ELASTIC-PLASTIC TRANSITION STRESSES IN A THIN ROTATING DISC WITH RIGID INCLUSION BY INFINITESIMAL DEFORMATION UNDER STEADY-STATE TEMPERATURE

by

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Stresses for the elastic-plastic transition and fully plastic state have been derived for a thin rotating disc with rigid shaft at different temperatures and results have been discussed and depicted graphically. It has been observed that at room temperature rotating disc made of compressible material and of smaller radii ratio yields at the internal surface at a higher angular speed as compared to rotating disc made of incompressible material. With the introduction of thermal effect rotating disc yields at the outer surface at a lesser angular speed as compared to rotating disc at room temperature. The circumferential stress is maximum at the outer surface of the rotating disc with further increases with the increase in temperature. It means that angular speed of the rotating disc is less than that of the temperature-loaded disc in the fully plastic case.

Key words: *elastic, plastic, stresses, rotating disc, inclusion, temperature, displacement, angular velocity*

Introduction

Rotating disc form an essential part of the design of rotating machinery, namely rotors turbines, compressors, fly wheel, and computer disc drives, *etc*. The use of rotating disc in machine and structural applications has generated considerable interest in many problems in domain of solid mechanics. Solution for thin isotropic discs can be found in most of the standard elasticity and plasticity text books [1-4]. Parmaksigoglu, *et al.* [5] found the "Plastic stress distribution in a rotating disc with rigid inclusion under a radial temperature gradient under the assumptions of Tresca's yield condition, its associated flow rule and linear strain hardening. To obtain the stress distribution, they matched the plastic stresses at the same radius $r = z$ of the disc. Seth's transition theory [6] does not acquire any assumptions like an yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deforming field and has been successfully applied to a large number of the problems [7-13]. Seth [7] has defined the generalized principal strain measure as:

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$$e_{ii} = \int_0^{e_{ii}^A} [1 - 2e_{ii}^A]^{(n/2)-1} \mathrm{d}e_{ii}^A = \frac{1}{n}[1 - (1 - 2e_{ii}^A)^{n/2}], \quad (i = 1, 2, 3) \tag{1}$$

where n is the measure.

In this paper, we investigate the problem of elastic-plastic transition in a thin rotating disc with inclusion under steady-state temperature, by using Seth's transition theory. Results have been discussed numerically and depicted graphically.

Governing equations

Consider a thin disc of constant density with central bore of radius a and external radius b. The annular disc is mounted on a shaft. Let a uniform temperature θ_0 be applied on the central bore of radius a of the disc. The disc is rotating with angular velocity ω about an axis perpendicular to its plane and passing through the centre as shown in fig. 1.The thickness of disc is assumed to be constant and is taken to be sufficiently small so that the disc is effectively in a state of plane stress that is, the axial stress T_{zz} is zero. The displacement components in cylindrical polar co-ordinate are given by [7]:

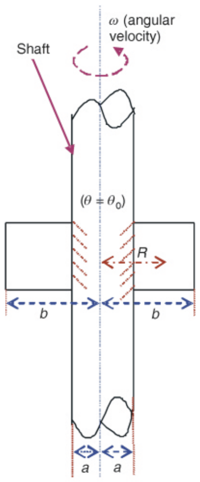

Figure 1. Geometry of rotating disc

$$u = r(1 - \beta); \; v = 0; \; w = dz, \tag{2}$$

where β is function of $r = (x^2 + y^2)^{1/2}$ only and d is a constant.

The strain components for infinitesimal deformation are given by [7, 15]:

$$\begin{aligned} e_{rr}^A &= \frac{\partial u}{\partial r} = [1 - (r\beta' + \beta)] \\ e_{\theta\theta}^A &= \frac{u}{r} = 1 - \beta \\ e_{zz}^A &= \frac{\partial w}{\partial z} = d \\ e_{r\theta}^A &= e_{\theta z}^A = e_{z2}^A = 0 \end{aligned} \tag{3}$$

Using eq. (3) in eq. (1), the generalized components of strain are:

$$\begin{aligned} e_{rr} &= \frac{1}{n}\{1 - [2(r\beta' + \beta) - 1]^{n/2}\} \\ e_{\theta\theta} &= \frac{1}{n}[1 - (2\beta - 1)^{n/2}] \\ e_{zz} &= \frac{1}{n}[1 - (1 - 2d)^{n/2}] \\ e_{r\theta} &= e_{r\theta} = e_{zr} = 0 \end{aligned} \tag{4}$$

where $\beta' = \mathrm{d}\beta/\mathrm{d}r$.

The stress-strain relations for isotropic media is given by [16]:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\theta\delta_{ij}, \quad (i, j = 1, 2, 3) \tag{5}$$

where $I_1 = e_{kk}\,(k = 1, 2, 3)$ and $\xi = \alpha(3\lambda + 2\mu)$; α being the coefficient of thermal expansion and θ is a temperature. Further θ has to satisfy heat equation which gives [16]:

$$\nabla^2\theta = 0 \tag{6}$$

Equation (5) for this problem become:

$$\begin{aligned} &T_{rr} \quad \frac{2\lambda\mu}{\lambda \quad 2\mu}(e_{rr} \quad e_{\theta\theta}) \quad 2\mu e_{rr} \quad \frac{2\mu\xi\theta}{\lambda \quad 2\mu} \\ &T_{\theta\theta} \quad \frac{2\lambda\mu}{\lambda \quad 2\mu}(e_{rr} \quad e_{\theta\theta}) \quad 2\mu e_{\theta\theta} \quad \frac{2\mu\xi\theta}{\lambda \quad 2\mu} \\ &T_{zz} \quad T_{zr} \quad T_{r\theta} \quad T_{\theta z} \quad 0 \end{aligned} \tag{7}$$

From eq. (5), strain components in terms of stresses are obtained as:

$$\begin{aligned} &e_{rr} \quad \frac{1}{E}(T_{rr} \quad \nu T_{\theta\theta}) \quad \alpha\theta \\ &e_{\theta\theta} \quad \frac{1}{E}(T_{\theta\theta} \quad \nu T_{rr}) \quad \alpha\theta \\ &e_{zz} \quad \frac{\nu}{E}(T_{rr} \quad T_{\theta\theta}) \quad \alpha\theta \\ &e_{rr} \quad e_{\theta z} \quad e_{zr} \quad 0 \end{aligned} \tag{8}$$

where $E = \mu(3\lambda + 2\mu)/(\lambda + \mu]$ and $\nu = \lambda/2(\lambda + \mu)$.

Substituting eq. (4) in eq. (7), the stresses are obtained as:

$$\begin{aligned} &T_{rr} \quad \frac{2\mu}{n} \quad 3 \quad 2C \quad [2\beta(P \quad 1) \quad 1]^{n/2}(2 \quad C) \quad (2\beta \quad 1)^{n/2}(1 \quad C) \quad \frac{nC\xi\theta}{2\mu} \\ &T_{\theta\theta} \quad \frac{2\mu}{n} \quad 3 \quad 2C \quad [2\beta(P \quad 1) \quad 1]^{n/2}(1 \quad C) \quad (2\beta \quad 1)^{n/2}(2 \quad C) \quad \frac{nC\xi\theta}{2\mu} \\ &T_{zz} \quad T_{zr} \quad T_{r\theta} \quad T_{\theta z} \quad 0 \end{aligned} \tag{9}$$

where $r\beta' = \beta P$ and $C = 2\mu/(\lambda + 2\mu)$.

The equations of equilibrium are all satisfied except:

$$\frac{\mathrm{d}}{\mathrm{d}r}(rT_{rr}) \quad T_{\theta\theta} \quad \rho\omega^2 r^2 \quad 0 \tag{10}$$

The temperature field satisfying eq. (6) and:

$$\begin{aligned} &\theta = \theta_0 \text{ at } r = a, \\ &\theta = 0 \text{ at } r = b. \end{aligned} \tag{11}$$

where θ_0 is constant, is given by:

$$\theta \quad \frac{\theta_0 \log \frac{r}{b}}{\log \frac{a}{b}} \tag{12}$$

Using eqs. (9) and (12) in eq. (10), we get a non-linear differential equation in β as:

$$\begin{aligned} &(2 \quad C)n\beta^2 P[2\beta(P \quad 1) \quad 1]^{(n/2) \quad 1}\frac{\mathrm{d}P}{\mathrm{d}\beta} \\ &\frac{n\rho\omega^2 r^2}{2\mu} \quad [2\beta(P \quad 1) \quad 1]^{n/2} \quad 1 \quad \frac{n\beta P(P \quad 1)(2 \quad C)}{2\beta(P \quad 1) \quad 1} \\ &(2\beta \quad 1)^{n/2} \quad 1 \quad \frac{n\beta P(1 \quad C)}{2\beta \quad 1} \quad \frac{nC\xi\overline{\theta_0}}{2\mu} \end{aligned} \tag{13}$$

where $\overline{\theta_0} \quad \theta_0/\log(a/b)$.

Transition points of β in eq. (13) are $P \to 0$ and $P \to \pm\infty$. $P \to 0$ gives nothing of importance. The boundary conditions are:

$$u = 0 \quad \text{at} \quad r = a \quad \text{and} \quad T_{rr} = 0 \quad \text{at} \quad r = b. \tag{14}$$

Solution through the principal stress

It has been shown [7-15] that the asymptotic solution through the principal stress lead from elastic state to plastic state at the transition point $P \to \pm\infty$. We define the transition function R as:

$$R = \frac{n}{2\mu}(T_{\theta\theta} - C\xi\theta) = 3 - 2C - [2\beta(P + 1) + 1]^{n/2}(1 - C) - (2\beta - 1)^{n/2}(2 - C) \tag{15}$$

Taking the logarithmic differentiation of eq. (15) with respect to ρ and using eq. (13), one gets:

$$\frac{\mathrm{d}(\log R)}{\mathrm{d}r} = -\frac{\dfrac{1-C}{2-C}\left\{\dfrac{n\rho\omega^2 r^2}{2\mu\beta^n}\left\{[2\beta(P+1)]^{n/2} - (2\beta-1)^{n/2}\right\} + n\beta P(1-C)(2\beta-1)^{n/2} + \dfrac{nC\xi\overline{\theta_0}}{2\mu\beta^n}\right\} - n\beta P(2-C)(2\beta-1)^{n/2}}{r\{3 - 2C - [2\beta(P+1)+1]^{n/2}(1-C) - (2\beta-1)^{n/2}(2-C)\}} \tag{16}$$

Taking the asymptotic value of eq. (16) as, $P \to \pm\infty$, one gets:

$$\frac{\mathrm{d}(\log R)}{\mathrm{d}r} = -\frac{1}{(2-C)r} \tag{17}$$

Integrating eq. (17), one gets:

$$R = A_1 r^{-1/(2-C)} \tag{18}$$

where A_1 is a constant of integration.

From eq. (15) and (18), we have:

$$T_{\theta\theta} = \frac{2\mu}{n} A_1 r^{-1/(2-C)} + \frac{C\xi\theta \log\dfrac{r}{b}}{\log\dfrac{a}{b}} \tag{19}$$

Substituting (19) in eq. (10) and integrating, one gets:

$$T_{rr} = \frac{2\mu(2-C)}{n(1-C)} A_1 r^{-1/(2-C)} + \frac{C\xi\theta_0 \log\dfrac{r}{b}}{\log\dfrac{a}{b}} - \frac{C\xi\theta_0}{\log\dfrac{a}{b}} - \frac{\rho\omega^2 r^2}{3} + \frac{B_1}{r} \tag{20}$$

where B_1 is a constant of integration, and $C\xi = 2\mu\alpha(3 - 2C)$.

From eq. (8), one gets:

$$\begin{aligned} e_{rr} \quad & \frac{\partial u}{\partial r} \quad \frac{1}{E}(T_{rr} \quad \nu T_{\theta\theta}) \quad \alpha\theta \quad \frac{1}{E} \; T_{rr} \quad \frac{1 \quad C}{2 \quad C} T_{\theta\theta} \quad \alpha\theta \\ e_{\theta\theta} \quad & \frac{u}{r} \quad \frac{1}{E}(T_{\theta\theta} \quad \nu T_{rr}) \quad \alpha\theta \quad \frac{1}{E} \; T_{\theta\theta} \quad \frac{1 \quad C}{2 \quad C} T_{rr} \quad \alpha\theta \\ e_{zz} \quad & \frac{\nu}{E}(T_{rr} \quad T_{\theta\theta}) \quad \alpha\theta \quad \frac{1 \quad C}{2 \quad C} \frac{1}{E}(T_{rr} \quad T_{\theta\theta}) \quad \alpha\theta \end{aligned} \tag{21}$$

where $\nu = (1 - C)/(2 - C)\backslash$ is the Poisson's ratio, and $E = 2\mu(3 - 2C)/(2 - C)$ is the Young's modulus.

Substituting eqs. (19) and (20) in equation (21), one gets:

$$\frac{\partial u}{\partial r} \quad \frac{1}{E} \; \frac{2\mu}{n} A_1 r^{\;\; 1/(2 \quad C)} \frac{3 \quad 2C}{(1 \quad C)(2 \quad C)} \quad \frac{\alpha E\theta_0 (2 \quad C)}{\log \frac{a}{b}} \quad \frac{\rho\omega^2 r^2}{3} \quad \frac{B_1}{r} \tag{22}$$

$$\frac{u}{r} \quad \frac{1 \quad C}{E(2 \quad C)} \; \frac{\rho\omega^2 r^2}{3} \quad \frac{\alpha E\theta_0 (2 \quad C)}{\log \frac{a}{b}} \quad \frac{B_1}{r} \tag{23}$$

Integrating eq. (22) with respect to r one gets:

$$u \quad \frac{1}{E} \; \frac{2\mu}{n} A_1 r^{\frac{1 \quad C}{2 \quad C}} \frac{3 \quad 2C}{(1 \quad C)^2} \quad \frac{\alpha E\theta_0 (2 \quad C) r}{\log \frac{a}{b}} \quad \frac{\rho\omega^2 r^3}{9} \quad B_1 \log r \quad D \tag{24}$$

where D is a constant of integration.

Comparing eqs. (23) and (24), one gets:

$$\begin{gathered} \frac{2\mu}{n} A_1 r^{\frac{1 \quad C}{2 \quad C}} \frac{3 \quad 2C}{(1 \quad C)^2} \\ \frac{\rho\omega^2 r^2}{9} \frac{5 \quad 4C}{2 \quad C} \quad \frac{\alpha E\theta_0 r (3 \quad 2C)}{\log \frac{a}{b}} \quad B_1 \frac{1 \quad C \quad (2 \quad C)\log r}{2 \quad C} \quad DE \end{gathered} \tag{25}$$

and

$$u \quad \frac{1 \quad C}{E(2 \quad C)} \; \frac{\rho\omega^2 r^3}{3} \quad \frac{\alpha E\theta_0 (2 \quad C) r}{\log \frac{a}{b}} \quad B_1 \tag{26}$$

Using boundary conditions (14) in eq. (26), one gets:

$$B_1 \quad \frac{\rho\omega^2 a^3}{3} \quad \frac{\alpha E\theta_0 a (2 \quad C)}{\log \frac{a}{b}} \tag{27}$$

Putting eq. (25) in eq. (20) and using boundary condition (14) and eq. (27), one gets:

$$\frac{\rho\omega^2 b^3}{9}\frac{5 \quad 4C}{2 \quad C} \quad \frac{\alpha E\theta_0 b(3 \quad 2C)}{\log\frac{a}{b}}$$

$$D \quad \frac{1}{E} \qquad \frac{\rho\omega^2 a^3}{3} \quad \frac{\alpha E\theta_0 a(2 \quad C)}{\log\frac{a}{b}} \quad \frac{1 \quad C}{2 \quad C} \quad \log b \tag{28}$$

$$\frac{3 \quad 2C}{(1 \quad C)(2 \quad C)} \quad \frac{\rho\omega^2(a^3 \quad b^3)}{3} \quad \frac{\alpha E\theta_0(2 \quad C)(b \quad a)}{\log\frac{a}{b}}$$

Using eqs. (25), (27), and (28) in eqs. (19) and (20), respectively, we get the stresses and displacement as:

$$T_{\theta\theta} \quad \frac{\rho\omega^2}{3r} \quad \frac{1}{3}\frac{5 \quad 4C}{2 \quad C}\frac{(1 \quad C)^2(r^3 \quad b^3)}{3 \quad 2C} \quad a^3\log\frac{r}{b}\frac{(1 \quad C)^2}{3 \quad 2C} \quad \frac{1 \quad C}{2 \quad C}(b^3 \quad a^3)$$

$$\frac{\alpha E\theta_0}{\log\frac{a}{b}} \quad \frac{(1 \quad C)^2(r \quad b)}{r} \quad \frac{a}{r}\log\frac{r}{b}\frac{(2 \quad C)(1 \quad C)^2}{3 \quad 2C} \quad \frac{(b \quad a)(1 \quad C)}{r} \quad (2 \quad C)\log\frac{r}{b} \tag{29}$$

$$T_{rr} \quad \frac{\rho\omega^2}{3r} \quad \frac{1}{3}\frac{5 \quad 4C}{3 \quad 2C}(1 \quad C)(r^3 \quad b^3) \quad a^3\log\frac{r}{b}\frac{(1 \quad C)(2 \quad C)}{3 \quad 2C} \quad b^3 \quad r^3$$

$$\frac{\alpha E\theta_0(2 \quad C)}{\log\frac{a}{b}} \quad C \quad \frac{b}{r} \quad 1 \quad \log\frac{r}{b} \quad \frac{a}{3r(2 \quad C)}\log\frac{r}{b} \tag{30}$$

and

$$u \quad \frac{1 \quad C}{2 \quad C} \quad \frac{\rho\omega^2}{3}(r^3 \quad a^3) \quad \frac{\alpha E\theta_0(2 \quad C)}{\log\frac{a}{b}}(r \quad a) \tag{31}$$

Initial yielding

From eq. (29), it is seen that $_{\theta\theta}$ is maximum at the external surface (that is at $r = b$), therefore yielding will take place at the external surface of the disc and eq. (29) gives:

$$|T_{\theta\theta}|_{r \quad b} \quad \left| \frac{\rho\omega^2(b^3 \quad a^3)}{3b}\frac{1 \quad C}{2 \quad C} \quad \alpha E\theta_0 \frac{(1 \quad C)(b \quad a)}{b\log\frac{b}{a}} \right| \quad Y$$

and the angular velocity necessary for initial yielding is given by:

$$\Omega_1^2 \quad \frac{\rho\omega_i^2 b^2}{Y} \quad \frac{3(2 \quad C)}{1 \quad \frac{a^3}{b^3} \quad (1 \quad C)} \quad 3(2 \quad C)\frac{\alpha E\theta_0}{Y} \quad \frac{1 \quad \frac{a}{b}}{1 \quad \frac{a^3}{b^3} \quad \log\frac{b}{a}} \tag{32}$$

and $\omega_i = (\Omega_i/b)(Y/\rho)^{1/2}$.

Fully-plastic state

The angular velocity ω_f for which the disc become fully-plastic ($C \quad 0$) at $r = a$ is given by eq. (29) as:

$$\left|T_{\theta\theta}\right|_{r \quad a} \quad \left|\frac{\rho\omega^2}{3b} \quad \frac{5}{18}(a^3 \quad b^3) \quad \frac{a^3}{3}\log\frac{a}{b} \quad \frac{1}{2}(b^3 \quad a^3) \quad \frac{4}{3}\alpha E\theta_0\right| \quad Y^*$$

The angular velocity ω_f for fully-plastic state is given by:

$$\Omega_f^2 \quad \frac{\rho\omega_f^2 b^2}{Y^*} \quad 3\frac{a}{b}\frac{1}{\frac{2}{9} \quad 1 \quad \frac{a^3}{b^3} \quad \frac{1}{3}\frac{a}{b}^3 \quad \log\frac{a}{b}}$$

$$4\frac{a}{b}\frac{\alpha E\theta_0}{Y^*}\frac{1}{\frac{2}{9} \quad 1 \quad \frac{a^3}{b^3} \quad \frac{1}{3}\frac{a}{b}^3 \quad \log\frac{a}{b}} \tag{33}$$

and $\omega_f = (\Omega_f/b)(Y^*/\rho)^{1/2}$.

We introduce the following non-dimensional components as:

$$R \quad \frac{r}{b}; \quad R_0 \quad \frac{a}{b}; \quad \sigma_r \quad \frac{T_{rr}}{Y}; \quad \sigma_\theta \quad \frac{T_{\theta\theta}}{Y}; \quad \theta_1 \quad \frac{\alpha E\theta_0}{Y}; \quad \bar{u} \quad \frac{uE}{Yb}$$

Elastic-plastic transitional stresses, angular velocity, and displacement from eqs. (29), (30), (31), and (32) in non-dimensional form become:

$$\sigma_\theta \quad \frac{\Omega_i^2}{3R} \quad \frac{1}{3}\frac{5 \quad 4C}{2 \quad C}\frac{(1 \quad C)^2}{3 \quad C}(R^2 \quad 1) \quad R_0^3 \log R\frac{(1 \quad C)^2}{3 \quad 2C} \quad \frac{1 \quad C}{2 \quad C}(1 \quad R_0^3)$$

$$\frac{\theta_1}{\log R_0} \quad \frac{(1 \quad C)^2(R \quad 1)}{R} \quad \frac{R_0}{R}\log R\frac{(2 \quad C)(1 \quad C)^2}{(3 \quad 2C)} \quad \frac{(1 \quad R_0)(1 \quad C)}{R} \quad (2 \quad C)\log R \tag{34}$$

$$\sigma_r \quad \frac{\Omega_i^2}{3R} \quad \frac{1}{3}\frac{5 \quad 4C}{3 \quad 2C}(1 \quad C)(R^3 \quad 1) \quad R_0^3 \log R\frac{(1 \quad C)(2 \quad C)}{3 \quad 2C} \quad 1 \quad R^3$$

$$\frac{\theta_1(2 \quad C)}{\log R_0} \quad C\frac{1 \quad R}{R} \quad \log R \quad \frac{R_0 \log R}{3R(2 \quad C)} \tag{35}$$

$$\bar{u} \quad \frac{1 \quad C}{2 \quad C} \frac{\Omega_i^2}{3}(R^3 \quad R_0^3) \quad \frac{\theta_1(2 \quad C)}{\log R_0}(R \quad R_0) \tag{36}$$

and

$$\Omega_i^2 \quad \frac{3(2 \quad C)}{(1 \quad R_0^3)(1 \quad C)} \quad \frac{3\theta_1(2 \quad C)}{1 \quad R_0^3} \frac{(1 \quad R_0)}{\log \frac{1}{R_0}} \tag{37}$$

Stresses and displacement for fully-plastic state (C 0), are obtained from eqs. (34), (35), (36), and (33) as:

$$\sigma_r \quad \frac{\Omega_f^2}{3R} \quad \frac{5}{9}(R^3 \quad 1) \quad \frac{2}{3}R_0^3 \log R \quad 1 \quad R^3 \quad \frac{2\theta_1^*}{\log R_0} \log R \quad \frac{R_0 \log R}{6R} \tag{38}$$

$$\sigma_\theta \quad \frac{\Omega_f^2}{3R} \begin{matrix} \frac{5}{8}(R^3 \quad 1) \\ \frac{R_0^3 \log R}{3} \quad \frac{1}{2}(1 \quad R_0^3) \end{matrix} \quad \frac{\theta_1^*}{\log R_0} \begin{matrix} \frac{R \quad 1}{R} \quad \frac{2}{3}\frac{R_0}{R}\log R \\ \frac{1 \quad R_0}{R} \quad 2\log R \end{matrix} \tag{39}$$

$$\bar{u}_f \quad \frac{\Omega_f^2}{6}(R^3 \quad R_0^3) \quad \frac{\theta_1^*}{\log R_0}(R \quad R_0) \tag{40}$$

and

$$\Omega_f^2 \quad \frac{3R_0}{\frac{2}{9}(1 \quad R_0^3) \quad \frac{R_0^3 \log R_0}{5}} \quad \frac{4R_0\theta_1^*}{\frac{2}{5}(1 \quad R_0^3) \quad \frac{R_0^3 \log R_0}{3}} \tag{41}$$

where $\theta_1^* \quad (\alpha E\theta_0)/Y^*; \bar{u}_f = uE/Y^*b$.

Rotating disc without thermal effect

The elastic-plastic transitional stresses, displacement, and angular velocity without thermal effect ($\theta_0 = 0$) are obtained from eqs. (33), (34), (35), and (36) as:

$$\sigma_\theta \quad \frac{\Omega_i^2}{3R} \frac{1}{3}\frac{5 \quad 4C}{2 \quad C}\frac{(1 \quad C)^2}{3 \quad 2C}(R^3 \quad 1) \quad R_0^3 \log R \frac{(1 \quad C)^2}{3 \quad 2C} \quad \frac{1 \quad C}{2 \quad C}(1 \quad R_0^3) \tag{42}$$

$$\sigma_r \quad \frac{\Omega_i^2}{3R}\frac{1}{3}\frac{5 \quad 4C}{3 \quad 2C}(1 \quad C)(R^3 \quad 1) \quad R_0^3 \log R \frac{(1 \quad C)(2 \quad C)}{3 \quad 2C} \quad 1 \quad R^3 \tag{43}$$

$$\bar{u} \quad \frac{\Omega_i^2(R^3 \quad R_0^3)(1 \quad C)}{3(2 \quad C)} \tag{44}$$

and

$$\Omega_i^2 \quad \frac{3(2 \quad C)}{(1 \quad R_0^3((1 \quad C)} \tag{45}$$

The eqs. (38), (39), (40), and (41) for fully-plastic state (C 0) become:

$$\sigma_\theta \quad \frac{\Omega_f^2}{3R} \frac{5}{18}(R^3 \quad 1) \quad \frac{R_0^3 \log R}{3} \quad \frac{1}{2}(1 \quad R_0^3) \tag{46}$$

$$\sigma_r \quad \frac{\Omega_f^2}{3R}[(1 \quad R_0^3)\sqrt{R} \quad R^3 \quad R_0^3] \tag{47}$$

$$\bar{u}_f \quad \frac{\Omega_f^2}{6}(R^3 \quad R_0^3) \tag{48}$$

and

$$\Omega_f^2 \quad \frac{3R_0}{\frac{2}{9}(1 \quad R_0^3) \quad \frac{R_0^3 \log R_0}{3}} \tag{49}$$

Numerical illustration and discussion

To see the combined effect of angular speed and temperature on the rotating disc, the problem has been solved by taking the following values: C = 0.00, 0.25, 0.75; and θ_1 = 0.0, 0.25, 0.75. Curves have been drawn in fig. 2, between angular speed Ω_i^2 required for initial yielding and various radii ratio ($R_0 = a/b$). In fig. 2, it has been seen that at room temperature rotating disc made of compressible material and of smaller radii ratio yields at the internal surface required higher angular speed as compared to rotating disc made of incompressible material. With the introduction of thermal effect rotating disc yields at the outer surface at a lesser angular speed as compared to rotating disc at room temperature. In fig. 3, curves have been drawn between angular speed ($\Omega_i^2 = \rho\omega_i^2 b^2/Y$) and temperature ($\theta_1 = \alpha E\theta_0/Y$) required for initial yielding of a rotating disc having radii ratio ($R_0 = 0.5$). It has been observed that the rotating disc without thermal effects and made of compressible material yields at the internal surface at a higher angular speed as compare to rotating disc made of incompressible material. With the introduction of thermal effect, rotating disc yields at the outer surface at a lesser angular speed. In fig. 4, curves have been drawn between stresses at elastic-plastic transition and radii ratio ($R = r/b$). It is seen that the circumferential stress is maximum at the internal surface. With the introduction of thermal effect, it is maximum at the outer surface of the rotating disc. Curves have been drawn in fig. 5, between angular speed Ω_f^2 and various radii ratio (R_0 = a/b) for the rotating disc to become fully plastic. The ro-

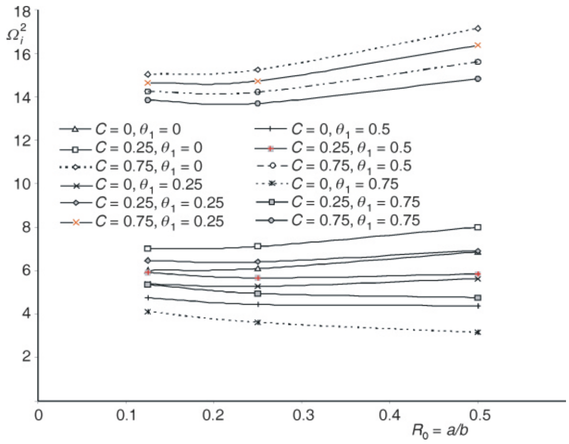

Figure 2. Angular speed required for initial yielding of the disc for various radii ratio

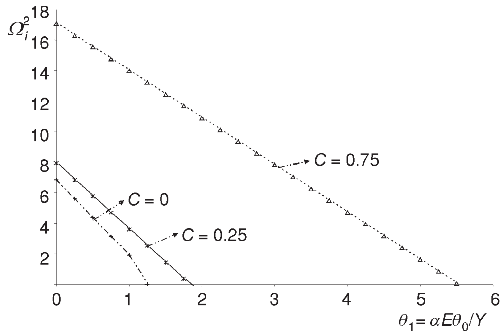

Figure 3. Relation between $\Omega_i^2 \quad \rho\omega_i^2 b^2/Y$ and $\theta_1 \quad \alpha E\theta_0/Y$ for yielding through the whole disc for R_0 = 0.5

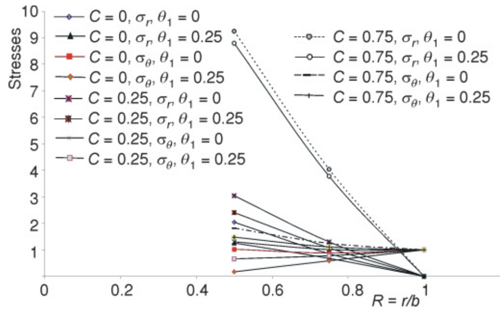

Figure 4. Stresses at the elastic-plastic transition of the rotating disc along the radius *R* = *r/b*

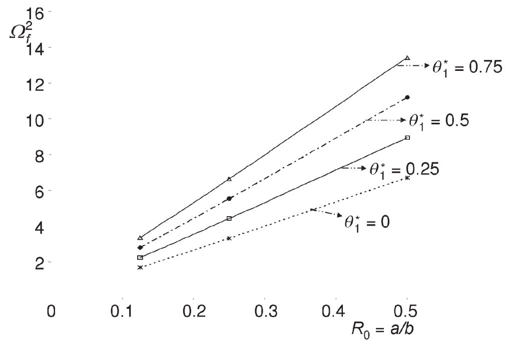

Figure 5. Angular speed required for the disc to be fully plastic at the various radii ration

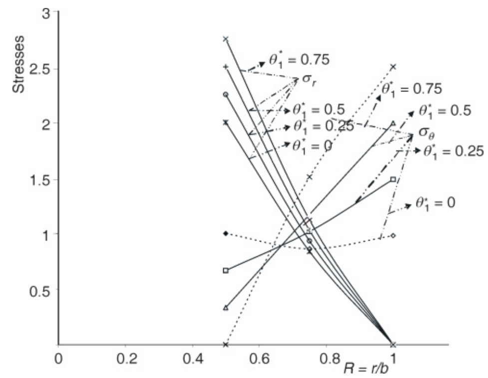

Figure 6. Stress distribution for fully plastic state of the disc along the radius *R* = *r/b*

tating disc of smaller radii ratio required higher angular speed to become fully plastic in comparison to rotating disc of higher thickness ratio, and this angular speed increases with the increase in temperature. From fig. 6, it can be seen that, with the introduction of thermal effect, the circumferential stress is maximum at the outer surface of the rotating disc with further increases with the increase in temperature. It means that angular speed of the rotating disc is less than that of the temperature-loaded disc in the fully plastic case.

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Nomenclature

a, b – internal and external radii of the circular cylinder, [m]

c – compressibility factor, [–]

K_1 – constant of integration, [–]

T_{ij}, e_{ij} – stress strain rate tensors, [$kgm^{-1}s^{-2}$]
R – radii ratio (= r/b, $R_0 = a/b$), [–]
u, v, w – displacement components, [m]
Y – yield stress, [$kgm^{-1}s^{-2}$]

Greek letters

θ – temperature, [°F]
σ_r – radial stress component (= T_{rr}/Y), [–]
σ_θ – circumferential stress component (= $T_{\theta\theta}/Y$), [–]

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