EFFICIENCY AND COST MODELLING OF THERMAL POWER PLANTS

by

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The proper characterization of energy suppliers is one of the most important components in the modelling of the supply/demand relations of the electricity market. Power generation capacity i. e. power plants constitute the supply side of the relation in the electricity market. The supply of power stations develops as the power stations attempt to achieve the greatest profit possible with the given prices and other limitations. The cost of operation and the cost of load increment are thus the most important characteristics of their behaviour on the market. In most electricity market models, however, it is not taken into account that the efficiency of a power station also depends on the level of the load, on the type and age of the power plant, and on environmental considerations. The trade in electricity on the free market cannot rely on models where these essential parameters are omitted. Such an incomplete model could lead to a situation where a particular power station would be run either only at its full capacity or else be entirely deactivated depending on the prices prevailing on the free market. The reality is rather that the marginal cost of power generation might also be described by a function using the efficiency function. The derived marginal cost function gives the supply curve of the power station. The load level dependent efficiency function can be used not only for market modelling, but also for determining the pollutant and CO_2 *emissions of the power station, as well as shedding light on the conditions for successfully entering the market. Based on the measurement data our paper presents mathematical models that might be used for the determination of the load dependent efficiency functions of coal, oil, or gas fuelled power stations (steam turbine, gas turbine, combined cycle) and IC engine based combined heat and power stations. These efficiency functions could also contribute to modelling market conditions and determining the environmental impact of power stations.*

Key words: *power plant, efficiency function, cost function, mathematical model*

Introduction

Fundamentals

It is indispensable to know the features of each individual power plant in order to simulate the power system as a whole. In a simulation model developed for examining load state,

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the role of each element is described by functions that give the changes to output features considering technology-specific parameters that vary according to changes in the input features. By input feature we mean either the load status (as a percent of nominal performance) or the allotted performance (in natural units, *i. e.* MW). Models of power plant units provide the following output on the ground of parameters referring to technologies of energy transformation: efficiency, fuel consumption, electricity; and with auxiliary parameters they are capable of determining fuel costs. Knowing the properties of the individual power plants is essential for the simulation of the power system. The elements/entities of a simulation model are described by its characteristics. These characteristics describe the changes in the outputs as a function of changes in the input parameters, considering the technology specific parameters.

For the purposes of the simulation the input parameter is the load state (for example in the percentage of the nominal load) or the desired power (in natural unit, *i. e.* MW). On the basis of the parameters relating to the energy conversion technologies employed in the various power plants, models of power plant units give the following output data: efficiency, fuel consumption, output heat, and power.

Classification of power plants

Extended power plant systems employ several types of power plant in various different versions. It would be impractical to devise different functions of efficiency and expense for all the various types of power plants, so for practical purposes we classified power plants into groups based on certain criteria. The behaviour of power plants belonging to the same group can be described by the same functions. The criteria used to classify the power stations should result in groups which are not too large or too numerous, but at the same time include the majority of power plants extant. In our study [1] the following groups were identified using data about European power plants (36,189 units) within which further subgroups can be defined:

(1) Gas turbines (GT): gas turbine in combined-cycle, gas turbine with heat recovery for desalination, gas turbine with heat recovery, gas turbine used for partial or complete steam-turbine repowering, gas turbine with steam sendout, gas turbine in topping configuration with existing conventional boiler and turbine/generator set;
(2) Hydro power plants (HY);
(3) Internal combustion engines (IC): internal combustion engine with heat recovery, internal combustion engine with steam sendout
(4) Steam power plants (ST): steam turbine, steam turbine in combined-cycle, steam turbine with steam sendout;
(5) Combined cycle power plants (CC): combined cycle (simple), combined cycle single shaft configuration;
(6) Wind turbines (WT): wind turbine generator, wind turbine generators located off-shore;
(7) Solar power plants (SOL): photovoltaic cells, solar collector including solar dish arrays, closed-cycle vapour turbogenerator;
(8) Other types of power plants (other): turbo expander/gas expander, reciprocating steam engine, fuel cell.

Figure 1 illustrates the capacity distribution by type of the operating power plants listed. It can be seen from the data displayed in the chart that in Europe power generation comes mostly from steam turbine thermal power plants and hydro power plants. A notable peculiarity of the statistical data collection – over which we had no influence – is that most combined cycle

type power plants are classified here as being of steam turbine type. This explains the low proportion of combined cycle type plants on the graph. Analysing the data on power plants we found that 98.8% of all power plants fall into the following categories:
– gas turbines,
– hydro power,
– steam turbines,
– combined cycles, and
– wind turbines.

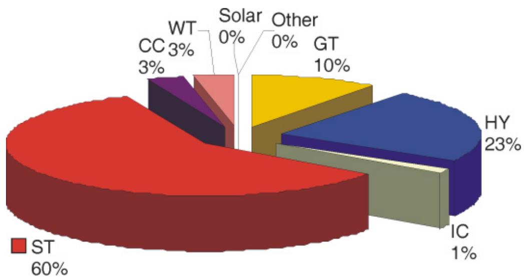

Figure 1. Operating capacity of power plants by type

However, this division has taken only the groups in the power plant database [1] into consideration. In addition to this grouping, further and more detailed division is required, and the creation of sub-groups. This division into sub-groups can be made primarily on the basis of technological characteristics:
(1) energy conversion technology,
(2) capacity,
(3) energy used (fuel type: coal, CH, biomass, biogas, other),
(4) main parameters (*e. g.* steam pressure and temperature),
(5) cooling technology, and
(6) auxiliary systems.

Of these possible properties, we will take numbers (1) to (5) into consideration in describing the efficiency functions.

Technology

Within each power generating technology several different types of plant may be distinguished. So, for instance, a variety of steam cycles exist (simple, reheat cycle, *etc.*) and combined cycle processes can also vary depending on the number of the steam generation pressure levels.

Capacity

Grouping by size is required because load dependent efficiency can essentially be discussed only when higher capacity blocks are installed. In the lower power range, complete power regulation is achieved by completely switching on and off blocks rather than switching to partial load operation; on the one hand, because these blocks operate at very low efficiency under partial load and on the other hand, because a power plant usually consists of a number of blocks of lower power. Based on the capacity of the block (nominal capacity), we distinguish the following power plants and blocks:
(1) large scale, above 250 MW (typically 300-1500 MW),
(2) medium scale, between 50 and 250 MW (typically 70-200 MW),
(3) small scale, between 1 and 50 MW (typically 5-20 MW), and
(4) micro scale, under 1 MW (typically 0.1-0.8 MW).

The load dependent efficiency of high capacity units should be reviewed first of all, where the heat output is not decisive. These large scale blocks play a decisive role in large systems, and their role may increase in future.

The medium size category can be said to include four basic models: older condensation power plants, old bus bar power plants, new industrial power plants (both delivering substantial heat supply) and back up power plants (mainly open cycle gas turbines). With the latter, it is not possible load dependent efficiency. Older model condensation power plants (with 100, 150 or 200 MW blocks) are still a decisive factor, so they also must be reviewed. Within the bus bar power plants a number of engine groups (20, 30, or 50 MW engines) operate in switch on and off mode, often operating according to heat demand, and the number of boilers does not significantly deviate from the number of engines. The internal load dispatch is a local issue (aligned to the minimum load of boilers). Back-up power plants are also operated at only 0% or 100% as they have practically no role in load dispatch. With small and micro scale power plants, the partial load of a small block does not play any role in the optimal load dispatch of the system, so modelling these is not necessary either. In terms of capacity we will provide the property curves of power plants in group (1) and perhaps (2), which may also be applied to smaller scale blocks, albeit with proper caution.

Fuel type/Energy source

The efficiency functions of power plant units should be considered primarily for power plants operating with primary fossil energy sources. Basically three sources are used: *black coal, brown coal* (*lignite*), and *natural gas.* In the case of the first two, the steam cycle is decisive while with newer plants natural gas is the fuel for gas cycles and co-generated cycles (practically gas turbines). The load dependency of efficiency will be reviewed in this group. Of course, *biomass* and *liquid fuel* (oil refining residues, orimulsion – bitumen based material – *etc.*) can also be included as a mainstream fuel, but much these are much less frequently employed, and are not characteristic units.

With *fissile substances*, the traditional nuclear power plant steam cycles, it would be misleading to talk about efficiency dependency because internationally established efficiencies are calculated and applied (at 33%). Incidentally, the incremental cost is also the lowest for non-combustible renewable sources. In the older nuclear power plants built with 200-250 MW turbine units, the technology similarly does not favour partial load. Only the more up-to-date, 900-1300 MW or the latest 1600 MW units could be subjected to partial load considerations.

With *renewable sources*, standard efficiencies are also calculated for the use of water, wind, solar, wave, tidal and geothermal heat (with the latter 10%, for the others 100%). Although efficiency may be important, the incremental cost anyway will be nil or almost nil since the energy carrier is practically free of charge. Only the load dependent fraction of the auxiliary materials and maintenance can be calculated.

Hydro power, which can be stored, but which is available mostly under constraints, should be given special consideration. In all the other cases when the available water replenishment is lower than the maximum discharge capacity of the equipment, the schedule of using the limited quantity within the load cycle (*e. g.* on a day) should be determined during the load dispatch. The pump-storage hydro power plant is a special case which is capable of operating in pump mode in the negative load range as well.

Accordingly, the following types of energy sources have been taken into account in determining the efficiency functions.
– fossil primary energy sources (coal and natural gas),
– nuclear fuel (fissile material), and
– renewable energy sources (wind and hydro).

Table 1 shows what efficiencies are used in compiling electrical industry statistics in Europe today for *nuclear power plants* and some of the *renewable sources* and users of non-fuel based sources.

Table 1. Harmonised efficiencies

Sources	Efficiency, η, [%]
Hydro, wind, and solar power plants	100
Geothermal power plants	10
Nuclear power plants	33

For *hydro, wind,* and *solar power plants* (and other, for instance tidal and wave power plants) 100% efficiency is taken for granted for the electrical energy generated. Of the renewable, an efficiency of lower than 10% is considered only for *geothermal* power plants and for ground source heat. With respect to biomass and other combustible renewable fuels, the technology of the particular cycle determines the efficiency.

The 33% efficiency of nuclear power plants is also harmonised, independent, for instance, of the measured energy consumption of steam generators. In third generation pressurised water reactor nuclear power plants, the actual efficiency can reach 37%. Calculating an efficiency dependency for incremental costs does not make any sense either although its size is greater than that of the renewable sources referred to. Therefore, the electrical power generation built on non-storable renewable sources surpasses nuclear power plant generation in load dispatch.

Regarding *secondary* energy sources, oil products (heating oil, fuel oil, diesel oil, petrol, *etc.*) would merit attention if frequent load changes could be expected. This is, however, quite rare: it may happen, for instance, in large scale blocks, when the operators are forced to use oil in the absence of natural gas. Larger backup units – such as open cycle gas turbines – also use oil refinery products but in a controlled mode only (0% or 100%) and so load dependency is not relevant. In engine solutions, liquid fuels are more applicable although control is mainly exercised based on heat. Furnace and chamber gas is a substantial fuel only in metallurgical power plants therefore the modelling does not cover it. Secondary gas like energy sources obtained from coal gasification, biomass, and various waste also belong here. *Hydrogen*, a promising secondary fuel for the future, can be mainly used in gas turbines, gas engines, or fuel cells. A system level assessment should take place only after 2030. The future energy role of materials from secondary renewable sources (biodiesel, bioethanol, biogas, *etc.*) is quite doubtful.

General system model

The method that we consider for determination of the characteristic functions of the power stations is demonstrated here by a condensation type thermal power station. The first practical step is to divide the power station into subsystems according to energy conversation technologies and processes (fig. 2):
– transformation of chemical or nuclear energy into heat (H),
– transformation of heat into mechanical energy (T),

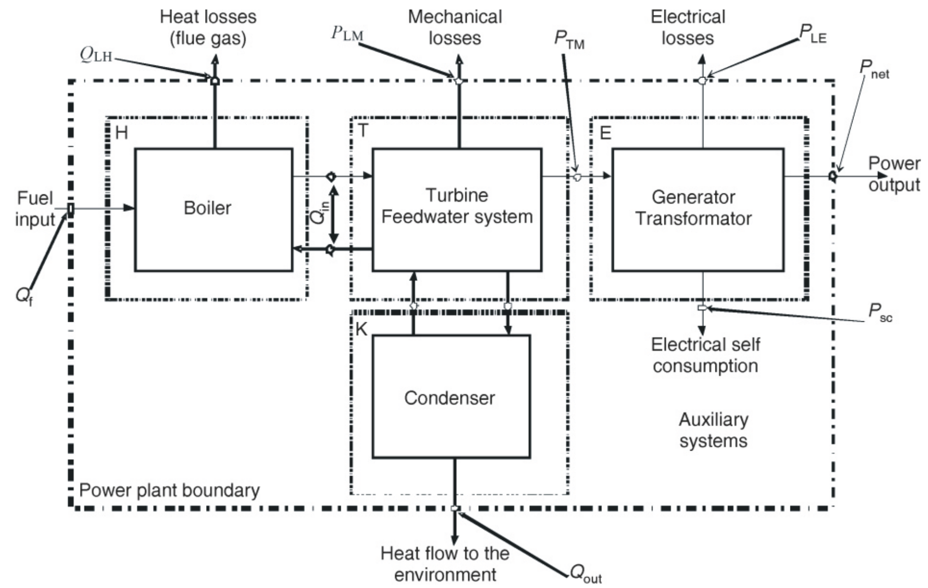

Figure 2. Subsystems of a thermal power plant

– transformation of mechanical energy into electricity (E), and
– heat removal from the cycle into the environment (K).

Every subsystem has its own efficiency that characterizes the subsystem. The subsystem efficiency depends on the type of main equipment and on the operation state (actual load). Applying the subsystems, the efficiency of the power plant may be calculated as follows:

$$\eta_{\mathrm{PP}} = \eta_{\mathrm{H}}\eta_{\mathrm{T}}\eta_{\mathrm{E}}\eta_{\mathrm{other}} = \frac{P_{\mathrm{net}}}{Q_{\mathrm{f}}} \tag{1}$$

where the η_{other} takes into account the other heat, mechanical and electrical losses, and self consumption. The efficiencies of the subsystems are defined in the following way:

– *Efficiency of subsystem H* in a direct way:

$$\eta_{\mathrm{H}} = \frac{Q_{\mathrm{in}}}{Q_{\mathrm{f}}} \tag{2}$$

or in an indirect way, by determination of the losses:

$$\eta_{\mathrm{H}} = 1 - \frac{Q_{\mathrm{LH}}}{Q_{\mathrm{f}}} \tag{3}$$

In most cases the indirect way is more effective.

– *Efficiency of subsystem T* can be determined according to the input and output energy flows:

$$\eta_{\mathrm{T}} = \frac{P_{\mathrm{TM}}}{Q_{\mathrm{in}}} \tag{4}$$

– *Efficiency of the subsystem E* can be determined in the direct way and the direct way is more effective:

$$\eta_{\mathrm{E}} = \frac{P_{\mathrm{net}}}{P_{\mathrm{TM}}} \tag{5}$$

In the previous formulas the folowing symbols: η – efficiency, [–]; Q – heat, [MW]; P – mechanical or electrical power, [MW], and subscripts: f – fuel; LH – heat losses; in – input, out – output heat flow of the T subsystem; TM – mechanical losses of the turbine; LM – other mechanical losses; LE – electrical losses; net – net output power; pp – power plant, are used.

The characteristic efficiencies of the subsystems vary according to the type of equipment and by the actual load. The typical characteristics are shown on fig. 3. In fig. 2 the E subsystem is divided into its two main pieces of equipment and the efficiency of the subsystem E will be the product of the efficiency of the generator (G) and the efficiency of the transformer (Tr):

$$\eta_{\mathrm{E}} = \eta_{\mathrm{G}}\eta_{\mathrm{Tr}} \tag{6}$$

The characterization of the other losses may be done by the product of the heat losses efficiency ($\eta_{\mathrm{other,0}}$) and the efficiency of self load (η_{sc}) as:

$$\eta_{\mathrm{other}} = \eta_{\mathrm{other,0}}\eta_{\mathrm{sc}} \tag{7}$$

The fuel factor may be defined as the converse of the efficiency:

$$q_{PP} \quad \frac{1}{\eta_{PP}} \tag{8}$$

For gas turbine and combined cycle power plants, the complex power plant procedures can be divided into parts and the efficiency function calculated in a similar way. An examination of these types of power plants and some suggested efficiency functions can be read in an article by Kim [2], and Zang *et al*. [3]. These authors suggest quadratic and bi-quadratic functions to describe efficiency functions. In our opinion quadratic functions offer sufficient precision. The next chapter introduces this in detail.

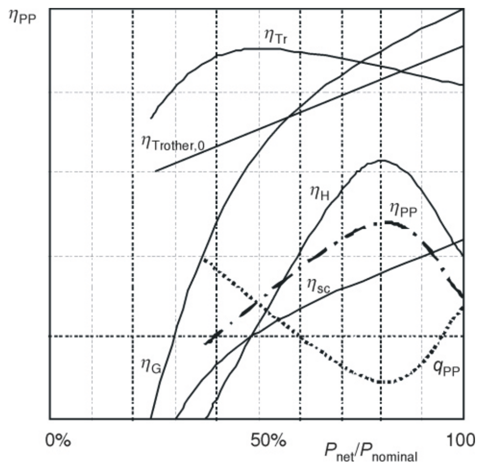

Figure 3. Efficiencies of power plant subsystems [4]

The empirical model

Definitions

A power plant unit operates in a steady or so-called *quasi stationary* condition with a power commensurate to the load, and this efficiency may vary according to the power. We examined how its efficiency changes as a function of the power of a unit. Both power – we mean only electric power here – and efficiency can be expressed with reference proportions, *i. e.* with quotients without a measurement unit. This can make the investigation more general.

With respect to *power*, it is practicable to refer the data to the nominal installed capacity (P_{nom}) of the power plant unit because this capacity is at the same time the name of the unit expressed in numbers. For instance, a 100 MW unit is meant when the nominal installed capacity recorded in certificates or the power is exactly 100 MW. This used to mean and sometimes may still mean a gross value, but lately it has been understood to mean net power, with reference to the output. To evaluate the load dependency of efficiency, it is more reasonable to use gross power because efficiency is mainly influenced by the power output of the turbo generating unit. The relative power can therefore be expressed with the ratio of P/P_{nom}. Its value in the nominal load condition is: 1.

The load condition of power plant units can be well characterised by a few typical levels of load such as:

– *maximal installed capacity* (P_{max}/P_{nom}), that is the overloadability of the unit,
– *optimal power* (P_{max}/P_{nom}) at the point of the best efficiency (η_{opt}), and
– *minimal power* (P_{min}/P_{nom}), at which it is able to continuously operate.

These load levels are jointly determined by the power plant technology, the condition of the equipment and the environment.

With *efficiency*, the values should be applied to the best efficiency (η_{opt}). Accordingly, at maximum efficiency the relative values of η_{max}/η_{opt}, at nominal η_{max}/η_{opt}, and at minimum (η_{min}/η_{opt}) will result, respectively.

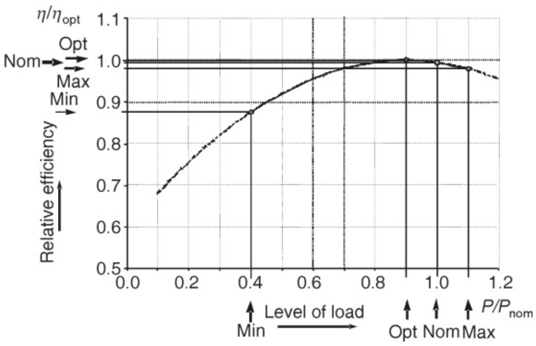

Figure 4. Unit efficiency as a function of the level of load

The efficiency of the power plant unit can be illustrated with these relative figures, with a slightly curving, upside down U-shaped line (fig. 4). In many cases (*e. g.* steam cycle power plants), it is only an approximate illustration because it does not take into account the impact of the so-called valve points. We examine here the various determinants of this approximate function for the given technology and other independent variables.

The relative efficiency curve may be represented by the 2nd ordered function where the relative power is the independent variable:

$$\frac{\eta}{\eta_{opt}} \quad 1 \quad a \frac{P_{opt}}{P_{nom}} \quad \frac{P}{P_{nom}} \quad b \frac{P_{opt}}{P_{nom}} \quad \frac{P}{P_{nom}}^{2} \tag{9}$$

where h is the efficiency in point of P load, [–]; η_{opt} – the efficiency in point of optimal load, [–]; P – the actual load of the block, [MW]; P_{opt} – the optimal load of the unit, [MW]; P_{nom} – the nominal capacity of the unit, [MW]; a – the first coefficient, [–]; b – the second coefficient, [–].

The a and b coefficients have to be determined for every typical unit. Every unit has a nominal power, optimal efficiency, and optimal power. These five characteristic properties may be used in modelling the operation of the units. Determination of the characteristic properties often raises some difficulties since in most the cases the optimum power does not equate with the nominal power. The units often run in a state of partial load, and the yearly average efficiency should be kept as high as possible, since the optimal point is somewhere around 75-85% of the nominal load in the case of schedule type operation, and 90-95% in the case of the base load power stations blocks. It is worth emphasizing that in the case of the small scale power plants, the nominal and the optimum points are practically the same. These types of blocks are controlled by a simple switch on and off mode while the greater power plants have continuous power control. The efficiency dependency on the level of load should be evaluated based on these facts.

It should be noted already at this point that in a lot of small scale power plants (with a nominal installed capacity less than 50 MW) the optimal and nominal power usually coincide ($P_{nom} = P_{opt}$), because these small scale power plants are not operated at partial load. They operate either at their nominal load or do not operate at all. In combined generation they will operate according to the heat demand. The joint power of the required small scale power plants is in every case set by switch on and off mode, that is the load is not restricted but they are controlled by switching the plant on and off as required. It is usually the plants which are smaller than 5 MW (*e. g.* gas fuelled) or micro scale power plants (<1 MW) which operate like this in one system.

It should be also emphasised that several electricity generation units *i. e.* co-generated local sources are intended in older power plants, or new industrial power plants for own consumption and even in large scale power plants ($P_{nom} > 50$ MW), where a number of sub-units are also switched on or off – depending mainly on the local demand – instead of running at partial load. This type of bus bar power plant no longer plays a significant role in the electrical power system however.

Factors and corrections

The *a* and *b* parameters should take into consideration the technical features and technology properties of the block in the efficiency function equation according to eq. (9). In addition to the technical properties, the operation of the power plant is influenced by a number of factors which also have a detectable impact on the load dependent efficiency. These influencing factors are:

- the *age of the power plant unit*, which influences its availability, reliability and from this the average efficiency applying to a particular period, as well as the average block level efficiency depending on the wear of the equipment, and
- external (ambient) parameters dependent on the *geographic location of the power plant unit* and *season of the year* (ambient air temperature, cooling water temperature, wind, *etc.*).

Based on this, the general efficiency function should be supplemented with two correction factors according to the following equation:

$$\frac{\eta}{\eta_{opt}} \quad \frac{\eta}{\eta_{opt}} \quad 1 \quad a \quad \frac{P_{opt}}{P_{nom}} \quad \frac{P}{P_{nom}} \quad b \quad \frac{P_{opt}}{P_{nom}} \quad \frac{P}{P_{nom}}^{\;2} \quad C_1\,(\textit{ambient conditions}) \;\; C_2\,(\tau) \tag{10}$$

where C_1 is the *ambient dependent correction factor*, depending on the location (geographic site) of the power plant $C_2(\tau)$ – and *age dependent correction factor*, where τ is the age of the power plant (block) measured from the year of being commissioned, calculated in years.

In eqs. (9) and (10) beside *a* and *b* factors two more quantities typical of power plants also appeared, which are given by the P_{opt}/P_{nom} quotient and the η_{opt} efficiency. These quantities differ in different power plants, so they can be defined by statistical processing of power plant attributes [1, 5]. The *a* and *b* coefficients of eqs. (9) and (10) are included in tab. 2.

P_{opt} optimal performance rate occurring in coherences (9) and (10) can be determined according to P_{opt}/P_{nom} rates shown in tab. 3 from the nominal performance for every type of power plant.

Table 2. Coefficients of eq. (10)

Type	Acronym	a	b
Gas turbines	GT	–0.052	–0.602
Steam turbines	ST	+0.051	–0.611
Combined cycles	CC	+0.080	–0,750
IC engines	IC	–0.455	+0.272

Table 3. P_{opt}/P_{nom} ratio

Type	Acronym	P_{opt}/P_{nom}
Gas turbines	GT	1.00
Steam turbines	ST	0.85
Combined cycles	CC	0.98
IC engines	IC	1.00

Based on the processing of the available data [1-3, 5], optimal efficiency can be written up as a logarithmic function of nominal capacity in the following general form:

$$\eta_{opt} = D_1 \ln(P_{nom}) + D_2 \tag{11}$$

where the constants marked D_1 and D_2 are the functions of the power plant type, and the numerical values must be substituted in the relationship according to tab. 4. As can be seen, for gas turbines two further groups can be formed on the basis of nominal performance.

Table 4. Coefficients for eq. (11)

Type of the power plant	Acronym	D_1	D_2
Steam power plant	ST	0.0288	0.2188
Gas turbine (small scale, 0-25 MW)	GT	0.0562	0.1576
Gas turbine (large scale, 25-250 MW)	GT	0.1287	–0.3183
Combined cycles	CC	0.0065	0.4230
IC engine	IC	0.0160	0.4083

When we follow the load dependency of efficiency of a power plant unit, it must not be forgotten that these curves constantly vary due to changes in the environment and the technology itself in the meantime (the weather gets warmer or colder and the equipment also ages). In multi-year studies, it is therefore not sufficient to track only the efficiency curve provided, but the various influence factors modifying efficiency must also be borne in mind, and they can be taken into consideration with the so called *correction factors* or *functions*.

Correction for ambient conditions

The temperature of the external environment fundamentally determines the electrical capacity (and thus efficiency) to be recovered from a power plant [6]. This influence:

- affects (increases or decreases) the efficiency through the pressure of the condenser in the condensation steam power plant,
- in the case of gas turbines, the density of the air induced, and thus its mass stream and consequently the power and efficiency of the block will change,
- in the case of gas engine blocks, the statements valid for gas turbines also prevail, and
- in combined cycle power plants, the statement jointly applies to the condensation steam power plant and gas turbines.

In a comprehensive descriptive model of the European system of power plants, this influence of ambient conditions can be taken into consideration by location. Based on the location, the annual average temperature typical of a particular region can be established from the meteorological data.

Since the efficiency values are usually provided for a condition according to ISO (15 °C temperature, 1.013 bar pressure, and 60% relative humidity), the correction also depends on a variance from these values. The C_1 correction factor in eq. (12) shall be defined with the following function:

$$C_1 \quad -0.0025\bar{t}_a \quad 1.0375 \tag{12}$$

where $\bar{t}_a$ is the annual average ambient temperature (function of location).

Correction for age (life time)

The efficiency of power plants gradually deteriorates with the passage of time and the extent of their wear. At the beginning, this deterioration is relatively small; however, approaching the end of their technical life cycle, this deterioration becomes increasingly significant. Based on the data available, we set up a correction function which provides the numeric value of the correction factor as a function of a plant's life span measured in years. The value of the correction factor also depends on the technological design of the power plant in addition to its age, which is essentially determined by the type of fuel. The general form of the function is according to eq. (13):

$$C_2 \quad s_2\tau^2 \quad s_1\tau \quad s_0 \tag{13}$$

where the coefficients s of the equation are presented as a function of the fuel in tab. 5.

Table 5. Coefficients of equation (13)

Fuel type	s_2	s_1	s_0
Coal, lignite, oil, and other solid fuels	–0.0000087	–0.00104	1
Natural gas or biogas	–0.000013	–0.00039	1

Heat rate and cost functions

Within the electrical power system the power is dispatched on the basis of the costs and marginal costs (in this case we deal with units in operation). Cost of production is basically determined by two factors: heat rate of power plant (HR) and heat cost of fuel. HR is defined as the reciprocal of efficiency:

$$HR(P) \quad \frac{1}{\eta(P)} \tag{14}$$

Substituting coherence of eq. (10) we get:

$$HR(P) \quad \frac{1}{\eta_{\text{opt}} \quad 1 + a \quad \dfrac{P_{\text{opt}}}{P_{\text{nom}}} \quad \dfrac{P}{P_{\text{nom}}} \quad b \quad \dfrac{P_{\text{opt}}}{P_{\text{nom}}} \quad \dfrac{P}{P_{\text{nom}}} \quad ^2 \quad C_1C_2} \tag{15}$$

Actual fuel cost can be defined by performance and fuel cost specified to efficiency according to the following coherence:

$$FC = P\ HR\ c_{\text{fuel}} \tag{16}$$

For power dispatch to be economical, specific marginal costs, or rather their equality between power plant units are used to determine the most favourable (resulting in the lowest cost) situation. Marginal cost can be defined based on marginal heat rate using eq. (17):

$$MC = MHR\ c_{\text{fuel}} \tag{17}$$

where marginal heat rate is:

$$MHR(P) \quad \frac{\mathrm{d}(P \quad HR)}{\mathrm{d}P} \quad P\frac{\mathrm{d}HR}{\mathrm{d}P} \quad HR \tag{18}$$

substituting expression occurring in eq. (15) into eq. (18) we get:

$$MHR(P) \quad HR(P) \quad \frac{\dfrac{P}{P_{\text{nom}}} \quad a \quad 2b\dfrac{P_{\text{opt}}}{P_{\text{nom}}}}{\eta_{\text{opt}} \quad 1 \quad a \quad \dfrac{P_{\text{opt}}}{P_{\text{nom}}} \quad \dfrac{P}{P_{\text{nom}}} \quad b \quad \dfrac{P_{\text{opt}}}{P_{\text{nom}}} \quad \dfrac{P}{P_{\text{nom}}} \quad ^{2} \quad ^{2} \quad C_1 C_2} \tag{19}$$

In optimal load state the specific heat rate function is at an extreme (a minimum), that is $\mathrm{d}HR/\mathrm{d}P = 0$, it follows that the rates of specific heat rate and marginal heat rate are equal: $HR(P_{\text{opt}}) = MHR(P_{\text{opt}})$

Summary

Testing the model

The completed efficiency functions were checked by applying them to the data from several Hungarian power plants, and it was found that they describe power plant behaviour with an adequate degree of accuracy and are suitable for modelling the demand of the electrical power market. Applications of the functions are shown by one of the units of the Dunamenti Power Plant. The year of installation was 1976, so it has been operating for 34 years. During this period retrofit or repowering did not take place. Its nominal installed capacity is 215 MW, the fuel is mostly natural gas, or in case of gas shortages light oil. The annual average temperature of ambient air is 10 °C. Efficiency rates calculated and measured by function (10) is shown in fig. 5. Correlation of the calculated and measured rates showed a correlation coefficient of 94% and thus the accuracy of the model can be considered acceptable.

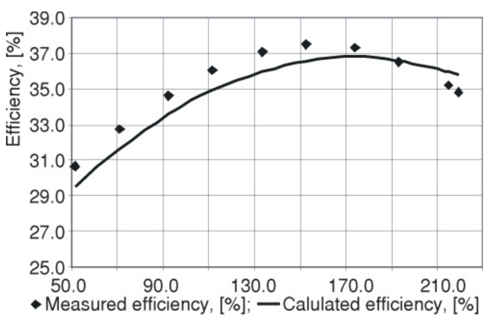

Figure 5. Measured and calculated efficiency values (Dunamenti power plant)

Monitoring was carried out on all the types of power plants above-mentioned, before using the data of typical Hungarian power plants. Results of calculation are included in tab. 6. On the basis of the results it can be stated that the completed model performs properly and is suitable for jointly modelling the units of a power plant system of great extension. The discrepancies that may appear at certain units will probably be compensated for due to the simultaneous modelling of a great number of units.

Table 6. Hungarian power plants

	Power plants			
	Mátra	Csepel	Dunamenti G1	Tatabánya
Fuel type	Lignite	Natural gas	Natural gas	Natural gas
Nominal (installed) capacity	212	160	146	6
Year of installation	1975	2000	1993	2004
Technology	Steam turbine	Combined cycle	Gas turbine (Large scale)	Internal combustion engine
Correlation coefficient	81%	95%	97%	96%

Taking the pollutant emission into consideration

Using these efficiency functions it is possible to model not only the offer side and the expenses of the electrical power market but also, if further data are considered, the emissions too [7, 8]. The instantaneous emission of the power plant referring to a given contamination can be written in this way:

$$E_i \quad {}_j(1 \quad \varphi_{i,j})\varepsilon_{i,j} \frac{P}{\eta(P)} \tag{20}$$

where E_i is the i^{th} emission of pollutant, $\varphi_{i,j}$ – the removal factor depending on the technology employed, and $\varepsilon_{i,j}$ – the specific emission factor by the pair of energy source-pollutant. Both the removal factor and emission factor can be considered independent of the load of the power plant. In view of these features not only instantaneous production but emission allocation can also be planned if the load duration [9] is known. Thus not only economical, but also environmental aspects can be proved and taken into account. The model that is completed by taking the pollutant emission into consideration partly internalizes the external costs thus applying it means that the environmental view points can also be taken into consideration. The model presented in our paper that also includes the handling of the emission can be applied as a sub-model in the system described in literature [10].

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