CREEP TRANSITION STRESSES IN A THIN ROTATING DISC WITH SHAFT BY FINITE DEFORMATION UNDER STEADY-STATE TEMPERATURE

by

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Creep stresses and strain rates have been derived for a thin rotating disc with shaft at different temperature. Results have been discussed and presented graphically. It has been observed that radial stress has maximum value at the internal surface of the rotating disc made of incompressible material as compared to circumferential stress and this value of radial stress further increase with the increase of angular speed. With the introduction of thermal effect, it has been observed that radial stress has higher maximum value at the internal surface of the rotating disc made of incompressible material as compared to circumferential stress, and this value of radial stress further increases with the increase of angular speed as compared to the case without thermal effect. Strain rates have maximum values at the internal surface for compressible material. Rotating disc is likely to fracture by cleavage close to the inclusion at the bore.

Key words: *creep stress, strain rates, displacement, angular speed, inclusion, rotating disc, temperature*

Introduction

Rotating discs are an essential part of the rotating machinery structure, *e. g.* rotors, turbines, compressors, flywheel, and computer's disc drive. The analytical procedures presently available are restricted to problems with simplest configurations. The use of rotating disc in machinery and structural applications has generated considerable interest in many problems in domain of solid mechanics. Solutions for thin isotropic discs can be found in most of the standard creep textbooks [1-6]. Wahl [7] has investigated creep deformation in rotating discs assuming small deformation, incompressibility condition, Tresca yield criterion, its associated flow rule and a power strain law. Seth's transition theory [8] does not acquire any assumptions like yield condition and incompressibility condition, thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out.

Seth [9] has defined the generalized principal strain measure as:

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$$e_{ii} \quad {}_0^{e_{ii}^A}[1 \quad 2e_{ii}^A]^{(n/2) \quad 1}\mathrm{d}e_{ii}^A \quad \frac{1}{n}[1 \quad (1 \quad 2e_{ii}^A)^{n/2}], \quad (i \quad 1, 2, 3) \tag{1}$$

where n is the measure and e_{ii}^A – the Almansi finite strain components [9]. For $n = -2, -1, 0, 1$, and 2 it gives Cauchy, Green Hencky, Swainger, and Almansi measures, respectively.

In this paper, the problem of creep transition stresses in a thin rotating disc with shaft by finitesimal deformation under steady-state temperature is investigated by using Seth's transition theory. Results have been discussed and presented graphically.

Governing equations

A thin annular disc with central bore of radius a and outer radius b is considered (fig. 1). The disc, produced of material of constant density, is mounted on a rigid shaft. The disc is rotating with angular speed ω about a central axis perpendicular to its plane. The thickness of disc is assumed to be constant and is taken sufficiently small so that the disc is effectively in a state of plane stress, that is, the axial stress T_{zz} is zero. The temperature at the central bore of the disc is Θ. The displacement components in cylindrical polar co-ordinate are given by [9]:

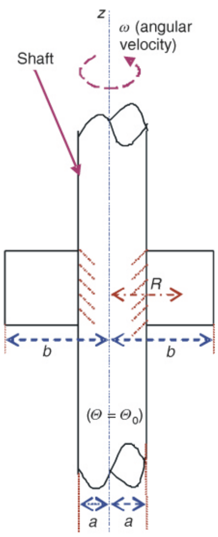


Figure 1. Geometry of rotating disc

$$u = r(1 - \beta), \quad v = 0, \quad w = dz \tag{2}$$

where β is the position function, depending on $r = (x^2 + y^2)^{1/2}$ only, and d is a constant.

The finite strain components are given by Seth [9] as:

$$\begin{aligned} e_{rr}^A &\quad \frac{1}{2}[1 \quad (r\beta \quad \beta)^2] \\ e_{\theta\theta}^A &\quad \frac{1}{2}[1 \quad \beta^2\lambda] \\ e_{zz}^A &\quad \frac{1}{2}[1 \quad (1 \quad d)^2] \\ e_{r\theta}^A &\quad e_{\theta z}^A \quad e_{zr}^A \quad 0 \end{aligned} \tag{3}$$

where $\beta' = \mathrm{d}\beta/\mathrm{d}r$ and meaning of superscripts A is Almansi.

By substituting eq. (3) in eq. (1), the generalized components of strain become:

$$\begin{aligned} e_{rr} &\quad \frac{1}{n}[1 \quad (r\beta \quad \beta)^n] \\ e_{\theta\theta} &\quad \frac{1}{n}(1 \quad \beta^n) \\ e_{zz} &\quad \frac{1}{n}[1 \quad (1 \quad d)^n] \\ e_{r\theta} &\quad e_{\theta z} \quad e_{zr} \quad 0 \end{aligned} \tag{4}$$

The stress-strain relations for thermo-elastic isotropic material are given by [10]:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\Theta\delta_{ij}, \quad (i, j = 1, 2, 3) \tag{5}$$

where T_{ij} are the stress components, λ and μ – the Lame's constants, $I_1 = e_{kk}$ – the first strain invariant, δ_{ij} – the Kronecker's delta, $\xi = \alpha(3\lambda + 2\mu)$, α – the coefficient of thermal expansion, and Θ – the temperature. Further, Θ has to satisfy:

$$ {}^2\Theta \quad 0 $$

or

$$ \frac{\mathrm{d}^2\Theta}{\mathrm{d}r^2} \quad \frac{1}{r}\frac{\mathrm{d}^2\Theta}{\mathrm{d}r^2} \quad \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} r \frac{\mathrm{d}\Theta}{\mathrm{d}r} \quad 0 $$

$$ \frac{\mathrm{d}\Theta}{\mathrm{d}r} \quad \frac{k}{r} $$

which has solution:

$$ \Theta = k_1(\log r + k_2) \tag{6} $$

where k_1 and k_2 are constant of integration and can be determined from the boundary condition.

Equation (5) for this problem becomes:

$$ \begin{aligned} &T_{rr} \quad \frac{2\lambda\mu}{\lambda \quad 2\mu}(e_{rr} \quad e_{\theta\theta}) \quad 2\mu e_{rr} \quad \frac{2\mu\xi\Theta}{\lambda \quad 2\mu} \\ &T_{\theta\theta} \quad \frac{2\lambda\mu}{\lambda \quad 2\mu}(e_{rr} \quad e_{\theta\theta}) \quad 2e_{\theta\theta} \quad \frac{2\mu\xi\Theta}{\lambda \quad 2\mu} \\ &T_{z\theta} \quad T_{\theta z} \quad T_{zr} \quad T_{zz} \quad 0 \end{aligned} \tag{7} $$

By substituting eq. (4) in eq. (7), the stresses are obtained as:

$$ \begin{aligned} &T_{rr} \quad \frac{2\mu}{n} \quad 3 \quad 2C \quad \beta^n \, 1 \quad C \quad (2 \quad C)\frac{r\beta}{\beta} \quad 1^{\ n} \quad \frac{nC\xi\Theta}{2\mu\beta^n} \\ &T_{\theta\theta} \quad \frac{2\mu}{n} \quad 3 \quad 2C \quad \beta^n \, 2 \quad C \quad (1 \quad C)\frac{r\beta}{\beta} \quad 1^{\ n} \quad \frac{nC\xi\Theta}{2\mu\beta^n} \\ &T_{r\theta} \quad T_{\theta z} \quad T_{zr} \quad T_{zz} \quad 0 \end{aligned} \tag{8} $$

where C is the compressibility factor of the material in term of Lame's constant, given by $C = 2\mu/\lambda + 2\mu$.

The equations of motion are all satisfied except:

$$ \frac{\mathrm{d}}{\mathrm{d}r}(T_{rr}) \quad T_{\theta\theta} \quad \rho\omega^2 r^2 \quad 0 \tag{9} $$

where ρ is the density of the material of the rotating disc.

The temperature is satisfying Laplace eq. (6) with boundary condition:

$$ \Theta = \Theta_0 \ \text{ at } \ r = a, $$

$$ \Theta = 0 \ \text{ at } \ r = b, $$

where Θ_0 is constant, given by [10]:

$$ k_1 \quad \frac{\Theta}{\log \frac{a}{b}} \quad \text{and} \quad k_2 \quad \log b $$

By substituting k_1 and k_2 from eq. (6), one gets:

$$\Theta = \frac{\Theta_0 \log\dfrac{r}{b}}{\log\dfrac{a}{b}} \tag{10}$$

By using eqs. (8) and (10) in eq. (9), one gets a non-linear differential equation for β:

$$(2-C)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta} = \frac{n\rho\omega^2 r^2}{2\mu} + \frac{nC\xi\overline{\Theta}_0}{2\mu} - \beta^n\{1-(P+1)^n - nP[1-C-(2-C)(P+1)^n]\} \tag{11}$$

where $\overline{\Theta}_0 = \Theta_0/\log(a/b)$ and $r\beta' = \beta P$ (P is function of β and β is function of r).

From eq. (11), the turning points of β are $P \to -1$ and $\pm\infty$.

The boundary conditions are:

$$u = 0 \quad \text{at} \quad r = a \quad \text{and} \quad T_{rr} = 0 \quad \text{at} \quad r = b \tag{12}$$

Solution through the principal stress difference

For finding the creep stresses, the transition function is taken through principal stress difference (see [9, 11-13] and Gupta *et al*. [14-21]) at the transition point $P \to -1$. The transition function R is defined as:

$$R = T_{rr} - T_{\theta\theta} = \frac{2\mu\beta^n}{n}[1-(P+1)^n] \tag{13}$$

Taking the logarithmic differentiating of eq. (13) with respect to r, one gets:

$$\frac{d}{dr}(\log R) = \frac{nP}{r[1-(P+1)^n]}\left[1-(P+1)^n - \beta(P+1)^{n-1}\frac{dP}{d\beta}\right] \tag{14}$$

By substituting the value $dP/d\beta$ of from eq. (11) into eq. (14) and by taking asymptotic value $P \to -1$, one gets:

$$\frac{d}{dr}(\log R) = -\frac{1}{r(2-C)}\left[n(3-2C)-1-\frac{n\rho\omega^2 r^{2+n}}{2\mu D^n} - \frac{nC\xi\overline{\Theta}_0}{2\mu\beta^n(2-C)}\right] \tag{15}$$

Asymptotic value of β as $P \to -1$ is D/r; D being a constant.

Integrating eq. (15) with respect to r, one gets:

$$R = T_{rr} - T_{\theta\theta} = K_1 r^k \exp(Fr^{n+2} + Gr^n) \tag{16}$$

where K_1 is a constant of integration, which can be determined by boundary condition and by:

$$\nu = \frac{1-C}{2-C}, \quad C\xi = \frac{\alpha E}{1-\nu}, \quad k = -[(n-1) - \nu(n+1)],$$

$$G = \frac{\alpha\overline{\Theta}_0(1-\nu)}{D^n}, \quad F = \frac{n\omega^2\rho(1-\nu)}{2\mu D^n(n+2)} = \frac{n\omega^2\rho(1-\nu^2)}{ED^n(n+2)}$$

From eq. (13) and (16), it follows:

$$T_{rr} - T_{\theta\theta} = K_1 r^k \exp(Fr^{n+2} + Gr^n) \qquad (17)$$

By substituting eq. (17) into eq. (9), one gets:

$$T_{rr} \quad K_1 \; r^{k \;\; 1} \exp(Fr^{n \;\; 2} \quad Gr^n)\,\mathrm{d}r \quad \frac{\rho\omega^2 r^2}{2} \quad K_2 \qquad (18)$$

where K_2 is a constant of integration, which can be determined by boundary condition.

By applying boundary condition (12) in eq. (18), one gets:

$$K_2 \quad K_1 \; r^{k \;\; 1} \exp(Fr^{n \;\; 2} \quad Gr^n)\,\mathrm{d}r_{\; r \;\; b} \quad \frac{\rho\omega^2 b^2}{2}$$

By substituting the value of K_2 into eq. (18), one gets:

$$T_{rr} \quad K_1 \, {}_r^b \, r^{k \;\; 1} \exp(Fr^{n \;\; 2} \quad Gr^n)\,\mathrm{d}r \quad \frac{\rho\omega^2 (b^2 \quad r^2)}{2} \qquad (19)$$

By substituting eq. (19) into eq. (17), one gets:

$$T_{\theta\theta} \quad K_1 \, {}_r^b \, r^{k \;\; 1} \exp(Fr^{n \;\; 2} \quad Gr^n)\,\mathrm{d}r \quad r^k \exp(Fr^{n \;\; 2} \quad Gr^n) \quad \frac{\rho\omega^2 (b^2 \quad r^2)}{2} \qquad (20)$$

From eq. (13) and (17), one gets:

$$\frac{2\mu}{n} \beta^n [1 \quad (P \quad 1)^n] \quad K_1 r^k \exp(Fr^{n \;\; 2} \quad Gr^n)$$

Taking asymptotic value $P \quad -1$, one gets:

$$\beta \quad \frac{n}{2\mu}(T_{rr} \quad T_{\theta\theta}) K_1 r^k \exp(Fr^{n \;\; 2} \quad Gr^n)^{1/n} \quad \frac{n(1 \quad \nu)}{E} K_1 r^k \exp(Fr^{n \;\; 2} \quad Gr^n)^{1/n} \qquad (21)$$

where $2\mu = E/(1 + \nu)$ is the Young's modulus in term of Poisson's ratio.

By using eq. (21) and eq. (2), one gets:

$$u \quad r \quad r \; \frac{n(1 \quad \nu)}{E} K_1 r^k \exp(Fr^{n \;\; 2} \quad Gr^n)^{1/n} \qquad (22)$$

By applying boundary condition (12) in equation (22), one gets:

$$K_1 \quad \frac{E}{n(1 \quad \nu) a^k \exp(Fa^{n \;\; 2} \quad Ga^n)}$$

By substituting the value of K_1 into eqs. (19), (20), and (22), one gets:

$$T_{\theta\theta} \quad \frac{E}{n(1 \quad \nu) a^k \exp(Fa^{n \;\; 2} \quad Ga^n)} \, {}_r^b \, r^{k \;\; 1} \exp(Fr^{n \;\; 2} \quad Gr^n)\mathrm{d}r \quad r^k \exp(Fr^{n \;\; 2} \quad Gr^n)$$
$$\frac{\rho\omega^2 (b^2 \quad r^2)}{2} \qquad (23)$$

$$T_{rr} \quad \frac{E}{n(1 \quad \nu) a^k \exp(Fa^{n \;\; 2} \quad Ga^n)} \, {}_r^b \, r^{k \;\; 1} \exp(Fr^{n \;\; 2} \quad Gr^n)\,\mathrm{d}r \quad \frac{\rho\omega^2 (b^2 \quad r^2)}{2} \qquad (24)$$

$$u \quad r \quad r \left[\frac{r^k \exp(Fr^{n\;\;2} \quad Gr^n)}{a^k \exp(Fa^{n\;\;2} \quad Ga^n)} \right]^{1/n} \tag{25}$$

Equations (23)-(25) define creep stresses and displacement for a thin rotating disc with inclusion at temperature Θ_0.

By introducing the following non-dimensional components as:

$$R \quad \frac{r}{b}, \quad R_0 \quad \frac{a}{b}, \quad \sigma_r \quad \frac{T_{rr}}{E}, \quad \sigma_\theta \quad \frac{T_{\theta\theta}}{E}, \quad \Omega^2 \quad \frac{\rho\omega^2 b^2}{E}, \quad \bar{u} \quad \frac{u}{b}, \quad \text{and} \quad \alpha\Theta_0 \quad \Theta_1$$

equations (23) to (25) in non-dimensional form become:

$$\sigma_\theta \quad \frac{1}{n(1 \quad \nu)R_0^k \exp(F_1 R_0^{n\;\;2} \quad G_1 R_0^n)} \int_R^1 R^{k\;\;1} \exp(F_1 R^{n\;\;2} \quad G_1 R^n)\mathrm{d}R \quad R^k \exp(F_1 R^{n\;\;2} \quad G_1 R^n) \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{26}$$

$$\sigma_r \quad \frac{1}{n(1 \quad \nu)R_0^k \exp(F_1 R_0^{n\;\;2} \quad G_1 R_0^n)} \int_R^1 R^{k\;\;1} \exp(F_1 R^{n\;\;2} \quad G_1 R^n)\mathrm{d}R \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{27}$$

$$\bar{u} \quad R \quad R \left[\frac{R^k \exp(F_1 R^{n\;\;2} \quad G_1 R^n)}{R_0^k \exp(F_1 R_0^{n\;\;2} \quad G_1 R_0^n)} \right]^{1/n} \tag{28}$$

where $F_1 \quad \frac{n\Omega^2(1 \quad \nu^2)b^n}{D^n(n \quad 2)}$, $k \quad [n \quad 1 \quad \nu(n \quad 1)]$ and $G_1 \quad \frac{\Theta_1(1 \quad \nu)b^n}{D^n \log R_0}$

For a disc made of incompressible material ($\nu \quad 1/2$) eqs. (26) to (28) become:

$$\sigma_\theta \quad \frac{2}{3nR_0^k \exp(F_2 R_0^{n\;\;2} \quad G_2 R_0^n)} \int_R^1 R^{k\;\;1} \exp(F_2 R^{n\;\;2} \quad G_2 R^n)\mathrm{d}R \quad R^k \exp(F_2 R^{n\;\;2} \quad G_2 R^n) \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{29}$$

$$\sigma_r \quad \frac{2}{3nR_0^k \exp(F_2 R_0^{n\;\;2} \quad G_2 R_0^n)} \int_R^1 R^{k\;\;1} \exp(F_2 R^{n\;\;2} \quad G_2 R^n)\mathrm{d}R \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{30}$$

$$\bar{u} \quad R \quad R \left[\frac{R^k \exp(F_2 R^{n\;\;2} \quad G_2 R^n)}{R_0^k \exp(F_2 R_0^{n\;\;2} \quad G_2 R_0^n)} \right]^{1/n} \tag{31}$$

where $F_2 \quad \frac{3n\Omega^2 b^n}{4D^n(n \quad 2)}$, $k^* \quad \frac{3n \quad 1}{2}$ and $G_2 \quad \frac{3\Theta_1 b^n}{2D^n \log R_0}$

Particular case

When there is no thermal effect ($\Theta_1 = 0$), creep stresses from eq. (26) to (28) become:

$$\sigma_\theta \quad \frac{1}{n(1 \quad \nu)R_0^k \exp(F_1 R_0^{n \quad 2})} \quad {}_R^{1} \frac{R^{k \quad 1}\exp(F_1 R^{n \quad 2})\mathrm{d}R}{R^k \exp(F_1 R^{n \quad 2})} \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{32}$$

$$\sigma_r \quad \frac{1}{n(1 \quad \nu)R_0^k (F_1 R_0^{n \quad 2})} \quad {}_R^{1} R^{k \quad 1}\exp(F_1 R^{n \quad 2})\mathrm{d}R \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{33}$$

$$u \quad R \quad R \left[\frac{R^k \exp(F_1 R^{n \quad 2})}{R_0^k \exp(F_1 R_0^{n \quad 2})}\right]^{1/n} \tag{34}$$

where $F_1 \quad \frac{n\Omega^2(1 \quad \nu^2)b^n}{D^n(n \quad 2)}, \quad k \quad [n \quad 1 \quad \nu(n \quad 1)]$

For incompressible material (ν 1/2) eqs. (32) to (34) become:

$$\sigma_\theta \quad \frac{2}{3nR_0^{k^*} \exp(F_2 R_0^{n \quad 2})} \quad {}_R^{1} \frac{R^{k^* \quad 1}\exp(F_2 R^{n \quad 2})\mathrm{d}R}{R^{k^*} \exp(F_2 R^{n \quad 2})} \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{35}$$

$$\sigma_r \quad \frac{1}{3nR_0^{k^*} \exp(F_2 R_0^{k^*})} \quad {}_R^{1} R^{k^* \quad 1}\exp(F_2 R^{n \quad 2})\mathrm{d}R \quad \frac{\Omega^2}{2}(1 \quad R^2) \tag{36}$$

$$u \quad R \quad R \left[\frac{R^{k^*} \exp(F_1 R^{n \quad 2})}{R_0^{k^*} \exp(F_1 R_0^{n \quad 2})}\right]^{1/n} \tag{37}$$

where $F_2 \quad \frac{3n\Omega^2 b^n}{4D^n(n \quad 2)}, \quad k^* \quad \frac{3n \quad 1}{2}$

These equations are the same as obtained by Gupta and Pankaj [20].

Strain rates

When the creep sets in, the strains should be replaced by strain rates. The stress-strain relations (5) become:

$$\dot{e}_{ij} \quad \frac{1 \quad \nu}{E} T_{ij} \quad \frac{\nu}{E}\delta_{ij} T \quad \alpha\Theta \tag{38}$$

where $\dot{e}_{ij}$ is the strain rate tensor with respect to flow parameter t.

By differentiating eq. (4) with respect to time t, one gets:

$$\dot{e}_{\theta\theta} \quad \beta^{n \quad 1}\dot{\beta} \tag{39}$$

For Swainger measure ($n = 1$), from eq. (39) it follows:

$$\dot{\varepsilon}_{\theta\theta} \quad \dot{\beta} \tag{40}$$

The transition value of eq. (13) as P -1 gives:

$$\beta \quad \frac{n}{2\mu}^{1/n} (T_{rr} \quad T_{\theta\theta})^{1/n} \quad \frac{n(1 \quad \nu)}{E}^{1/n} (T_{rr} \quad T_{\theta\theta})^{1/n} \tag{41}$$

By substituting eq. (39), (40), and (41) into eq. (38), one gets:

$$\begin{aligned} &\dot{\varepsilon}_{rr} \quad [n(\sigma_r \quad \sigma_\theta)(1 \quad \nu)]^{1/n \quad 1} (\sigma_r \quad \nu\sigma_\theta \quad \alpha\Theta) \\ &\dot{\varepsilon}_{\theta\theta} \quad [n(\sigma_r \quad \sigma_\theta)(1 \quad \nu)]^{1/n \quad 1} (\sigma_r \quad \nu\sigma_\theta \quad \alpha\Theta) \\ &\dot{\varepsilon}_{zz} \quad [n(\sigma_r \quad \sigma_\theta)(1 \quad \nu)]^{1/n \quad 1} (\nu(\sigma_r \quad \sigma_\theta) \quad \alpha\Theta) \end{aligned} \tag{42}$$

where $\Theta = \Theta_0 \log R/\log R_0$.

For incompressible material ($\nu \quad 1/2$) eq. (42) becomes:

$$\begin{aligned} &\dot{\varepsilon}_{rr} \quad \frac{3n(\sigma_r \quad \sigma_\theta)}{2}^{1/n \quad 1} \frac{2\sigma_r \quad \sigma_\theta \quad 2\alpha\Theta}{2} \\ &\dot{\varepsilon}_{\theta\theta} \quad \frac{3n(\sigma_r \quad \sigma_\theta)}{2}^{1/n \quad 1} \frac{2\sigma_\theta \quad \sigma_r \quad 2\alpha\Theta}{2} \\ &\dot{\varepsilon}_{zz} \quad \frac{3n(\sigma_r \quad \sigma_\theta)}{2}^{1/n \quad 1} \frac{1}{2}(\sigma_r \quad \sigma_\theta) \quad \alpha\Theta \end{aligned} \tag{43}$$

These constitutive equations are same as obtained by Odqvist [5] provided $n = 1/N$.

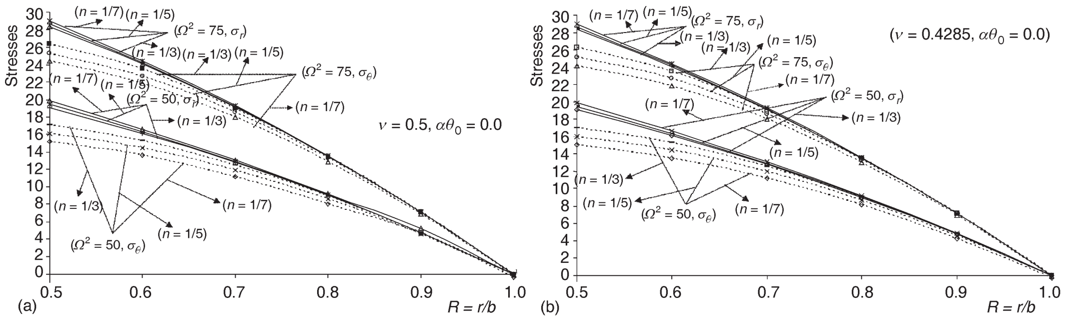
Results and discussion

For calculating stresses, strain-rates and displacement based on the above analysis, the following values have been taken: $\Omega^2 = \rho\omega^2 b^2/E = 50$ and 75, $\nu = 0.5$ (incompressible material [21], $\nu = 0.42857$ and 0.333 (compressible materials [20]), $n = 1/3$, 1/5, and 1/7 (*i. e.* $N = 3$, 5, and 7), $\alpha = 5.0 \cdot 10^{-5}$ deg F^{-1} (for methyl methacrylate) [22], $\Theta_0 = 0$ and 700 °F, $\Theta_1 = \alpha\Theta_0 = 0.00$ and 0.035, and $D = 1$.

In classical theory measure N is equal to $1/n$. Definite integrals in eqs. (26) and (27) have been solved by using Simpson's rule .

Curves have been drawn in figs. 2(a)-(c) and 3(a)-(c) representing relations between stresses and radii ratio $R = r/b$ for a rotating disc made of compressible/incompressible material with and without thermal effect at different angular speeds. It has been observed, from figs. 2(a)-(c), that in the absence of thermal effect the radial stress has maximum value at the internal surface of disc as compared to the circumferential stress. It is also observed that the radial stress has maximum value at the internal surface of the rotating disc with inclusion made of incompressible material as compared to compressible material for measure $n = 1/7$ (or $N = 7$) at angular speed $\Omega^2 = 50$, whereas circumferential stress has maximum value at the internal surface for measure $n = 1/3$ (or $N = 3$) at this angular speed. The values of radial/circumferential stress further increases at the internal surface with the increase of angular speed ($\Omega^2 = 75$) for measure $n = 1/7$ (or $N = 7$) and $n = 1/3$ (or $N = 3$), respectively.

With the introduction of thermal effects it can be seen from figs. 3(a)-(c) that much higher angular speed is required for yielding to appear at the internal surface as compared to the case without thermal effect. It can also be seen that maximum value of radial stress occurs at the internal surface.



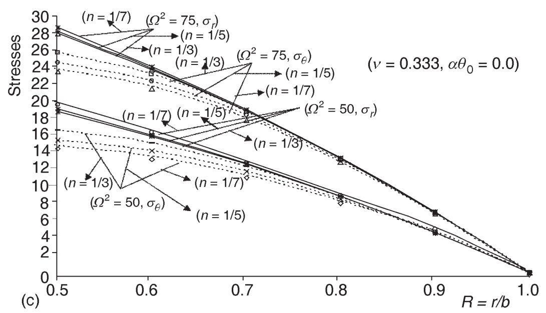
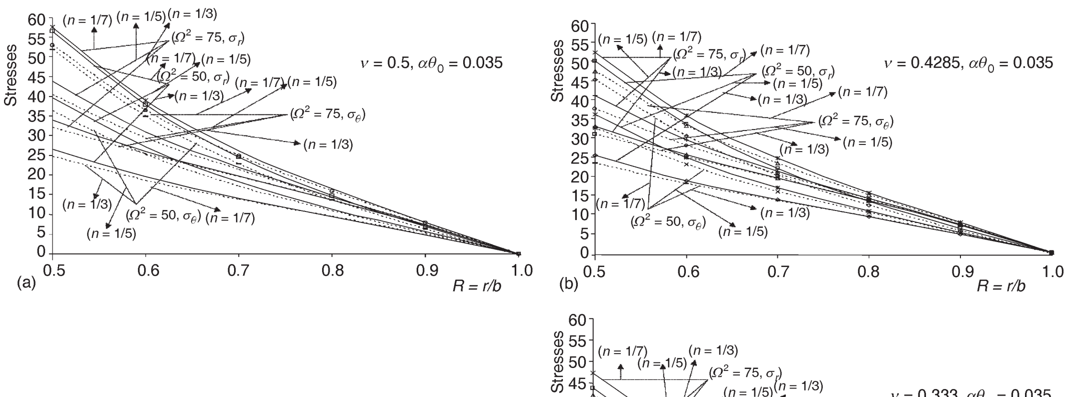


Figure 2. Creep stresses for a thin rotating disc with inclusion along the radius *R* = *r/b*

Curves have been drawn in figs. 4(a)-(f) representing relations between creep strain rates and radius $R = r/b$ at angular speed $\Omega^2 = 50$ and 75 and measures $n = 1/7$, 1/5, and 1/3 (or $N = 7$, 5, and 3). It can been seen that rotating disc made of compressible material has higher maximum value at the internal surface as compared to incompressible material at the angular speed $\Omega^2 = 50$. The values of strain rates further increases at the internal surface with the increase of the angular speed $\Omega^2 =$



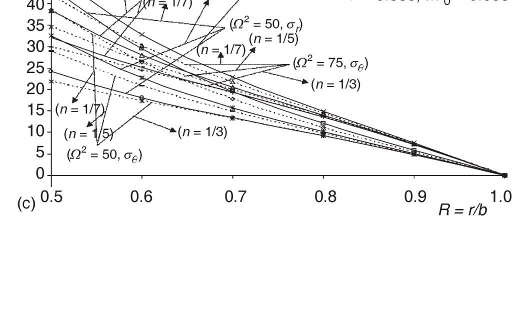


Figure 3. Thermal creep stresses for a thin rotating disc with inclusion along the radius *R* = *r/b*

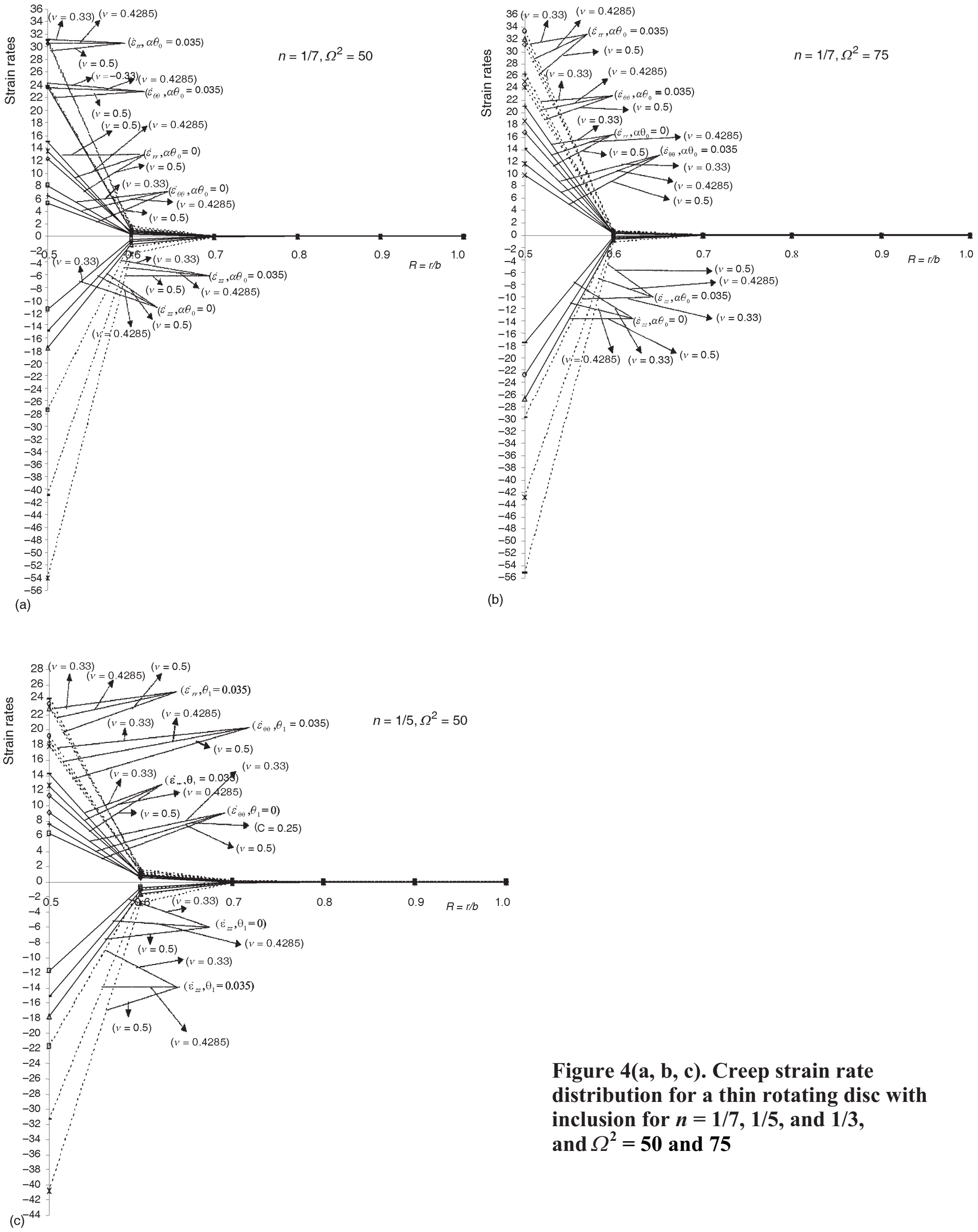
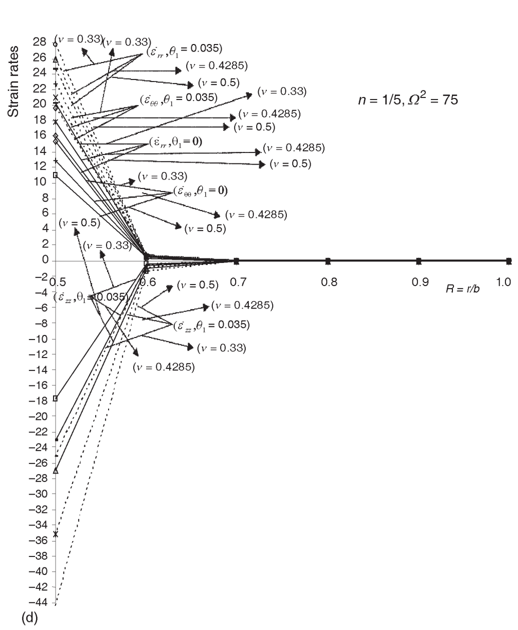


Figure 4(a, b, c). Creep strain rate distribution for a thin rotating disc with inclusion for *n* = 1/7, 1/5, and 1/3, and Ω^2 = 50 and 75

75. With the introduction of thermal effects, the maximum value of strain rates at the internal surface is higher as compared to the case without thermal effect.



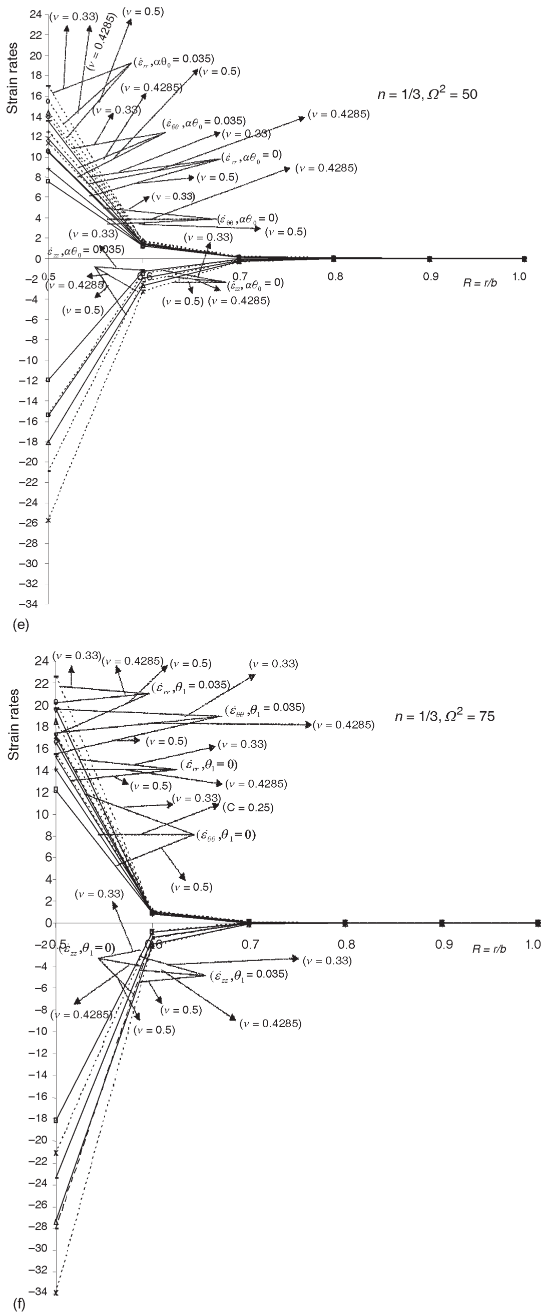


Figure 4(d, e, f). Creep strain rate distribution for a thin rotating disc with inclusion for *n* = 1/7, 1/5, and 1/3, and Ω^2 = 50 and 75

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Nomenclature

a, b – internal and external radii of the disc, [m]
C – compressibility factor, [–]
e_{ij}, T_{ij}, – stress strain rate tensors, [kgm^{-1}s^{-2}]
K_1,K_2,
k_1, k_2 – constants of integration, [–]
R – radii ratio (= *r/b*), [–]
R_0 – radii ratio(= *a/b*), [–]
u, v, w – displacement components, [m]

Greek letters

ε_{ij} – Swainger strain components, [–]
Θ,θ – temperature, [K]; $\Theta_1 = \alpha\Theta_0$, [–]
ν – Poisson's ratio, [–]
ρ – density of material, [kgm^{-3}]
σ_r – radial stress component (= T_{rr}/E), [–]
σ_θ – circumferential stress component (= $T_{\theta\theta}/E$), [–]
Ω^2 – speed factor (= $\rho\omega_2 b^2/E$), [–]
ω – angular speed of rotation, [s^{-1}]

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