EFFECTIVE THERMAL CONDUCTIVITY MODELING WITH PRIMARY AND SECONDARY PARAMETERS FOR TWO-PHASE MATERIALS

by

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In this article, the collocated parameter models are used to estimate the effective thermal conductivity of the two-phase materials including the effect of various inclusions in the unit cell. The algebraic equations are derived using unit cell based isotherm approach for two dimensional spatially periodic medium. The geometry of the medium is considered as a matrix of touching and non-touching in-line octagon and hexagon cylinders. The models are used to predict the thermal conductivity of numerous two-phase materials (maximum conductivity ratio of 1000 and concentration ranging between 0 and 1). The estimated thermal conductivity data is in good agreement with the experimental data within ±15.84%, ±18.14% maximum deviation, respectively, from octagon and hexagon cylinders for various two-phase systems. The obtained results are compared with a wide range of experimental data for various geometrical configurations to estimate the effective thermal conductivity of two-phase materials.

Key words: *effective thermal conductivity, concentration, conductivity ratio, unit-cell approach, inclusions, two-phase materials*

Introduction

The importance of two-phase materials such as ceramics, metal foams, emulsion and suspended systems, and granular materials lies in many of the applications in microelectronic chip cooling, spacecraft structure, catalytic reactors, heat recovery process, heat exchangers, heat storage systems, petroleum refineries, nuclear reactors, electronic packaging, and food processing. Many researchers have spent an enormous amount of effort on developing various analytical methods for modeling and calculating two-phase homogeneous materials with imbedded inclusions and surrounding inter phase. Moreover, this problem has importance because of its analogy with the general susceptibility of dispersed media such as dielectric constant, refractive index, magnetic permittivity, electrical conductivity, elastic modulus, and diffusion coefficient. The problem is one of the long standing issues and has been treated in many papers on the basic

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of unit cell approach by considering the primary parameters such as concentration of the dispersed phase (ν), conductivity ratio (α), and secondary parameters (contact resistance, heat transfer through radiation, Knudsen effect, and geometrical configurations). Numerous models were developed to find out the effective thermal conductivity (ETC) of the mixtures, but one of the major limitations of the models is its suitability for specific applications.

Maxwell's work [1] predicting the magnetic permittivity of a dilute suspension of spheres is the earliest reported work in the modeling of transport properties of two-phase media. But one of the limitations of the model is applicable for lower concentration of the dispersed phase. The Maxwell and phase inverted Maxwell [2] models are the minimum and maximum bounds for predicting the thermal conductivity of the two-phase system. These are the most restrictive bounds proposed and every model should incorporate these bounds as a minimum and maximum. The upper and lower limits to the conductivity of two-phase materials based on parallel and series resistances were given by Wiener [3].

Zehner *et al*. [4] proposed a model considering the effect of particle contact as well as the effect of secondary parameters such as thermal radiation, pressure dependence, particle flattening, shape, and size distribution for cylindrical unit cell containing spherical inclusions. An important deficiency in the model is that the deformation of the flux field is taken only as a function of concentration, not as a function of the conductivity ratio. Hsu, *et al*. [5] obtained algebraic expressions for effective thermal conductivities of porous media by applying lumped parameter method, which is based on an electric resistance analogy. Models were developed to describe the effective thermal conductivity of randomly packed granular materials based on the unit cell method, by Crane *et al.* [6]. A review of thermal conductivity of packed beds at no-flow condition was described by the Tsotsas *et al*. [7]. Bruggeman [8] extended Maxwell's result for lower concentration of the dispersed phase to the full range of concentration by assuming the mixture to be quasi-homogeneous. Raghavan *et al.* [9] proposed a unit cell model that agreed exactly with field solutions of Maxwell and provided the basis for a fundamentally correct approach in the modeling of conductivity. Numerical study for effective conductivity based on a model made up of spheres in cubic lattice has been carried out by Krupiczka [10]. Krischer [11] described the unit cube thermal conductivity model. A review of conduction in heterogeneous systems was studied by Meredith *et al.* [12]. The purpose of this work was correcting, modifying and extending the Rayleigh [13] formula for interactions of higher order between particles. Bauer [14] developed an analytical model for the effect of randomly distributed inclusions or pores on the solution of Laplace's heat conduction equation for prediction of thermal conductivity of packed beds. The effective thermal conductivity of packed beds based on field solution approach was carried out by Dietz [15]. A review of various methods for predicting the effective thermal conductivity of composite materials was proposed by Progelhof *et al*. [16]. The thermal conductivity of a saturated porous medium was calculated for a two-layer model representing as electrical resistance in an electrical circuit (Deisser *et al*. [17]). Kunii *et al.* [18] proposed a unit cell model. The electrical conductivity of binary metallic mixtures was investigated by Landauer [19]. Samantray *et al*. [20] proposed a comprehensive conductivity model by considering the primary parameters based on unit cell and field solution approaches. Later, the validity of the model was extended to predict the effective conductivity of various binary metallic mixtures with a high degree of accuracy [21]. Reddy *et al.* [22] developed the collocated parameter model based on the unit cell approach for predicting the effective thermal conductivity of the two-phase materials.

The aim of this paper is to clarify the situations by providing general guidelines for selecting suitable effective thermal conductivity model. In this paper, a collocated parameter model is proposed based on the unit cell approach with parallel isotherms to estimate the thermal conductivity of various inclusions. An effort is made to develop the effective thermal conductivity equations for hexagon and octagon cylinders and compare with experimental results [23-42].

Effective thermal modeling of two-dimensional spatially periodic two-phase medium

The development of collocated-parameter model for estimating the effective thermal conductivity based on the material micro and nano-structure is extremely important for thermal design and analysis of two-phase systems. The electric resistance analogy leads to algebraic expressions for stagnant thermal conductivity of the two-phase materials. The resistance method is referred as the collocated parameter model. The main feature of resistance method is to assume one-dimensional heat conduction in a unit cell. The unit cell is divided into three parallel layers, namely, solids, fluid, or composite layers normal to the temperature gradient. The effective thermal conductivity of two-phase system is determined by considering equivalent electrical resistances of parallel and series in the collocated parameter unit cell model. The thermal conductivity of the composite layer is obtained using the series model.

Octagon cylinder

The ETC of the two-dimensional medium can be estimated by considering an octagon cylinder with cross-section $a \cdot a$ having a connecting bar width of c as shown in fig. 1(a). The stagnant thermal conductivity of the two-dimensional periodic medium is the finite contact between the spheres by connecting plates with c/a denoting the contact parameter. Because of the symmetry of the plates, one fourth of the square cross-section has been considered as a unit cell and is shown in fig. 1(b). The unit cell consists of three rectangular layers normal to the direction of heat flow. The thermal conductivity of the solid and fluid layer is obtained based on a series model. The first rectangular layer is fully occupied by the solid with a dimension of $(l/2)$ $(c/2)$ and other two rectangular layers consists of solid and fluid phases with a di-

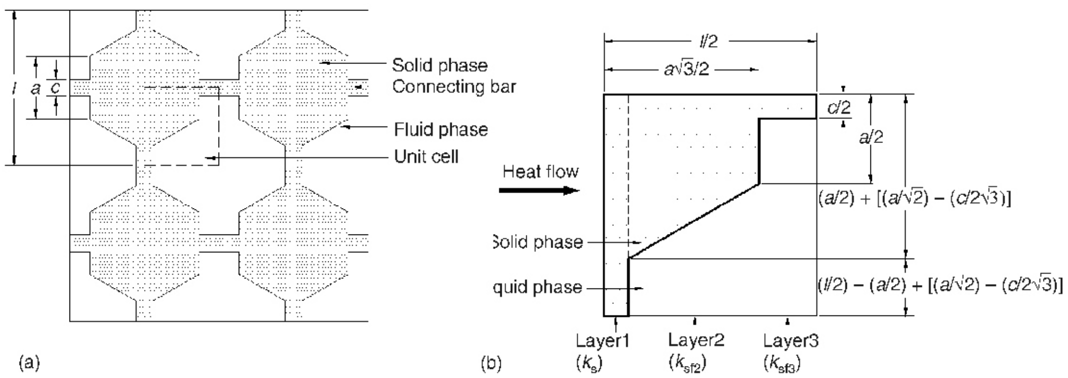

Figure 1. Two-dimensional spatially periodic two-phase system
(a) touching octagon cylinder; (b) unit, cell of octagon cylinder

mension of $(l/2)$ $[(a/2 + a/2^{1/2}) - c/2]$ and $(l/2)$ $[l/2 - (a/2 + a/2^{1/2})]$, respectively. The model is based on the one dimensional heat conduction in the unit cell. The temperature gradient in the three layers is normal to the direction of heat flow. The ETC of two-dimensional octagon cylinder is calculated for parallel isotherm conditions as follows:
total resistance offered by the octagon cylinder in the unit cell is given as:

$$R_{\mathrm{total}} = R_1 + R_{\mathrm{eft2}} + R_{\mathrm{eff3}} \tag{1}$$

where
– the resistance offered by the solid layer I,

$$R_1 \quad \frac{\varepsilon\lambda}{\alpha} \tag{2}$$

– the total resistance offered by the layer II,

$$R_{\mathrm{eff2}} \quad \frac{1}{R_{2\mathrm{s}}} \quad \frac{1}{R_{2\mathrm{sf}}}^{1} \quad \frac{\frac{1}{\alpha}\ \frac{1\quad \alpha}{1\quad \sqrt{2}}\ \frac{2\sqrt{2}}{2\quad \sqrt{2}}\ \frac{2\varepsilon[(1\quad \lambda)\quad \sqrt{2}]}{\frac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}}\{[2(1\quad (\varepsilon\quad \varepsilon\sqrt{2}))]\quad [\varepsilon(1\quad \lambda)\quad \sqrt{2}]\}}}{\frac{1}{\alpha}\ \frac{1\quad \alpha}{1\quad \sqrt{2}}\ \frac{2\sqrt{2}}{2\quad \sqrt{2}}\ \frac{2\varepsilon[(1\quad \lambda)\quad \sqrt{2}]}{\frac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}}\{[2(1\quad (\varepsilon\quad \varepsilon\sqrt{2}))]\quad [\varepsilon(1\quad \lambda)\quad \sqrt{2}]\}}} \tag{3}$$

– the total resistance offered by the layer III,

$$R_{\mathrm{eff3}} \quad \frac{1}{R_{3\mathrm{s}}} \quad \frac{1}{R_{3\mathrm{sf}}}^{1} \quad \frac{[1\quad (\varepsilon\quad \varepsilon\sqrt{2})][\alpha\quad (1\quad \alpha)\varepsilon\lambda]}{\alpha\{(1\quad \varepsilon\lambda)\quad \varepsilon\lambda[\alpha\quad (1\quad \alpha)\varepsilon\lambda]\}} \tag{4}$$

The thermal conductivities of the composite layers can be obtained based on the series model. The thermal conductivity of the composite layer II is given by:

$$\frac{1}{k_{\mathrm{sf2}}} \quad \frac{\dfrac{\varepsilon\quad \frac{1}{2}\quad \frac{1}{\sqrt{2}}\ (2\quad \lambda)}{\frac{1\quad \lambda}{2}\quad \frac{1}{\sqrt{2}}}}{k_{\mathrm{s}}} \quad \frac{1\quad \dfrac{\varepsilon\quad \frac{1}{2}\quad \frac{1}{\sqrt{2}}\ (2\quad \lambda)}{\frac{1\quad \lambda}{2}\quad \frac{1}{\sqrt{2}}}}{k_{\mathrm{f}}} \tag{5}$$

eq. (5) is re-written as:

$$\frac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}} \quad \frac{\alpha\ \frac{1\quad \lambda}{2}\quad \frac{1}{\sqrt{2}}}{\varepsilon\ \frac{1}{2}\quad \frac{1}{\sqrt{2}}\ (2\quad \lambda)\quad \alpha\ \frac{1\quad \lambda}{2}\quad \frac{1}{\sqrt{2}}\quad \varepsilon\alpha\ \frac{1}{2}\quad \frac{1}{\sqrt{2}}\ (2\quad \lambda)} \tag{6}$$

Similarly, the thermal conductivity of the composite layer III is given by:

$$\frac{k_{\mathrm{sf3}}}{k_{\mathrm{f}}} \quad \frac{\alpha}{\alpha\quad (1\quad \alpha)\varepsilon\lambda} \tag{7}$$

where $\alpha = k_s/k_f$, $\varepsilon = a/l$, $\lambda = c/a$, and $\varepsilon\lambda = c/l$.

The solid phase fraction of the unit cell is represented in terms of concentration (v), and is given by:

$$v \quad \frac{\text{Volume of the solid phase}}{\text{Total volume of the unit cell}} \quad \frac{cl \quad (a \quad c)(a \quad a\sqrt{2}) \quad a^2\sqrt{2} \quad a^2 \quad [l \quad (a \quad a\sqrt{2})]c}{l^2} \tag{8}$$

eq. (8) can be written as:

$$v \quad 2\varepsilon^2[(1 \quad \sqrt{2})(1 \quad \lambda)] \quad 2\varepsilon\lambda \tag{9}$$

Substituting eqs. (2-9) in eq. (1), the total resistance of unit cell is given as:

$$R_{\text{total}} \quad \frac{\varepsilon\lambda}{\alpha} \quad \frac{\frac{1}{\alpha} \quad \frac{1 \quad \lambda}{1 \quad \sqrt{2}} \quad \frac{2\sqrt{2}}{2 \quad \sqrt{2}} \quad \frac{2\varepsilon[(1 \quad \lambda) \quad \sqrt{2}]}{\frac{k_{\text{sf2}}}{k_{\text{f}}}\{[2(1 \quad (\varepsilon \quad \varepsilon\sqrt{2}))] \quad [\varepsilon(1 \quad \lambda) \quad \sqrt{2}]\}}}{\frac{1}{\alpha} \quad \frac{1 \quad \lambda}{1 \quad \sqrt{2}} \quad \frac{2\sqrt{2}}{2 \quad \sqrt{2}} \quad \frac{2\varepsilon[(1 \quad \lambda) \quad \sqrt{2}]}{\frac{k_{\text{sf2}}}{k_{\text{f}}}\{[2(1 \quad (\varepsilon \quad \varepsilon\sqrt{2}))] \quad [\varepsilon(1 \quad \lambda) \quad \sqrt{2}]\}}}$$

$$\frac{[1 \quad (\varepsilon \quad \varepsilon\sqrt{2})][\alpha \quad (1 \quad \alpha)\varepsilon\lambda]}{\alpha\{(1 \quad \varepsilon\lambda) \quad \varepsilon\lambda[\alpha \quad (1 \quad \alpha)\varepsilon\lambda]\}} \tag{10}$$

The non-dimensional thermal conductivity of two-dimensional octagon cylinder is given as:

$$K \quad \frac{k_{\text{eff}}}{k_{\text{f}}} \quad \frac{1}{R_{\text{total}}} \tag{11}$$

Hexagon cylinder

The ETC of the two-dimensional medium can be estimated by considering a hexagon cylinder with cross-section $a{\cdot}a$ having a connecting bar width of c as shown in fig. 2(a). The stagnant thermal conductivity of the two-dimensional periodic medium is the finite contact between the spheres by connecting plates with c/a denoting the contact parameter. Because of the symmetry of the plates, one fourth of the square cross-section has been considered as a unit cell and is shown in fig. 2(b). The unit cell consists of three rectangular layers normal to the direction of heat flow. The thermal conductivity of the solid and fluid layer is obtained based on a series model. The first rectangular layer is fully occupied by the solid with a dimension of $(l/2)(c/2)$ and other two rectangular layers consists of solid and fluid phases with a dimension of $(l/2)[(a3^{1/2}/2) - c/2^{1/2})]$ and $(l/2)[(l/2) - a3^{1/2}/2)]$, respectively. The model is based on the one dimensional heat conduction in the unit cell. The temperature gradient in the three layers is normal to the direction of heat flow. The ETC of two-dimensional hexagon cylinder is calculated for parallel isotherm conditions as follows:

– the total resistance offered by the hexagon cylinder in the unit cell is given as:

$$R_{\text{total}} = R_1 + R_{\text{eff2}} + R_{\text{eff3}} \tag{12}$$

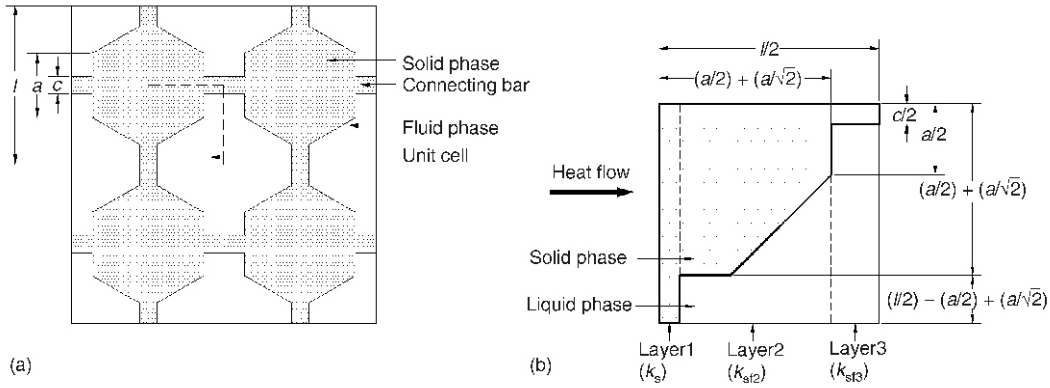

Figure 2. Two-dimensional spatially periodic two-phase system
(a) touching hexagon cylinder; (b) unit cell of hexagon cylinder

where
– the resistance offered by the solid layer I,

$$R_1 \quad \frac{\varepsilon\lambda}{\alpha} \tag{13}$$

– the total resistance offered by the layer II,

$$R_{\mathrm{eff2}} \quad \frac{1}{R_{2\mathrm{s}}} \quad \frac{1}{R_{2\mathrm{sf}}}^{1} \quad \frac{\dfrac{2\sqrt{3}\varepsilon(\sqrt{3} \quad \lambda)}{[(\sqrt{3} \quad \lambda) \quad 2\sqrt{3})]\dfrac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}} \quad 2\sqrt{3}\;1 \quad \varepsilon \quad \varepsilon \quad \dfrac{\varepsilon\lambda}{\sqrt{3}} \quad \varepsilon(\sqrt{3} \quad \lambda)}^{1}}{\dfrac{1}{[(\sqrt{3} \quad \lambda) \quad 2\sqrt{3})]} \quad \dfrac{\varepsilon}{\dfrac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}}\dfrac{1}{\alpha} \quad 2\sqrt{3}\;1 \quad \varepsilon \quad \varepsilon \quad \dfrac{\varepsilon\lambda}{\sqrt{3}} \quad \varepsilon(\sqrt{3} \quad \lambda)}} \tag{14}$$

– the total resistance offered by the layer III,

$$R_{\mathrm{eff2}} \quad \frac{1}{R_{3\mathrm{s}}} \quad \frac{1}{R_{3\mathrm{sf}}}^{1} \quad \frac{\dfrac{1 \quad \varepsilon\sqrt{3}}{\dfrac{k_{\mathrm{sf3}}}{k_{\mathrm{f}}}[\varepsilon\lambda(1 \quad \varepsilon\lambda)}}{\dfrac{1}{\varepsilon\lambda} \quad \dfrac{1}{\dfrac{1}{\alpha}\dfrac{k_{\mathrm{sf3}}}{k_{\mathrm{f}}}(1 \quad \varepsilon\lambda)}} \tag{15}$$

The thermal conductivities of the composite layers can be obtained based on the series model. The thermal conductivity of the composite layer II is given by:

$$\frac{1}{k_{\mathrm{sf2}}} \quad \frac{\varepsilon \quad \dfrac{\varepsilon}{2} \quad \dfrac{\varepsilon\lambda}{2\sqrt{3}}}{k_{\mathrm{s}}} \quad \frac{1 \quad \varepsilon \quad \dfrac{\varepsilon}{2} \quad \dfrac{\varepsilon\lambda}{2\sqrt{3}}}{k_{\mathrm{f}}} \tag{16}$$

eq. (16) is re-written as:

$$\frac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}} \quad \frac{\alpha}{\alpha \quad \varepsilon \quad \dfrac{\varepsilon}{2} \quad \dfrac{\varepsilon\lambda}{2\sqrt{3}} \quad (1 \quad \alpha)} \tag{17}$$

Similarly, the thermal conductivity of the composite layer III is given by:

$$\frac{k_{\mathrm{sf3}}}{k_{\mathrm{f}}} \quad \frac{\alpha}{\alpha \quad (1 \quad \alpha)\varepsilon\lambda} \tag{18}$$

where $\alpha = k_s/k_f$, $\varepsilon = a/l$, $\lambda = c/a$, and $\varepsilon\lambda = c/l$.

The solid phase fraction of the unit cell is represented in terms of concentration (v), and is given by:

$$v \quad \frac{\text{Volume of the solid phase}}{\text{Total volume of the unit cell}} \quad \frac{cl \quad (a\sqrt{3} \quad c) \quad a \quad \dfrac{a}{2} \quad \dfrac{c}{2\sqrt{3}} \qquad c^2}{l^2} \tag{19}$$

eq. (19) can be written as:

$$v \quad \varepsilon^2\lambda^2 \quad \varepsilon^2(\sqrt{3} \quad \lambda) \; 1 \quad \frac{1}{2} \quad \frac{\lambda}{2\sqrt{3}} \qquad \varepsilon\lambda \tag{20}$$

Substituting eqs. [13-20] in eq. [12], the total resistance of unit cell is given as:

$$R_{\mathrm{total}} \quad \frac{\varepsilon\lambda}{\alpha} \quad \frac{\dfrac{2\sqrt{3}\varepsilon(\sqrt{3} \quad \lambda)}{[(\sqrt{3} \quad \lambda) \quad 2\sqrt{3})]\dfrac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}} \quad 2\sqrt{3}\;1 \quad \varepsilon \quad \varepsilon \quad \dfrac{\varepsilon\lambda}{\sqrt{3}} \qquad \varepsilon(\sqrt{3} \quad \lambda)}}{\dfrac{1}{[(\sqrt{3} \quad \lambda) \quad 2\sqrt{3})]} \quad \dfrac{\varepsilon}{\dfrac{k_{\mathrm{sf2}}}{k_{\mathrm{f}}}\dfrac{1}{\alpha} \quad 2\sqrt{3}\;1 \quad \varepsilon \quad \varepsilon \quad \dfrac{\varepsilon\lambda}{\sqrt{3}} \qquad \varepsilon(\sqrt{3} \quad \lambda)}}$$

$$\frac{\dfrac{1 \quad \varepsilon\sqrt{3}}{\dfrac{k_{\mathrm{sf3}}}{k_{\mathrm{f}}}[\varepsilon\lambda(1 \quad \varepsilon\lambda)]}}{\dfrac{1}{\varepsilon\lambda} \quad \dfrac{1}{\dfrac{1}{\alpha}\dfrac{k_{\mathrm{sf3}}}{k_{\mathrm{f}}}(1 \quad \varepsilon\lambda)}} \tag{21}$$

The non-dimensional thermal conductivity of two-dimensional hexagon cylinder is given as:

$$K \quad \frac{k_{\mathrm{eff}}}{k_{\mathrm{f}}} \quad \frac{1}{R_{\mathrm{total}}} \tag{22}$$

Results and discussions

The effective thermal conductivity of a two-phase system mainly depends on concentration, conductivity ratio, size, shape, and thermal contact between solid-solid and solid-fluid interface. The effect of concentration (ν) on the non-dimensional thermal conductivity of two-dimensional (octagon and hexagon cylinder) geometry's have been studied and are shown in figs. 3 and 4. The present model with various inclusions lies between parallel and series lines for conductivity ratio $\alpha = 20$ and contact ratio $\lambda = 0\text{-}0.2$. For octagon cylinder, the present correlation is applicable for concentration varying from 0 to 0.7, for further increment in the concentration; the non-dimensional thermal conductivity is increasing beyond the upper bound. Similarly for hexagon cylinder the present equation is applicable, if the concentration varying from 0 to 0.8. Both the models are not applicable for concentration beyond 0.8, because the limitations in the shape of the models.

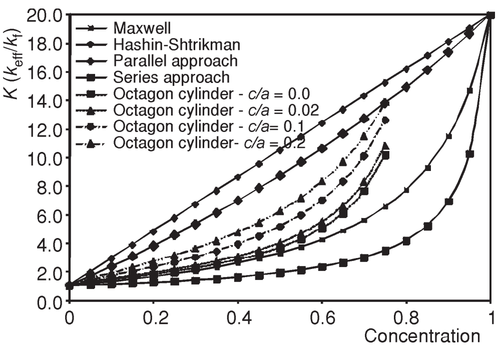

Figure 3. Variation of non-dimensional thermal conductivity with concentration of 2-D spatially periodic two-phase systems with octagon cylinder for α = 20

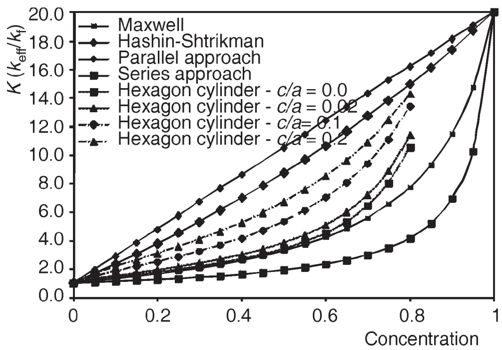

Figure 4. Variation of non-dimensional thermal conductivity with concentration of 2-D spatially periodic two-phase systems with hexagon cylinder for α = 20

The variation of non-dimensional thermal conductivity with conductivity ratio (α) for low ($\nu = 0.3$) and high ($\nu = 0.8$) concentration two-dimensional spatially periodic medium with various contact ratios are, respectively, shown in figs. 5 and 6. The predicted non-dimensional

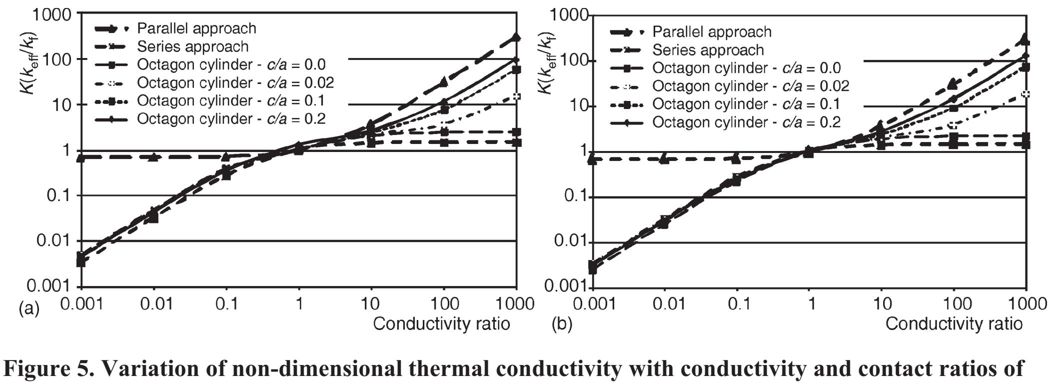

Figure 5. Variation of non-dimensional thermal conductivity with conductivity and contact ratios of various inclusions for lower concentration (n = 0.3) two-phase systems

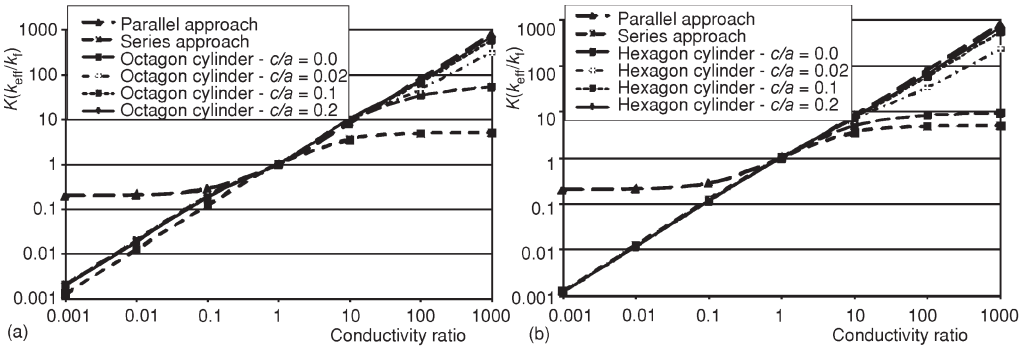

Figure 6. Variation of non-dimensional thermal conductivity with conductivity and contact ratios of various inclusions for higher concentration (n = 0.8) two-phase systems

thermal conductivity increases with the conductivity ratio and contact ratios. For lower ($\nu = 0.3$) concentrations, the deviation between all models are almost same. For higher concentration ($\nu = 0.8$) and higher conductivity ratios, the deviation is more within the models. For $\lambda = 0.1$, the variation in the non-dimensional thermal conductivity has been considerable. It can be seen that the contact ratio (λ) is the deterministic parameter when the conductivity ratio (α) is high where as concentration is deterministic parameter when α is approaching to one. Similarly, for lower conductivity ratios ($\alpha < 1$), the non-dimensional thermal conductivity is insensitive to the contact ratios, but it is sensitive to the higher conductivity ratios ($\alpha > 1$). From the iso-conductance point, $\alpha = 1$, the non-dimensional thermal conductivity approaches to unity for all the models with the same slope. The present model shows a good trend for the concentrations 0.3 and 0.8. For low values of α, the thermal conductivity estimations of all the models are comparable, but they deviate when the conductivity ratio approaches to 1000.

The predicted theoretical expression values for various inclusions has been compared on a large number of samples cited in the literature and found that the values predicted are quite close to the experimental results. A comparison of present models with experimental data for various concentrations has been made for various two-phase systems such as porous-granular materials, suspension systems, emulsion systems, and solid-solid mixtures. For porous granular materials (ν = 0.2 to 0.866 and α = 1.812 to 398.7), the octagon cylinder has good agreement with the experimental data. The range of accuracy appears quite good in consideration with the variety of sources of data selected and the wide range of shapes included. It is observed that the octagon cylinder has an average deviation of 10.25% from experimental data as against 14.12% hexagon cylinder respectively (tab. 1). The non-dimensional thermal conductivity of suspension systems (solid/liquid phase) is shown in (tab. 2). All the inclusions show a good agreement with experimental values within the range of 20% maximum deviation, because all the experimental values are low concentration with higher conductivity ratio. The variation of non-dimensional thermal conductivity of emulsion systems with comparison of experimental values is shown in (tab. 3). The experimental data considered in emulsion systems have low concentration and low conductivity ratio. Therefore all the inclusions are estimated in the order of accuracy. It is observed that, the octagon cylinder model has an average deviation 12.19% from experimental data as against 14.32% of hexagon cylinder. The non-dimensional thermal conductivity prediction for solid-solid mixtures is shown in (tab. 4). It is observed that, the hexagon cylinder has an average deviation of 9.63% from experimental data as against 10.47% of

Table 1. Non-dimensional thermal conductivity of porous granular materials

ν	α	K_{exp}	λ	K_{hex}	Deviation [%]	K_{oct}	Deviation [%]	System/Source
0.2	45.79	1.708	0.01	2.055	20.33	2.032	18.99	Glass sphere/air [23]
0.41	56.96	4.06	0.02	4.241	4.46	4.078	0.44	Wassau and/helium [24]
0.456	127.47	7.34	0.03	8.674	18.18	7.681	4.65	Miami silf foam/air [25]
0.47	95.285	5.714	0.02	6.324	10.68	5.919	3.59	Zircona powder/air [26]
0.485	64.91	5.596	0.02	5.572	0.43	5.374	3.96	Wassau sand/n-heptane [24]
0.495	61.91	5.9614	0.03	6.492	8.89	6.120	2.66	Stainless steel/eth. alcohol [27]
0.507	128.6	8.879	0.03	10.377	16.87	9.280	4.52	Air/calcite [28]
0.511	138.1	6.328	0.01	6.563	3.71	6.345	0.27	He/steel [27]
0.535	19.7	4.051	0.01	3.951	2.46	4.225	4.31	Etoh/calcite [29]
0.547	6.1	2.515	0.01	2.710	7.76	3.211	27.66	H_2O/silica [10]
0.552	127.47	9.6	0.02	9.802	2.10	9.161	4.58	Miami silt foam/air [25]
0.56	398.7	15.336	0.01	13.947	9.06	12.669	17.39	Air/quartz [26]
0.561	17.9	3.963	0.01	4.127	4.15	4.477	12.96	H_2O/silica [27]
0.563	16	5.244	0.8	5.193	0.97	5.292	0.91	Air/coal [26]
0.563	2.2	1.524	0.9	1.764	15.73	1.995	30.91	H_2/coal [26]
0.563	16	5.23	0.07	5.045	3.53	5.176	1.04	Air/coal [10]
0.563	2.17	1.53	0.9	1.746	14.15	1.977	29.23	H_2/coal [10]
0.569	21.18	4.341	0.01	4.440	2.28	4.789	10.32	Silica sphere/water [10]
0.569	17.868	4.494	0.01	4.218	6.14	4.590	2.14	Water/silica [10]
0.569	7.648	2.859	0.01	3.113	8.88	3.630	26.95	IC8/glass [10]
0.57	7.368	2.8194	0.01	3.071	8.91	3.598	27.60	Glass sphere/iso-octane [10]
0.572	2.03	1.5832	0.01	1.579	0.24	2.224	40.45	Glycerin/glass [10]
0.575	104.37	5.724	0.01	7.389	29.08	7.422	29.67	H_2/SiC [30]
0575	104.4	5.7	0.01	7.390	29.64	7.423	30.23	H_2/SiC [10]
0.576	290.5	9.876	0.01	12.199	23.52	11.413	15.57	Air/SiO [27]
0.577	3.023	1.891	0.01	2.001	5.82	2.665	40.95	Etoh/glass [27]
0.58	66.7	7.66	0.02	7.761	1.32	7.679	0.25	Zircona power/air [31]
058	7.824	2.862	0.01	3.216	12.38	3.762	31.45	He/glass [27]

ν	α	K_{exp}	λ	K_{hex}	Deviation [%]	K_{oct}	Deviation [%]	System/Source
0.58	2.06	1.572	0.9	1.696	7.88	1.942	23.53	Glycerol/glass [27]
0.58	1.812	1.384	0.9	1.546	11.68	1.782	28.75	H_2O/glass [27]
0.6	57.617	7.387	0.01	7.816	5.80	7.877	6.63	Lead/water [26]
0.6	37.62	6.206	0.01	5.806	6.45	6.191	0.24	Glass beds/air [32]
0.6	43.46	6.769	0.02	6.912	2.11	7.093	4.78	Glass/air [33]
0.6	124.2	7.213	0.01	8.746	21.25	8.788	21.84	Glycerin/lead [34]
0.6	161.4	8.86	0.01	9.839	11.05	9.715	9.65	Air/sand [35]
0.603	191.1	8.025	0.01	10.833	34.99	10.585	31.90	Etoh/lead [27]
0.612	253.3	12.775	0.01	13.142	2.87	12.642	1.04	Glycerin/Cu [27]
0.612	253.3	12.8	0.01	13.142	2.67	12.642	1.23	Cu/glycerol solution [10]
0.62	233.65	14.55	0.01	13.013	10.56	12.652	13.04	Lead shots/helium [24]
0.62	191.88	13.569	0.01	11.657	13.79	11.536	14.99	Lead shots/hydrogen [24]
0.62	54.77	8.618	0.02	8.247	4.30	8.422	2.27	Lead shots/water [24]
0.639	7.864	3.398	0.01	3.674	8.12	4.451	30.98	Microbeads/soltrol [36]
0.64	66.7	9.36	0.02	9.834	5.06	10.031	7.17	Zircona powder/air [31]
0.64	56.96	9	0.02	9.106	1.18	9.385	4.28	Ottawa sand/helium [24]
0.65	42.89	7.857	0.01	7.320	6.84	7.988	1.66	Glass beds/air [37]
0.65	8.578	3.571	0.01	3.935	10.19	4.756	33.18	Glass beads/benzene [37]
0.65	40.23	7.423	0.01	7.165	3.47	7.838	5.60	Micro beads/air [36]
0.655	9.4	5.7	0.1	4.885	14.82	5.534	2.90	Air/Cr/Al catalyst [10]
0.655	11.6	5.8	0.05	5.097	12.11	5.792	0.14	Air/Cr/Al catalyst [10]
0.676	8.069	3.759	0.01	4.068	8.21	5.053	34.42	Quartz sand/water [37]
0.7	66.7	12.13	0.01	10.859	10.48	12.189	0.49	Zircona powder/air [31]
0.7	6.8	4.2	0.01	3.914	6.81	5.076	20.86	Air/Pt/Al_2O_3/catalyst [10]
0.71	7.8	4.45	0.01	4.351	2.23	5.600	25.84	Air/Co/Mo catalyst [10]
0.725	8.1	6.6	0.2	5.431	17.71	6.589	0.17	Behmite [30]
0.74	45.79	9.458	0.01	11.686	23.56	14.064	48.70	Glass sphere/air [23]
0.77	14.5	9.8	0.01	7.697	21.46	10.115	3.22	Air/Ni/W catalyst [10]
0.866	8.1	8.3	0.9	6.223	25.03	8.459	1.91	Powder [30]
Average deviation [%]				14.12		10.25		

Table 2. Non-dimensional thermal conductivity of suspension systems

v	α	K_{exp}	λ	K_{hex}	Deviation [%]	K_{oct}	Deviation [%]	System/Source
0.1	37.08	1.286		1.547	20.33	1.536	19.48	Selenium/polypropylene glycol [37]
0.2	37.08	1.564		1.992	27.35	1.982	26.70	Selenium/polypropylene glycol [37]
0.24	241	2.887	0.01	12.690	24.08	3.582	12.69	Graphite/water [38]
0.3	37.08	2.25		2.523	12.12	2.525	12.22	Selenium/polypropylene glycol [37]
0.4	37.08	3.014		3.220	6.81	3.258	8.10	Selenium/polypropylene glycol [37]
Average deviation [%]				18.14		15.84		

Table 3. Non-dimensional thermal conductivity of emulsion systems

v	α	K_{exp}	λ	K_{hex}	Deviation [%]	K_{oct}	Deviation [%]	System/Source
0.1	3.313	1.445	0.1	1.214	16.02	1.283	11.18	Cellosize/flexol plasticier [39]
0.1	3.02	1.1	0.9	1.295	17.76	1.212	10.19	Cellosize/flexol plasticizer [39]
0.1	3.66	1.213	0.9	1.366	12.58	1.253	3.30	Cellosize/polypropylene glycol [39]
0.1	3.72	1.168	0.9	1.372	17.46	1.257	7.60	Cellosize/polypropylene glycol [39]
0.2	3.318	1.4615	0.9	1.603	9.70	1.486	1.65	Water/petroleum solvent [40]
0.2	3.826	1.366	0.9	1.708	25.02	1.555	13.81	Water/petroleum solvent [40]
0.2	4.087	1.57	0.1	1.478	5.84	1.577	0.41	Water/mineral oli [40]
0.3	3.313	1.415	0.9	1.844	30.29	1.762	24.55	Cellosize/flexol plasticizer [39]
0.3	3.021	1.347	0.9	1.754	30.23	1.696	25.87	Cellosize/flexol plasticizer [39]
0.3	3.66	1.56	0.1	1.640	5.11	1.821	16.73	Cellosize/polypropylene glycol [39]
0.4	3.52	1.798	0.1	1.831	1.84	2.118	17.80	Water/petroleum solvent [40]
0.4	4.1	1.959	0.1	1.960	0.03	2.218	13.23	Water/mineral oil [40]
Average deviation [%]				14.32		12.19		

Table 4. Non-dimensional thermal conductivity of solid-solid-mixtures

ν	α	K_{exp}	λ	K_{hex}	Deviation [%]	K_{oct}	Deviation [%]	System/Source
0.05	90.14	1.203	0.01	1.449	20.41	1.413	17.46	Lead powder/silicon rubber [41]
0.05	21.63	1.125		1.275	13.33	1.279	13.71	Bismuth powder/silicon rubber [41]
0.1	9.517	1.307		1.346	3.01	1.384	5.91	Silica powder/dimethyl [42]
0.15	132.3	2.16		2.206	2.13	2.093	3.09	Zinc oxide/methyl vinyl [42]
0.15	9.626	1.44		1.494	3.75	1.543	7.17	Silica powder/methyl vinyl [42]
0.16	90.14	1.726		2.083	20.66	2.011	16.51	Lead powder/silicon rubber [41]
0.16	21.63	1.536		1.691	10.12	1.703	10.87	Bismuth powder/silicon rubber [41]
0.24	21.63	1.906		2.019	5.95	2.044	7.25	Bismuth powder/silicon rubber [41]
0.25	9.626	1.684		1.807	7.30	1.890	12.25	Silica powder/methyl vinyl [42]
Average deviation [%]				9.63		10.47		

octagon cylinder, respectively. So, the present model can be used to estimate the effective thermal conductivity of wide range of two-phase systems.

Conclusions

The collocated parameter models are developed with the effect of various inclusions for estimating the effective thermal conductivity of the two-phase materials. The effects of concentration, conductivity and contact ratios on the non-dimensional thermal conductivity of various inclusions have been investigated. The present models are also compared with experimental data's for various concentration and conductivity ratio. The present models are predicting effective thermal conductivity with a maximum deviation of 20% from the experimental data's for the various two-phase systems. The present models can be extensively used for predicting the effective thermal conductivity of two-phase materials used in the engineering applications.

Nomenclature

- a – length of the octagon and hexagon cylinders
- c – width of the connecting plate in the octagon and hexagon cylinders
- K – non-dimensional thermal conductivity of the two-phase materials ($k_{\text{eff}}/k_{\text{f}}$)
- k_{eff} – effective thermal conductivity of two-phase materials, [$\text{Wm}^{-1}\text{K}^{-1}$]
- k_{f} – fluid or continuous thermal conductivity, [$\text{Wm}^{-1}\text{K}^{-1}$]
- k_{s} – solid or dispersed thermal conductivity, [$\text{Wm}^{-1}\text{K}^{-1}$]
- k_{sf} – equivalent thermal conductivity of a composite layer, [$\text{Wm}^{-1}\text{K}^{-1}$]
- l – length of the unit cell, [m]
- R – thermal resistance, ($\text{m}^2\text{K}^{-1}\text{W}^{-1}$]

Greek letters

α – conductivity ratio (k_s/k_f)
ε – length ratio (*a*/*l*)
λ – contact ratio (*c*/*a*)
ν – concentration

Subscripts

eff – effective
exp – experimental
hex – hexagon
oct – octogon

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