Ra}_s = g\beta_c \Delta C d^3 / \kappa \nu$, the coefficient of separation $\psi = -(\beta_c / \beta_T)(D_T / D) C_0(1 - C_0)$, the Prandtl number $\text{Pr} = \nu / \kappa$, the Lewis number $\text{Le} = \kappa / D$, the extinction coefficient $\mu_a = \mu_e d$, and the fraction of extracted heat flux f . The associated dimensionless boundary conditions read as:

$$\begin{aligned}
 C^* &= \frac{1}{2} \frac{\partial T^*}{\partial Z^*} \frac{(1-f)\mu_a \exp(\mu_a)}{1 - (1-f)\mu_a \exp(\mu_a)} \quad Z^* = 0, \quad X^* \\
 C^* &= \frac{1}{2} \frac{\partial T^*}{\partial Z^*} \frac{\mu_a [1 - f \exp(\mu_a)]}{1 - (1-f)\mu_a \exp(\mu_a)} \quad Z^* = 1, \quad X^* \\
 \frac{\partial C^*}{\partial X^*} &= \frac{\partial T^*}{\partial X^*} = 0 \quad \text{and} \quad \bar{V}^* = \bar{0} \quad X^* = 0, A \quad Z^* \\
 w^* &= 0 \quad \text{and} \quad \frac{\partial u^*}{\partial Z^*} = 0 \quad Z^* = 0, 1 \quad X^*
 \end{aligned} \tag{18}$$

The corresponding dimensionless temperature and concentration fields for the steady-state solution are:

$$T_0^*(Z^*) = \frac{0.5 \exp(\mu_a) \exp(\mu_a Z^*) - 0.5 - f(Z^* - 0.5)\mu_a}{\exp(\mu_a) - 1 - f\mu_a} \tag{19}$$

$$C_0^*(Z^*) = \psi \frac{Ra_t \exp(\mu_a) \{ \exp(\mu_a Z^*) - Z^* \exp[\mu_a - 1] - 1 \}}{Ra_s [1 - (1-f)\mu_a \exp(\mu_a)]} \quad Z^* = \frac{1}{2} \tag{20}$$

In the following, the dimensionless quantities will be no starry in order to simplify the notation.

3. Linear stability analysis

We introduce into the system of eqs. (17) and (18) the perturbed fields:

$$\bar{V} = \bar{v}, \quad T = T_0 + \theta, \quad C = C_0 + c, \quad P = P_0 + p \tag{21}$$

where $\bar{v}, \theta, c,$ and p are infinitesimal perturbations, and second order terms are neglected in the perturbed equations. The non-dimensionalized equations involving the perturbed variables take the form:

$$\begin{aligned}
 \frac{\partial \bar{v}}{\partial t} &= -p - \text{Pr}(Ra_t \theta - Ra_s c) \bar{k} \\
 \frac{\partial \theta}{\partial t} &= w \frac{\partial T_0}{\partial Z} - \theta \\
 \frac{\partial c}{\partial t} &= w \frac{\partial C_0}{\partial Z} - \frac{1}{Le} c - \psi \frac{Ra_t}{Ra_s} \theta
 \end{aligned} \tag{22}$$

where w and C_0/Z are functions of the variable Z . Taking the rotational of the Navier-Stokes equation and keeping only the stationary transition, eqs. (22) can be rewritten as:

$$\begin{aligned}
 4w \frac{\partial^2}{\partial X^2} (Ra_t \theta - Ra_s c) &= 0 \\
 w \frac{\partial T_0}{\partial Z} - \theta &= 0 \\
 w \frac{\partial C_0}{\partial Z} - \frac{1}{Le} c - \psi \frac{Ra_t}{Ra_s} \theta &= 0
 \end{aligned} \tag{23}$$

with the following associated boundary conditions:

$$\begin{aligned} c &= 0 \text{ for } Z = 0 \text{ and } Z = 1 \\ \frac{\partial \theta}{\partial Z} &= 0 \text{ for } Z = 0 \text{ and } Z = 1 \\ w &= 0 \text{ for } Z = 0 \text{ and } Z = 1 \text{ and } \frac{\partial u}{\partial X} = 0 \end{aligned} \quad (24)$$

A cell of infinite extension in the x-direction is considered. The velocity, temperature and concentration perturbations are developed in normal modes as:

$$(w, \theta, c) = (w(Z), \theta(Z), c(Z)) \exp(ikX - \sigma t) \quad (25)$$

where $w(Z)$, $\theta(Z)$, and $c(Z)$ are functions which depend on only the variable Z , k is the wave number along x , and σ the temporal expansion rate of the perturbation which is generally complex, i. e., $\sigma = \sigma_r + i\omega$ and $i^2 = -1$ (the neutral stability curve corresponds to $\sigma_r = 0$). The perturbations are then expanded using polynomial series as:

$$\begin{aligned} w(Z) &= a_1(Z - 2Z^3 + Z^4) + a_2(Z - 2Z^3 + 6Z^4 - 3Z^5) + \dots + a_i(1 - Z)^3 Z^i \\ \theta(Z) &= b_1 - b_2 Z^2 - 2b_2 \frac{Z^3}{3} + \dots + b_{i-2}(Z - 1)^2 Z^{i-1} \\ c(Z) &= \dots + c_i(1 - Z) Z^i \end{aligned} \quad (26)$$

Substituting expansions (26) in (23), we obtain a new set of equations which satisfy the whole required boundary conditions and which we solve using the well-known Galerkin method.

Results and discussion

To test the validity of the used linear stability analysis procedure, we study in a first time, the case of a pure fluid ($\psi = 0$) and our results are depicted in fig. 2. This case allows us to

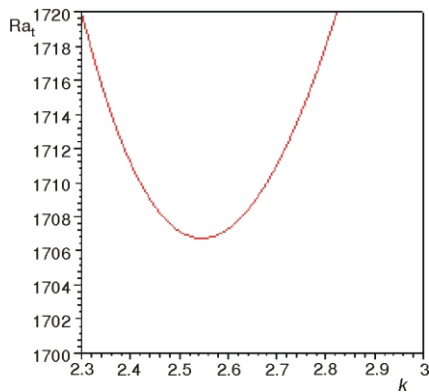


Figure 2. Rayleigh number vs. wave number for $\mu_a = 0.2$, $Le = 94.3$, and $\psi = 0$ (pure fluid case)

make a comparison with the results already obtained by Chandrasekhar [30].

As is evident from fig. 2, the marginal stability leading to an initiation of the oscillatory motion (onset of natural convection) is around $Ra_t = 1706.98$ and $k = 2.6$ (the minimum of the curve). This result agrees very well with the one calculated by Chandrasekhar [30] and which is equal to 1707.76 for impermeable and infinites walls, implying that the expansion polynomials which we propose are appropriate.

Figure 3 presents the temperature and concentration profiles in the steady-state case, with $f = 0$ (without extracting the heat from the pond). These profiles are in good agreement with those found experimentally [31-33].

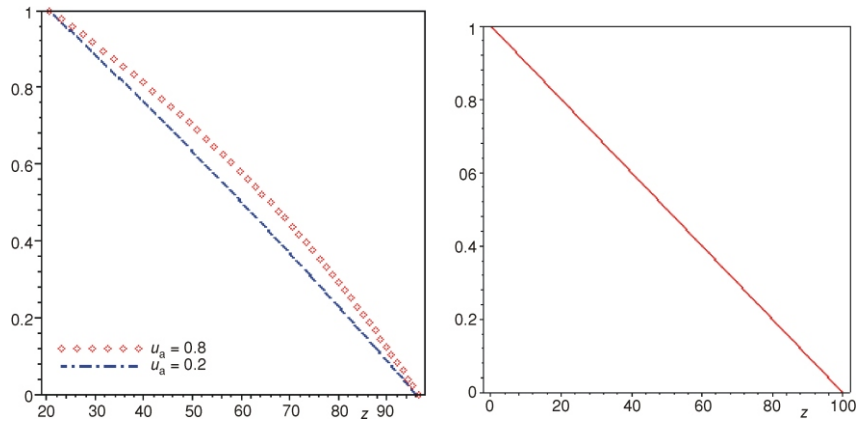


Figure 3. Steady-state temperature profile (left) and concentration profile (right) in the case of $f = 0$

The thermodiffusion in the solar pond

To the best of our knowledge, there are few data about thermodiffusion in solar ponds. Rothmeyer [34] calculated salt fluxes for the University of New Mexico solar pond and found that the flux due to Soret effect rises to almost 30% in the summer when the temperature gradients are strong. However, his estimation comes from a rough analysis of various terms of the diffusion equations, without any solution of differential equations. Angeli *et al.* [29] introduced the thermodiffusion effect into the salt diffusion and solved numerically a 1-D mathematical model. The later has been dimensioned as the one operating at El Paso (Texas). Their results showed that the thermodiffusion can reach an annual averaged percentage of the total flux close to 10%, with picks of about 15% in the summer at the bottom of the NCZ. In this work and to get better insight onto the problem, we use the study of the linear stability reported above to describe the thermodiffusion contribution to the salt diffusion within a gradient layer. First, we vary the coefficient of separation ψ according to the critical Rayleigh number corresponding to the marginal state conditions for the stability of the gradient zone in the infinite extension solar ponds.

Positive separation coefficient case ($\psi > 0$)

In view of the results reported in tab. 1, the convection occurs latter than in the pure fluid case ($\psi = 0$). The wave numbers decrease with increasing ψ , in contrast to the Rayleigh-Benard problem, where the critical Rayleigh number associated with the first bifurcation decreases. This means that the number of cells contained in the gradient layer of the solar pond decreases, indicating therefore that the system is more stable when ψ is increased (fig. 4).

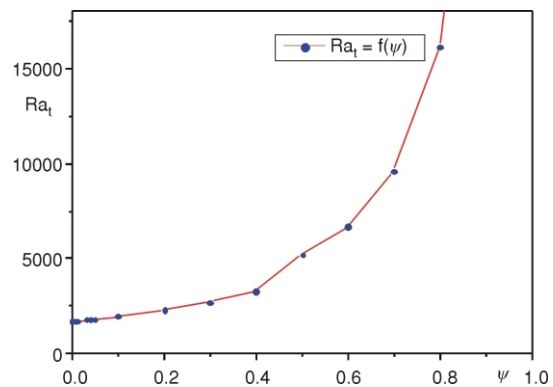


Figure 4. Critical Rayleigh number vs. separation factor ($\psi > 0$), $\mu_a = 0.2$, $Le = 100$, and $f = 0.5$

Table 1. Critical values of the thermal Rayleigh number and the wave number for different positive values of ψ , with $\mu_a = 0.2$, $Le = 100$, and $f = 0.5$

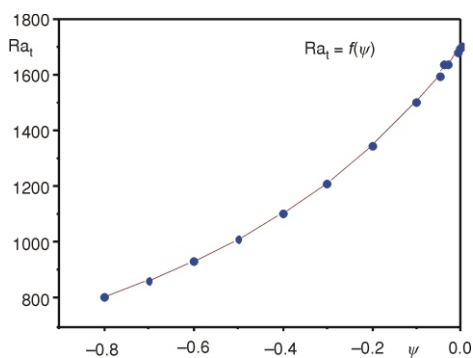
ψ	0	0.0005	0.005	0.01	0.03	0.04	0.05
Ra_t	1706.72	1707.8	1717.97	1729.35	1776.26	1800.5	1825.4
k	2.545	2.543	2.54	2.534	2.51	2.49	2.486
ψ	0.1	0.2	0.3	0.4	0.5	0.6	0.8
Ra_t	1959.76	2286.9	2718.9	3293.2	5217.66	6695.3	16161
k	2.424	2.294	2.154	1.199	1.4	1.43	1.189

Negative separation coefficient case ($\psi < 0$)

In tab. 2, when changing the ψ values, the evolution of the critical Rayleigh number with the wave numbers, is opposite to the case studied ($\psi > 0$). However, the convection is advanced with decreasing $\psi < 0$ to compare to the pure fluid, this may explain the system instability for negative values of ψ and the waves number increases producing thermosolutal cells formation in the gradient layer, fig. 5.

Table 2. Critical values of the thermal Rayleigh number and the wave number for different negative values of ψ , with $\mu_a = 0.2$, $Le = 100$, and $f = 0.5$

ψ	0	-0.0005	-0.01	-0.04	-0.1	-0.2
Ra_t	1706.7	1705.6	1684.6	1641.9	1506.7	1345.5
k	2.545	2.546	2.557	2.581	2.66	2.767
ψ	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8
Ra_t	1213.4	1103.4	1010.7	931.72	863.6	804.39
k	2.86	2.96	3.048	3.129	3.2	3.27

**Figure 5. Critical Rayleigh number vs. the separation factor ($\psi < 0$), with $\mu_a = 0.2$, $Le = 100$, and $f = 0.5$** **Determination of the critical values**

In this part, we determine numerically the critical value of the thermal Rayleigh number for the relative values of a solar pond, namely, $\mu_a = 0.8$, $f = 0.5$, $Le = 100$ and for two different values of ψ . In order to detect the onset of the convective movement, the manner consists in varying the Rayleigh number around the value given by the stability of the linear analysis and to observe the temporal evolution of the different fields. After that, we can refine the range of this critical value leading to a more fine determination of this critical value of the Rayleigh number (see fig.6). A 2-D

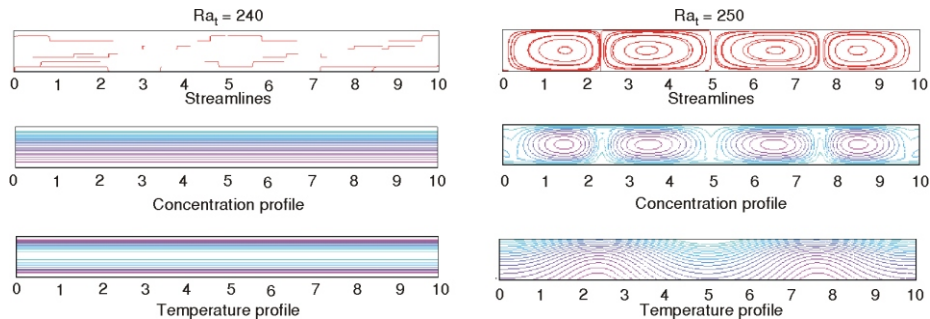


Figure 6. Projection of streamlines, isoconcentration and isotherms for $Le = 100$, $\mu_a = 0.8$, $f = 0.5$, and $\psi = -0.7$

Femlab [35] model is performed, using the built-in thermal and concentration calculations, which uses a finite element method for high Lewis number. The system of eqs. (17) is then integrated numerically taking into account the appropriate boundary conditions (18). The following parameters $\mu_a = 0.8$, $f = 0.5$, $\psi = -0.7$, $\psi = 0.5$, and $A = L/H = 10$ (large enough to simulate the convective motions in an infinite length along the x-direction) have been chosen. In fig. 6, we present for the case $\psi = -0.7$, the streamlines, isotherms and isoconcentrations before and after the transition towards a convection.

On tab. 3, we show an acceptable agreement between the results obtained by the numerical simulation and those obtained by the linear stability analysis.

Table 3. Critical values of thermal Rayleigh number and the wave number for two different values of ψ , with $\mu_a = 0.2$, $Le = 100$, and $f = 0.5$

	Linear stability	Numerical results
	Ra_t	Ra_t
$\psi = 0.5$	319	290
$\psi = -0.7$	259.57	250

Conclusions

The double diffusive problem has been extensively studied by different authors in the context of solar ponds. Linear profiles of salt and temperature were generally assumed and stability criteria for this situation has been determined considering a layer of fluid heated from below.

Nevertheless, due to the solar radiation absorption in the gradient zone of a solar pond, the initial profile and the boundary conditions for temperature in the upper and the lower of the gradient zone are not *a priori* known such as the concentration; their values depend on others conditions such as the depth Z of the pond, the heat extracted from bellow and the thermodiffusion contribution acting in the salt diffusion equation which cannot be neglected.

In the present paper, we have considered the thermosolutal gradient layer of a solar pond with more realistic boundary conditions for temperature and concentration profiles. The analysis of this problem is performed by solving the Boussinesq approximation of Navier Stokes equations with two sources-terms due to the absorption of the solar radiation in the energy equation and the thermodiffusion effect in the species equation.

The stability of these equations is analysed and the marginal states for the onset of convection are obtained using a Galerkin method based on a weak formulation of the governing

equations for perturbations. The results are in good agreement with the previous results obtained by authors for $\psi = 0$, in the case of a fluid heated from below. Also, upon studying the effect of the coefficient of separation ψ for positive and negative case, we showed that the convection occurs late when increasing ψ in the positive case and the wave number decreases. However, the convection is advanced with decreasing $\psi < 0$ in comparison to the pure fluid case. A brief comparison was also performed in order to determine the critical value of the Rayleigh number and to visualize the temporal evolution of the different fields, namely, velocity, temperature, and concentration. The results were shown to be in a satisfactory agreement with those obtained from the linear analysis.

Nomenclature

A	– aspect ratio ($=d/L$)	X	– dimensional horizontal co-ordinate, [m]
C	– salt concentration, [$\text{kg}^{-1}\text{m}^{-1}$]	Z	– dimensional vertical co-ordinate, [m]
c	– perturbation of concentration	<i>Greek letters</i>	
C_p	– specific heat of the solution, [$\text{Jkg}^{-1}\text{°C}^{-1}$]	β_T	– coefficient of thermal expansion, [°C^{-1}]
D	– coefficient of salt diffusion, [m^2s^{-1}]	β_c	– coefficient of salt expansion, [°C^{-1}]
d	– depth of gradient zone, [m]	κ	– thermal diffusivity of the fluid, [$\text{Wm}^{-1}\text{°C}^{-1}$]
f	– the fraction of the heat flux extracted from the storage zone	λ	– thermal conductivity of water, [$\text{Wm}^{-1}\text{°C}^{-1}$]
g	– acceleration of gravity, [ms^{-2}]	ρ	– fluid density, [kgm^{-3}]
h_d	– convection heat transfer coefficient, [$\text{Wm}^{-2}\text{°C}^{-1}$]	ψ	– separation coefficient
K	– wave number	μ_a	– dimensionless coefficient of extinction ($=\mu_e d$)
\vec{k}	– unit vector pointing upwards	μ_e	– dimensional coefficient of extinction, [m^{-1}]
L	– length of gradient zone, [m]	θ	– perturbation of temperature
Le	– Lewis number, [–]	<i>Indices</i>	
Pr	– Prandtl number, [–]	0	– initial
\dot{q}	– heat generated per unit time and volume, [Wm^{-3}]	<i>Acronyms</i>	
Ra_t	– thermal Rayleigh number, [–]	LCZ	– lower convective zone
Ra_s	– solutal Rayleigh number, [–]	NCZ	– non-convective zone
S_T	– Soret coefficient, [°C^{-1}]	UCZ	– upper convective zone
T	– temperature, [°C]		
T_∞	– upper convective zone temperature, [°C]		
u	– horizontal component of velocity, [m^2s^{-1}]		
w	– vertical component of velocity, [ms^{-1}]		

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