MATHEMATICAL MODELING AND OPTIMIZATION OF TRI-GENERATION SYSTEMS WITH RECIPROCATING ENGINES

by

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Tri-generation systems are used to simultaneously produce electrical, heating, and cooling energy. These systems are usually more efficient than conventional systems for separate production and have smaller distribution losses since they are often located closer to the consumer. For achievement of the best technical and/or financial results, tri-generation plants have to be properly, i. e. optimally designed and operated. Operational optimization is used for short term production planning, control of tri-generation systems operation and as a part of design level optimization. In this paper an approach to operational optimization of tri-generation plants with reciprocating engines is presented with the following mathematical model. It is also explained how this algorithm might be embedded in some larger optimization procedure. In this approach, the importance of the part load performance of different units of the tri-generation systems is emphasized, especially of co-generation unit, i. e. engine generator set and thus it relies on manufacturers' data and is characterized with relatively high level of details examined. Mathematical model is based on the equipment performance based constraints and demand satisfaction based constraints with the possibility to add more equations if appropriate. Objective function for optimization is benefit-cost function. Optimal operation regimes for typical days for each month are obtained and analyzed. Impact of electrical energy price on pay-back period and primary energy saving is analyzed. Primary energy savings are determined and compared to maximal value that could be obtained.

Key words: *co-generation, tri-generation, optimization, pay-back period, primary energy savings*

Introduction

Co-generation systems produce simultaneously electrical and thermal energy and are generally more efficient than conventional systems for separate production. Moreover, on site production is characterized with lower distribution losses in both electric and heating systems. In order to further improve efficiency of such systems, thermal energy is often used in absorp-

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tion refrigeration units for cooling purposes. Systems that produce electrical, heating, and cooling energy are called tri-generation systems.

Achievement of the best or nearly the best financial or technical indicators of co-generation and/or tri-generation systems requires these systems to be properly designed and operated, *i. e*. to be optimized. Optimization of co-generation systems in general is discussed in details in [1]. The topic of this paper is operational optimization, *i. e*. determination of values related to operation of tri-generation plants for previously defined and sized equipment of the facility.

Operational optimization of the co-generation and tri-generation plants is subject of a number of papers. Different authors used different objective functions and different approaches. In [2-12] objective functions are related to financially optimal operation, in [13, 14] quantity known as primary energy savings is emphasized as an objective function, while Osman *et al*. [15] optimize three quantities: primary energy consumption, global warming potential, and tropospheric ozone precursor potential.

Dotzauer [2, 3] shows that operational optimization could be used for optimal short term (24 hours to 7 days) energy production planning based on energy demand forecasts in order to achieve the best financial results. Kalina *et al*. [4] and Cho *et al.* [5] emphasize the importance of operational optimization for optimal control of the equipment of co-generation plants. Operational optimization is also important as a part of design level optimization [1] and decision making in the feasibility stage of the co-generation/tri-generation projects.

While Cho *et al*. [5], Lahdelma *et al.* [6], Rong *et al.* [7] and Rong *et al*. [10] use linear or linearized models, Dotzuer [2, 3] formulates problem as strongly non-linear, mostly due to usage of efficiencies of the equipment and their quadratic dependencies on part loads. Kalina *et al*. [4] use genetic algorithm, Chicco *et al*. [13] use sequential quadratic programming, while Bojić *et al*. [12] and Osman *et al*. [15] present mixed integer linear programming problems.

Operational optimization related to this paper is performed using the TRIGEN MSO software developed by the authors of this paper at the Faculty of Mechanical Engineering Niš [14]. The approach used in this paper has certain common characteristic with models described in some of the mentioned references. This model is also partly inspired by models presented in [16-18]. TRIGEN MSO allows the user to decide whether the objective function is financial or related to primary energy savings, or even the weighted average of both. In this model, part load curves and other details are used and the level of details examined is similar as in referent modules of the leading software for energy systems simulations like DOE-2 [16], eQUEST or ENERGY PLUS [17].

In this paper, operational optimization problem is described, as well as the following mathematical model relying on part load performance curves of the equipment and the condition that electrical, heating, and cooling demands must be satisfied at the same time. Model described in this paper considers co-generation unit as a central one, taking into account variations in its performance as a function of part load ratio, ambient temperature, altitude, *etc*. and examining these relations in details. Co-generation unit is assumed to be a natural gas fired internal combustion reciprocating engine generator set. With some modifications, the model could be applied to gas turbines based plants. Other units are also modeled in details.

Problem formulation

The objective of the algorithm and mathematical model presented in this paper is to determine financially optimal operational regime of a tri-generation facility. This model might be also adjusted to maximize primary energy savings or minimize environmental impact, but that is beyond the scope of this paper.

Tri-generation facility considered in this paper, schematically shown in fig. 1, contains the following units: (1) co-generation units, *i. e.* engine generator sets that simultaneously produce electrical and thermal energy and consume fuel, *i. e.* natural gas, (2) supplementary heating units, *i. e.* hot water boilers, that produce additional amount of thermal energy, and (3) refrigeration units, *i. e.* absorption chillers that use thermal energy and small amount of electrical energy as an input and compression chillers that consume only electrical energy. Engine generator sets and hot water boilers also consume small amounts of auxiliary electrical energy required for operation. The model is defined in such way that additional electrical energy might be imported from the national electrical grid or excess electricity produced in the engine generator sets might be exported to the grid. Electrical, heating, and cooling demands to be satisfied by the tri-generation plant have to be predefined and these three values are inputs into mathematical model.

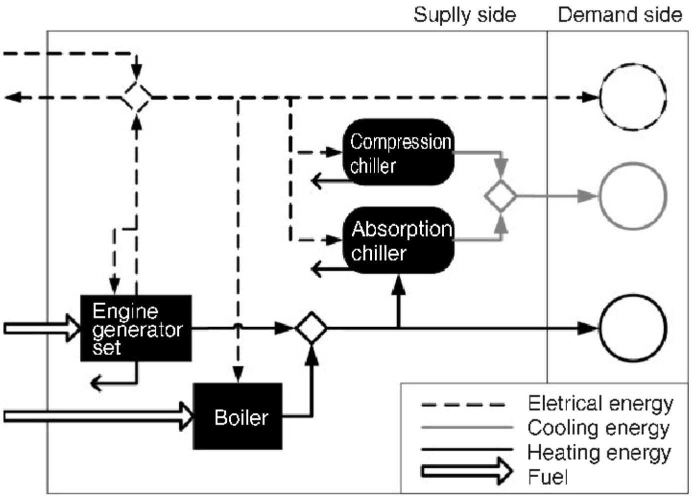

Figure 1. Energy flow diagram of a tri-generation plant

This model is based on the following assumptions: (1) the assumption that the operation parameters for each time interval are independent of the parameters for any other interval and (2) steady-state operation assumption, *i. e.* it is assumed that during each time interval observed energy demands are constant, as well as all quantities related to tri-generation plant operation. The exception might be performance during start-up and shut down periods for equipment, but treatment of these influences is beyond the scope of this paper. Length of the intervals of time observed might be specified. It is usually taken to be 1 hour, like in this paper, but the model allows longer or shorter intervals to be defined, depending on the necessity for precision and having in mind that larger number of shorter time intervals require more precise inputs and longer time required for calculation.

Optimization algorithm is shown in fig. 2. Based on previously defined equipment and manufacturers' data, climate data, and energy demands for each interval of time observed (*e. g.* each hour), initial set of feasible integer variables defining number of units of each kind (co-generation, supplementary heating, and refrigeration) running is defined. Start-up and shut down energy consumption can then be determined for all kinds of equipment on a daily basis. For each interval of time operational optimization is performed and values of decision variables and objective function(s) are found. The model is also capable to perform operational optimization for more intervals (*i. e.* 24 or 48 hours) at once if needed. Another feasible set of integer variables is than defined, *etc*. Different sets of integer variables are defined using the combinatorial algorithms until the optimal final daily value(s) of the objective function(s) are found. If only one

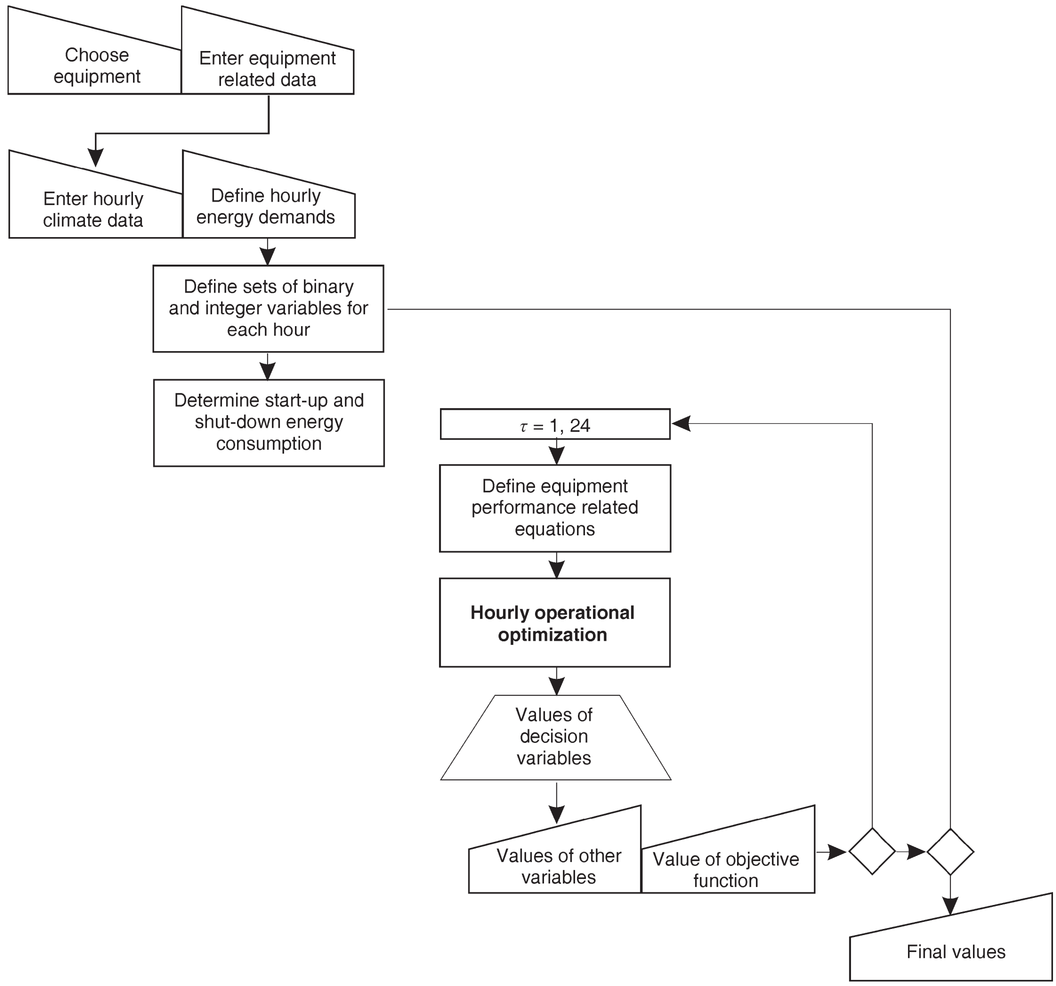

Figure 2. Optimization algorithm for 24 hours period with time intervals of 1 hours

unit of each kind is foreseen, then integer variables become binary variables showing whether the unit is on or off during the observed interval. In this paper, mathematical model for operational optimization for an interval of time is described, while determination of integer variables and other aspects are beyond its scope.

Objective function

There are several important and often used financial parameters defined in [1, 19]. Although dynamic parameters like net present value, internal rate of return, and dynamic pay-back period are more sophisticated for financial analyses, the objective function considered in this article is simple pay-back period because it is widely used and recognized in engineering analyses. Thus the objective is to minimize simple pay-back period, SPB:

$$\text{Minimize } SPB \quad \frac{Z^{\mathrm{CI}}}{\Delta CF} \tag{1}$$

where Z^{CI} is the amount of capital investment of the co-generation project (land, equipment, licenses, *etc.*) and ΔCF – the difference between the annual cash flow forecasts for referent, baseline scenario (without construction of the co-generation facility) and project scenario where co-generation implementation is foreseen. Mentioned cash flows are contained of operating costs (fuel, electrical energy, maintenance, labor, administrative costs, *etc.*), operating revenues from selling products (electrical and heating energy) and other costs. Choice of cash flows included and referent years for consideration might vary depending on the approach.

The objective is to minimize simple pay-back period of the co-generation project subject to the technical operating parameters, thus it is convenient to introduce new function that should be maximized:

$$\text{Maximize } F_{\mathrm{B\text{-}C}} \quad \Delta CF \quad F_{\mathrm{NA}} \tag{2}$$

where $F_{\mathrm{B\text{-}C}}$ is the annual benefit-cost function contained of all the operating costs and revenues that depend on operational regimes of the facility for that year and F_{NA} – the function of the costs non-related on the operation modes and, thus impossible to influence with operational optimization. Having this in mind $F_{\mathrm{B\text{-}C}}$ is defined as:

$$\begin{aligned} F_{\mathrm{B\text{-}C}} \quad & \left[{}_{\tau\ 1}^{\frac{8760}{\Delta\tau}} (b_{\mathrm{e}}\dot{W}_{\mathrm{G,S}}^{\tau} \quad c_{\mathrm{e}}\dot{W}_{\mathrm{G,P}}^{\tau} \quad c_{f}\dot{Q}_{\mathrm{EGS}}^{I\tau} \quad c_{\mathrm{f}}\dot{Q}_{\mathrm{B}}^{I\tau} \quad \dot{Z}_{\mathrm{CG}}^{\mathrm{OMv}\tau})\Delta\tau \quad Z_{\mathrm{CG}}^{\mathrm{OMc}\tau} \right]_{\mathrm{CG}} \\ & \left[{}_{\tau\ 1}^{\frac{8760}{\Delta\tau}} (\quad c_{\mathrm{e}}\dot{W}_{\mathrm{G,P}}^{\tau} \quad c_{\mathrm{f}}\dot{Q}_{\mathrm{B}}^{I\tau} \quad \dot{Z}_{\mathrm{RF}}^{\mathrm{OMv}\tau})\Delta\tau \quad Z_{\mathrm{RT}}^{\mathrm{OMc}\tau} \right]_{\mathrm{RF}} \end{aligned} \tag{3}$$

where τ is time, $\Delta\tau$ is duration of each interval, in hours, b_{e}, c_{e}, and c_{f} are prices of sold and purchased electrical energy and fuel, *i. e.* natural gas, in € per kWh, $\dot{W}_{\mathrm{G,S}}$ and $\dot{W}_{\mathrm{G,P}}$ are electrical power sold to and purchased from the national grid, respectively, in kW, $\dot{Q}_{\mathrm{EGS}}^{I}$ and $\dot{Q}_{\mathrm{B}}^{I}$ are time rates of primary energy (fuel) consumption of the engine generator set and boiler, respectively, in kWh of the fuel net calorific value per unit of time. Z^{OMc} is the sum of fixed operation and maintenance costs, except fuel costs, *i. e.* the costs not depending on the operation for all the units, while $\dot{Z}^{\mathrm{OMv}}$ – the time rate of the sum of variable operation and maintenance costs for all the units, *i. e.* costs dependent on the current electrical power output of the engine generator set, thermal power output of boiler, time of operation of the units, *etc*. Subscript CG refers to co-generation implementation case, *i. e.* co-generation project scenario, while subscript RF indicates referent case, *i. e.* baseline scenario. For the purpose of this article it is assumed that electrical energy price is 10.59 c€/kWh when exported to the grid, according to the valid feed-in tariff in Serbia (according to the size of co-generation unit) and from 7 to 10 c€/kWh when imported from the grid during the high price tariff (from 8 AM to midnight) and 4 times less during the low price tariff. Natural gas price, based on the net calorific value, is 4 c€/kWh. Equation (3) might take more complicated forms if some tariff systems are considered.

Maximization of the benefit-cost function leads to the minimal value of the simple pay-back period subject to technical operating parameters, as already shown, but also to optimal values of some dynamic financial indicators like net present value or dynamic pay-back period.

Following the assumption that the operation variables for any time interval observed, τ, are independent of the operation variables in other intervals, annual benefit-cost function given in eq. (3) is defined as a sum of benefit-cost functions for each time interval observed during the year. Maximization of annual benefit-cost function, thus, is achieved by separate maximization of benefit-cost functions for each interval (*e. g.* 1 hour) or more consecutive intervals (*e. g.* 24 hours).

Mathematical model

Mathematical model represents the set of constraints related to decision variables and the ability of the plant to satisfy electrical, heating, and cooling energy demands in the observed interval. It also might include satisfaction of high efficiency co-generation criteria related to primary energy savings.

Co-generation unit

Mathematical model for co-generation unit, *i. e.* engine generator set is very detailed and is mostly based on the manufacturer's data. Simplified scheme of this unit is given in fig. 3.

Decision variables related to the engine generator set part load operation, are: (1) electrical power output during the observed time interval, $\dot{W}^{\tau}_{\mathrm{EGS}}$, constrained with minimal and maximal values if the engine is on and equal to 0 if the engine is off, (2) thermal power generated during exhaust gasses cooling, $\dot{Q}^{\mathrm{XG}\tau}_{\mathrm{EGS}}$, also constrained with its maximal value, $\dot{Q}^{\mathrm{XGtot}\tau}_{\mathrm{EGS}}$, that is the function of electrical power generated, and (3) thermal power rejected during engine cooling, $\dot{Q}^{0\tau}_{\mathrm{EGS}}$. Instead of electrical power, part load ratio of the engine generator set or mechanical power output, *i. e.* shaft power might be used also.

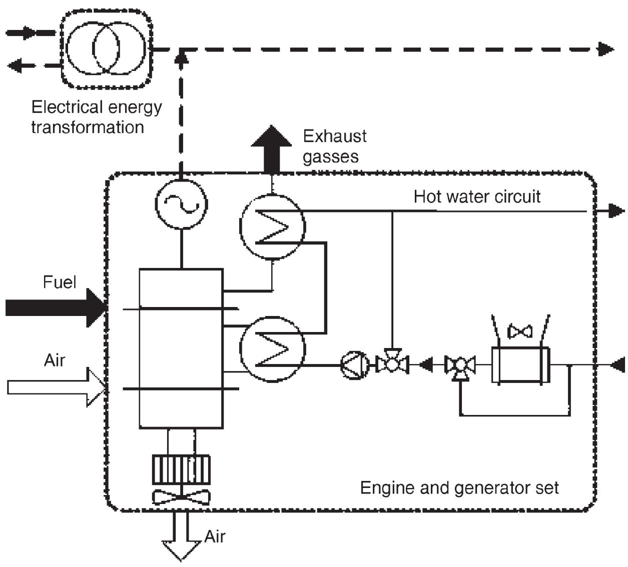

Figure 3. Scheme of the engine generator set with electrical energy transformer

Primary energy (fuel, natural gas) consumed by the engine generator set during part load operation in the observed interval of time, $\dot{Q}^{\mathrm{I}\tau}_{\mathrm{EGS}}$, is calculated as a function of electrical power generated (or part load ratio), ambient temperature and altitude, based on manufacturer's data. Start-up and/or shut down energy consumption might be added to primary energy (fuel) consumption if the engine has been off in the previous interval of time or is planned to be off in the following interval. However, this value is not so important for the results of the optimization procedure since it depends only on predefined integer variables and engine properties, *i. e.* does not depend on any of the mentioned decision variables.

Thermal energy available during the engine part load operation is divided in four parts: (1) high temperature energy of exhaust gasses, $\dot{Q}_{\mathrm{EGS}}^{\mathrm{XGtot}\tau}$, (2) high temperature energy at approximately 85 to 95 °C, $\dot{Q}_{\mathrm{EGS}}^{\mathrm{HTC}\tau}$, from the engine cooling system, (3) low temperature energy at approximately 40 to 50 °C, $\dot{Q}_{\mathrm{EGS}}^{\mathrm{LTC}\tau}$, from the engine cooling system, and (4) very low temperature energy radiated from different parts of the engine generator set to the environment. All mentioned types of energy depend on electrical power output (*i. e.* part load ratio). The fourth type is not going to be examined further, since it is not usable, while three others are represented in the function of electrical energy generated (or part load ratio), ambient temperature, and altitude, based on manufacturer's data. Low temperature energy, for the purpose of this paper, is going to be assumed to be entirely rejected to the environment. Rejection of thermal energy to the environment requires auxiliary electrical energy to be consumed for operating the fans, since the energy is rejected either using water-air heat exchangers with forced convection or ventilating the engine container. One of the advantages of this model is that it recognizes the difference between different thermal energy sources of the engine generator set. One of the positive consequences is that the optimization procedure allows energy from exhaust gasses to be partly used and partly rejected to the environment, not requiring auxiliary electrical energy for the rejection.

Auxiliary electrical power required by the engine generator set, $\dot{W}_{\mathrm{EGS}}^{\mathrm{x}\tau}$, for excess energy rejection, container ventilation, providing air, *i. e.* oxygen for combustion, for pumps, control devices and other auxiliary electrical devices is modeled as a function of the part load mode, ambient temperature and amounts of rejected high and low temperature power.

For predefined ambient temperature and altitude, as well as having in mind the assumption that low temperature thermal power available, which is defined as the function of electrical output, is completely rejected to the environment, engine generator set related constraints might be written in the form of eqs. (4)-(7):

$$\dot{Q}_{\mathrm{EGS}}^{I\tau} \quad \dot{Q}_{\mathrm{EGS}}^{I\tau}(\dot{W}_{\mathrm{EGS}}^{\tau}, T_0^{\tau}, \mathrm{Altitude}) \quad \dot{Q}_{\mathrm{EGS}}^{\mathrm{I}\tau}(\dot{W}_{\mathrm{EGS}}^{\tau}) \tag{4}$$

$$\dot{Q}_{\mathrm{EGS}}^{\mathrm{XGtot}\tau} \quad \dot{Q}_{\mathrm{EGS}}^{\mathrm{XGtot}\tau}(\dot{W}_{\mathrm{EGS}}^{\tau} T_0^{\tau}, \mathrm{Altitude}) \quad \dot{Q}_{\mathrm{EGS}}^{\mathrm{XGtot}\tau}(\dot{W}_{\mathrm{EGS}}^{\tau}) \tag{5}$$

$$\dot{Q}_{\mathrm{EGS}}^{\mathrm{HTC}\tau} \quad \dot{Q}_{\mathrm{EGS}}^{\mathrm{HTC}\tau}(\dot{W}_{\mathrm{EGS}}^{\tau}, T_0^{\tau}, \mathrm{Altitude}) \quad \dot{Q}_{\mathrm{EGS}}^{\mathrm{HTC}\tau}(\dot{W}_{\mathrm{EGS}}^{\tau}) \tag{6}$$

$$\dot{W}_{\mathrm{EGS}}^{\mathrm{x}\tau} \quad \dot{W}_{\mathrm{EGS}}^{\mathrm{x}\tau}(\dot{W}_{\mathrm{EGS}}^{\tau}, T_0^{\tau}, \dot{Q}_{\mathrm{EGS}}^{0\tau}, \dot{Q}_{\mathrm{EGS}}^{\mathrm{LTC}\tau} \_ \quad \dot{W}_{\mathrm{EGS}}^{\mathrm{x}\tau}(\dot{W}_{\mathrm{EGS}}^{\tau}, \dot{Q}_{\mathrm{EGS}}^{0\tau}) \tag{7}$$

Supplementary heating unit

Decision variable related to the heating unit, *i. e.* boiler part load operation is thermal power output from the boiler, $\dot{Q}_{\mathrm{B}}^{\tau}$, constrained with minimal and maximal values if the boiler is on and equal to 0 if the boiler is off. Part load ratio might be used instead.

Primary energy (fuel, natural gas) consumed, $\dot{Q}_{\mathrm{B}}^{I\tau}$, might be represented as a function of thermal power output and water temperature at the exit of the boiler, T_{W}^{τ}. This temperature is usually defined in the form of the regulation curve, depending only on the external temperature, which is predefined in this case. Thus primary energy consumption might be represented like:

$$\dot{Q}_{\mathrm{B}}^{I\tau} \quad \dot{Q}_{\mathrm{B}}^{I\tau}(Q_{\mathrm{B}}^{\tau}, T_{\mathrm{W}}^{\tau})\dot{Q}_{\mathrm{B}}^{I\tau}(\dot{Q}_{\mathrm{B}}^{\tau}) \tag{8}$$

Start-up and/or shut down energy consumption discussion for engine generator set is applicable for the case of hot water boilers also.

Auxiliary electrical power required for boiler operation, $\dot{W}_{\mathrm{B}}^{\mathrm{x}\tau}$, is defined as a function of the thermal power output.

Cooling components

Decision variables for cooling components, *i. e.* single stage absorption chiller and compression chiller are refrigeration power outputs from these units: $\dot{Q}_{\mathrm{ABC}}^{\mathrm{r}\tau}$ and $\dot{Q}_{\mathrm{CPC}}^{\mathrm{r}\tau}$, respectively. These variables are constrained with their minimal and maximal values. Part load ratios might be used instead.

Capacities and part load performance of cooling units depend on the temperature of chilled water leaving the chiller, evaporator outlet temperature and condenser inlet temperature. These dependencies are considered, but for the optimization procedure thermal and electrical power inputs are given as the functions of refrigeration power outputs as:

$$\dot{Q}_{\mathrm{ABC}}^{\tau} \quad \dot{Q}_{\mathrm{ABC}}^{\tau}(\dot{Q}_{\mathrm{ABC}}^{\mathrm{r}\tau}); \quad \dot{W}_{\mathrm{ABC}}^{\mathrm{x}\tau} \quad \dot{W}_{\mathrm{ABC}}^{\mathrm{x}\tau}(\dot{Q}_{\mathrm{ABC}}^{\mathrm{r}\tau}); \quad \dot{W}_{\mathrm{CPC}}^{\tau} \quad \dot{W}_{\mathrm{CPC}}^{\tau}(\dot{Q}_{\mathrm{CPC}}^{\mathrm{r}\tau}) \tag{9}$$

where $\dot{Q}_{\mathrm{ABC}}^{\tau}$ is the thermal power input into absorption chiller, $\dot{W}_{\mathrm{ABC}}^{\mathrm{x}\tau}$ – the auxiliary electrical power required by the absorption chiller, and $\dot{W}_{\mathrm{CPC}}^{\tau}$ – the electrical power input into compression chiller.

Demand related constraints

The model described in this paper must contain at least three more constraints, *i. e.* constraints, shown in eqs. (10)-(12), that ensure that the demands of all the forms of energy (electrical, heating and cooling) are satisfied by the tri-generation plant output consisting of the co-generation unit(s), supplementary heating unit(s), and refrigeration units:

$$(\dot{W}_{\mathrm{G,S}}^{\tau} \quad \dot{W}_{\mathrm{G,P}}^{\tau}) \quad \gamma_{\mathrm{EGS}}^{\tau}(\dot{W}_{\mathrm{EGS}}^{\tau} \quad \dot{W}_{\mathrm{EGS}}^{\mathrm{x}\tau}) \quad \gamma_{\mathrm{B}}^{\tau}\dot{W}_{\mathrm{B}}^{\mathrm{x}\tau} \quad \gamma_{\mathrm{ABC}}^{\tau}\dot{W}_{\mathrm{EGS}}^{\mathrm{x}\tau} \quad \gamma_{\mathrm{CPC}}^{\tau}\dot{W}_{\mathrm{CPC}}^{\tau} \tag{10}$$

$$\gamma_{\mathrm{EGS}}^{\tau}(\dot{Q}_{\mathrm{EGS}}^{\mathrm{xG}\tau} \quad \dot{Q}_{\mathrm{EGS}}^{\mathrm{HTC}\tau} \quad Q_{\mathrm{EGC}}^{0\tau}) \quad \gamma_{\mathrm{B}}^{\tau}\dot{Q}_{\mathrm{B}}^{\tau} \quad \gamma_{\mathrm{ABC}}^{\tau}\dot{Q}_{\mathrm{ABC}}^{\tau} \quad \dot{Q}_{\mathrm{HD}}^{\tau} \tag{11}$$

$$\gamma_{\mathrm{ABC}}^{\tau}\dot{Q}_{\mathrm{ABC}}^{\mathrm{r}\tau} \quad \gamma_{\mathrm{CPC}}^{\tau}\dot{Q}_{\mathrm{CPC}}^{\mathrm{r}\tau} \quad \dot{Q}_{\mathrm{CD}}^{\tau} \tag{12}$$

where $\dot{W}_{\mathrm{D}}^{\tau}$, $\dot{Q}_{\mathrm{HD}}^{\tau}$ and $\dot{Q}_{\mathrm{CD}}^{\tau}$ are electrical, heating, and cooling demands, respectively, in kW, $\gamma_{\mathrm{EGS}}^{\tau}$, $\gamma_{\mathrm{B}}^{\tau}$, $\gamma_{\mathrm{ABC}}^{\tau}$, and $\gamma_{\mathrm{CPC}}^{\tau}$ are non-negative integer variables indicating number of units of each component running during the observed time interval, related to the engine generator sets, hot water boilers, absorption and compression chillers, respectively.

It is possible to define other, case specific constraints and variables in addition to ones presented in eqs. (10)-(12), if appropriate.

Primary energy savings

Primary energy savings function represents the difference between primary energy consumed by the co-generation plant and primary energy consumed by the referent, non-cogeneration plant that produces the same amounts of energy of each kind. European Commission defines primary energy savings for cogeneration plants in [20, 21]. Similar indicator, called fuel energy savings, is defined in [22]. Chicco *et al.* [13] define tri-generation primary energy savings. In this paper, primary energy savings function is defined as:

$$F_{\mathrm{PES}} \quad \frac{\dot{Q}_{\mathrm{HD}}}{\eta_{\mathrm{t,HRF}}} \quad \frac{\dfrac{\dot{Q}_{\mathrm{CD}}}{COP_{\mathrm{RF}}} \quad \dot{W}_{\mathrm{G,S}}\eta_{\mathrm{D}} \quad \dot{W}_{\mathrm{D}}}{\eta_{\mathrm{G,RF}}\eta_{\mathrm{T}}\eta_{\mathrm{D}}} \, \mathrm{d}\tau \qquad \dot{Q}_{\mathrm{EGS}}^{I} \quad \dot{Q}_{\mathrm{B}}^{I} \quad \frac{\dot{W}_{\mathrm{G,P}}}{\eta_{\mathrm{G,RF}}\eta_{\mathrm{T}}\eta_{\mathrm{D}}} \, \mathrm{d}\tau \tag{13}$$

where $\dot{W}_{\mathrm{D}}$, $\dot{Q}_{\mathrm{HD}}$, and $\dot{Q}_{\mathrm{CD}}$ are electrical, heating, and cooling demands, respectively, in kW, $\eta_{\mathrm{G,RF}}$, η_{T}, and η_{D} are referent efficiencies of electrical energy generation, transmission and distribution, respectively, while $\eta_{\mathrm{t,RF}}$ and COP_{RF} are referent thermal energy generation and distribution efficiency and referent refrigeration coefficient of performance. As a consequence of steady-state operation assumption, eq. (13) might be written in non-integral form as:

$$F_{\mathrm{PES}} \quad \sum_{\tau \; 1}^{\frac{8760}{\Delta\tau}} \quad \frac{\dot{Q}_{\mathrm{HD}}^{\tau}}{\eta_{\mathrm{t,RF}}} \quad \frac{\dfrac{\dot{Q}_{\mathrm{CD}}^{\tau}}{COP_{\mathrm{RF}}} \quad \dot{W}_{\mathrm{G,S}}^{\tau}\eta_{\mathrm{D}} \quad \dot{W}_{\mathrm{D}}^{\tau}}{\eta_{\mathrm{G,RF}}\eta_{\mathrm{T}}\eta_{\mathrm{D}}} \qquad \dot{Q}_{\mathrm{EGS}}^{I\tau} \quad Q_{\mathrm{B}}^{I\tau} \quad \frac{\dot{W}_{\mathrm{G,P}}^{\tau}}{\eta_{\mathrm{G,RF}}\eta_{\mathrm{T}}\eta_{\mathrm{D}}} \quad \Delta\tau \tag{14}$$

Some referent values for efficiencies are suggested in [21] and [13]. In this paper referent value for total electrical energy generation efficiency is 0.4, for boiler plant efficiency 0.82, while referent *COP* for electric chillers is 3.

Annual primary energy saving function might be represented as the sum of primary energy savings for all time intervals .

European legislative [20] defines the condition high efficiency co-generation production should satisfy. It is related to primary energy savings and assumes that high efficiency co-generation production is one that has primary energy savings of at least zero for small units (less than 1 $\mathrm{MW_e}$).

Results

In the case presented here, the objective was to define operation regimes that would result with the lowest simple pay-back period for the project of improvement of energy supply system for heating and air conditioning by implementation of natural gas fired reciprocating engine for co-generation of thermal and electrical energy and absorption refrigeration units. In the baseline scenario, the facility provides thermal energy for heating with two natural gas fired boilers of 490 kW each and for cooling with compression chiller of 444 kW. Application of one engine generator set of 315 $\mathrm{kW_e}$ is considered. For each month, one typical day is chosen for which hourly profiles of electricity, heating, and cooling demands are obtained. After simulations and optimization, optimal operational regimes are defined, depending on the demands and energy prices. In fig. 4, hourly demand curves and optimal operational regimes for co-generation and refrigeration units are shown for typical days in April, August, and December for high tariff electrical energy price of 9 c€/kWh.

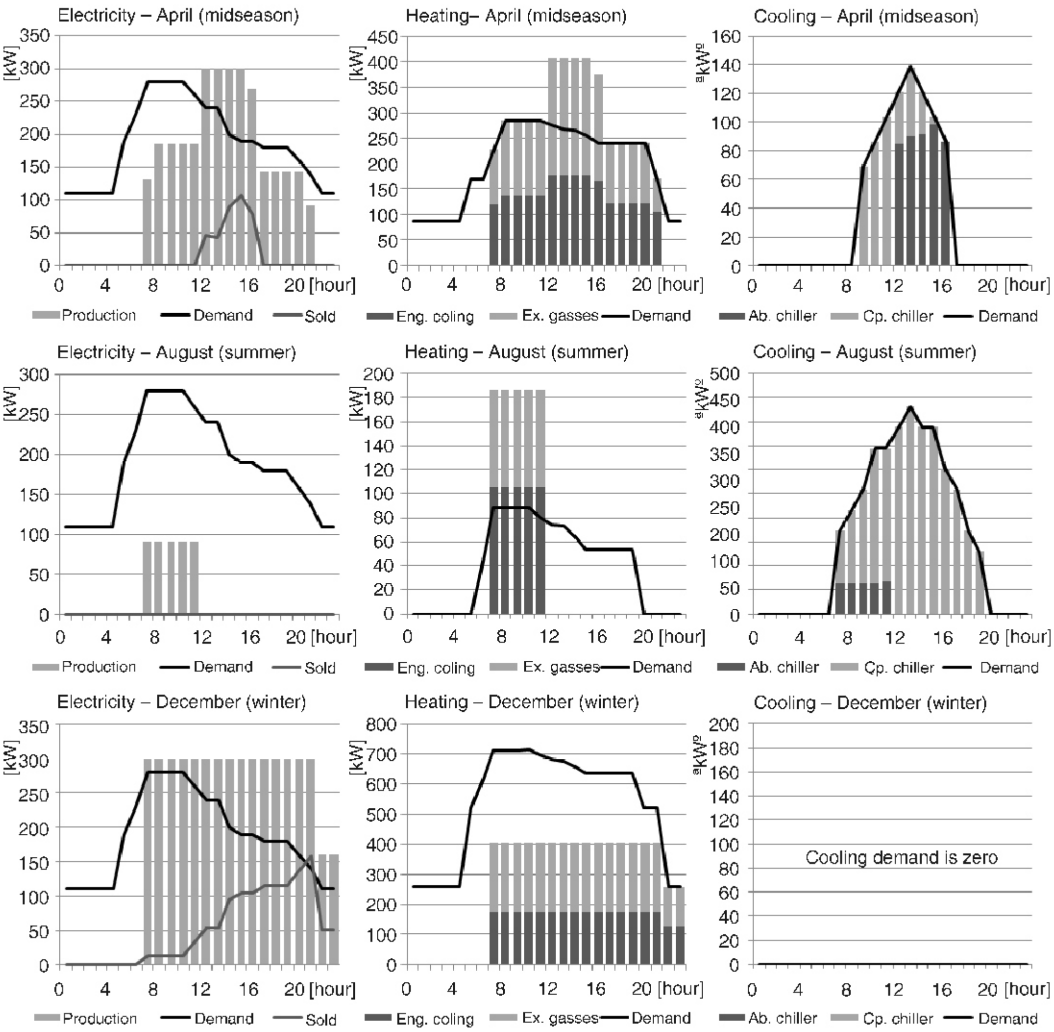

Figure 4. Hourly energy demand curves and production

Values of decision variables, as well as chosen operational regimes are strongly dependent on electrical, heating, and cooling demand and prices of energy commodities.

Optimal operation regime during most of the time assumes following the heating load, as can be seen in fig. 5. When cooling load exists, it is sometimes rational to satisfy part of cooling demand from the absorption chillers that use thermal output from the co-generation unit. When electrical energy price is higher, periods when usage of absorption refrigeration units is rational are more frequent, since electrical energy input into compression chillers is more expensive, making these units less financially attractive to use. It is concluded that for this facility, under above defined conditions, maximal capacity of absorption refrigeration unit that is optimal for use is approximately 236 kW, when high tariff electrical energy price is 10 c€/kWh, while for lower electrical energy prices this values decreases down to 204 kW for 9 c€/kWh and 99 kW for c€/kWh.

When electrical energy price is 7 c€/kWh, absorption unit is not used at all in optimal operation scenarios.

Higher price of electrical energy purchased from grid also makes import of electricity more expensive and production of electrical energy in co-generation unit more attractive. Thus, more electrical and heating energy is produced in engine generator set when electrical energy price is higher. It is also obvious that in most cases during the intervals when electrical energy from the grid can be purchased for low tariff price (from midnight to 8 AM) running of co-generation unit and production of electrical energy is not optimal.

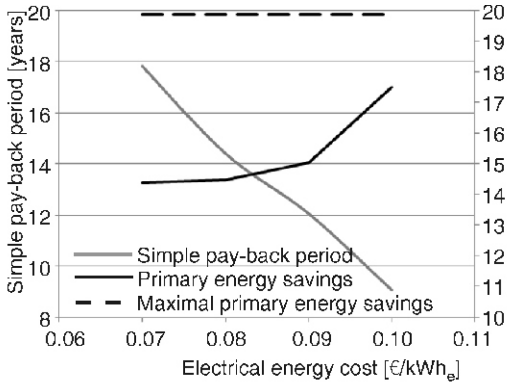

Figure 5. Simple payback period and primary energy savings of tri-generation plant in the function of electrical energy price (high tariff)

From the previous discussion it is logical to conclude that financial indicators are the best when electrical energy price is highest, as shown in fig. 5 and tab. 1. Increase of simple pay-back period with electrical energy price is obvious. It is the consequence of the fact that electricity import alternative for satisfaction of electrical demand, as well as for cooling purposes becomes more expensive, *i. e.* less attractive comparing to co-generation and absorption cooling using co-generation thermal output.

Table 1. Variations in primary energy savings, thermal output costs, and simple pay-back period with electrical energy price

Electrical energy price (high price tarrif)	€/kWh$_e$	0.07	0.08	0.09	0.10
Primary energy savings	MWh (ncv)	1110	1120	1172	1344
Primary energy savings	%	14.37	14.47	15.05	17.49
Maximal primary energy savings	%	19.87	19.87	19.87	19.87
Co-generation thermal output costs	€/kWh$_t$	6.61	6.04	5.21	3.95
Simple payback period	years	17.81	14.39	12.05	9.07

Primary energy savings calculated using eq. (14) is in the range from 14.37% to 17.49%. With the increase in the number of hours of running engine generator set and absorption chiller primary energy savings also increase approaching its maximal value of 19.87%. For this case, *i. e.* for this equipment and such part load performance, as well as for the referent conditions defined above, results show that the best primary energy savings is achieved mostly running the co-generation unit in heat match mode, *i. e.* regime during which engine is operated to follow heating energy consumption of both the heat consumer and absorption refrigeration unit. Heating demand is covered by the engine generator set and cooling demand is also covered using the engine thermal output. Engine exhaust gasses are mostly cooled down to the minimal al-

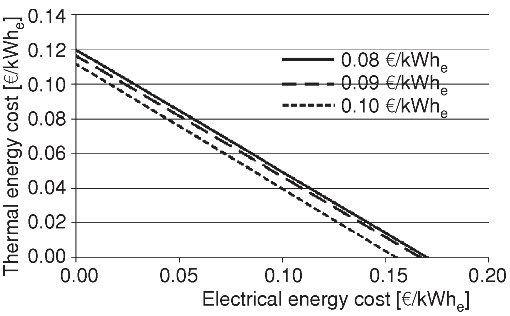

Figure 6. Triangles of prices for co-generation unit for different prices of electrical energy

lowed temperature, in this case 120 °C. If engine thermal output is not high enough to cover consumer and absorption chiller demand, additional cooling energy is provided by the compression refrigeration unit. Electrical energy from the engine generator set is used for satisfying consumer demand and to run chillers. Excess electrical energy, if any, is exported to the grid, while for the cases when additional electrical energy is required, it is imported from the grid, keeping the engine generator set at the part load operation high enough to satisfy heating energy consumption. This is concluded using the hourly primary energy savings function as the objectives in optimization. Maximal value of primary energy savings logically does not depend on the prices of energy commodities.

For engine generator set, assigning electrical energy price and feed-in tariffs value to electrical energy costs values, thermal energy cost is determined according to [23] and presented in tab. 1. Also triangle of prices for engine generator set is constructed and shown in fig. 6.

Conclusions

In this paper an approach to operational optimization of tri-generation plants with natural gas fired reciprocating engines is presented. Operational optimization of such facilities is important for short term energy production planning based on energy demand forecasts, for optimal control of the equipment of a tri-generation plant, but also as a part of design level optimization and decision making in the feasibility stage of the co-generation/tri-generation projects.

Results of one of the analyses performed using this model are presented. It is shown that values of decision variables that determine operational regime depend on electrical, heating, and cooling demand, as well as on energy commodities prices.

Increase in electrical energy price influence higher costs related to baseline scenario (without co-generation) for both satisfaction of electrical demand and cooling purposes. At the same time, co-generation of heating and electrical energy becomes more financially attractive, as well as using thermal output of co-generation unit for absorption cooling. pay-back period of co-generation projects decreases, as well as cost of thermal energy produced in co-generation unit.

It can also be concluded that small scale (up to 1 MW$_e$) tri-generation plants consume less primary energy compared to conventional plants for separate production due to higher overall efficiencies and small distances between facilities and end users. Higher electrical energy price results in the increase in the number of hours of running engine generator set and absorption chiller influencing primary energy savings also to increase from 14.4% to 17.5% approaching its maximal value that in this case is almost 20%.

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