EFFECT OF THERMOPHORESIS ON NATURAL CONVECTION BOUNDARY LAYER FLOW OF A MICROPOLAR FLUID

by

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The present investigation deals with obtain the solution natural convection boundary layer flow of a micropolar fluid with thermophoresis. The similarity method is used to obtain solution for the governing equation. Four different cases of flows have been studied namely a vertical isothermal surface, vertical surface with uniform heat flux, a plane plume and flow generated from a horizontal surface. Numerical computations are carried out for the non-dimensional physical parameter. The results are analyzed for the effect of different physical parameters such as thermophoresis, Prandtl number, microrotation parameter, buoyancy parameter and Shmidt number of the fluid.

Key words: *thermophoresis, micropolar, heat and mass transfer, natural convection, boundary layer, similarity solution*

Introduction

The theory of micropolar fluids and thermomicropolar fluids developed by Erigen [1, 2] has been of much interest because it can be used to explain the characteristics in certain fluids such as colloidal suspensions, polymeric fluids, and liquid crystals for the microscopic effects due to the local structure and micromotions of the fluid elements.

Thermophoresis is a phenomenon by which submicron sized particles suspended in a non-isothermal gas acquire a velocity relative to the gas in the direction of decreasing temperature. The velocity acquired by the particles is called thermophoretic velocity and the force experienced by the suspended particles due to the temperature gradient is known as thermophoretic force. The first analysis of thermophoretic deposition in geometry of engineering interest appears to be that of [3-6]. They solved the laminar boundary layer equations for simultaneous aerosol and steam transport to an isothermal vertical surface situated adjacent to a large body of an otherwise quiescent air-steam-aerosol mixture. Thermophoresis in laminar flow over a horizontal flat plate has been studied theoretically by Goren [7] where the analysis covered both cold and hot plate conditions. Epstein *et al.* [8] consider the effect of surface mass transfer on mixed convection flow past a heated vertical flat permeable surface in the presence of thermophoresis. Previous work on this topic includes papers, who carried out a thermophoretic

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analysis of small particles in a free convection boundary layer adjacent to a cold vertical surface, and Mills *et al*. [9] and Tsai [10], who reported correlations for the deposition rate in the presence of thermophoresis and wall suction in laminar flow over a flat plate. Jia *et al.* [11] also investigated numerically the interaction between radiation and thermophoresis in forced convection laminar boundary layer flow and natural convective laminar flow over a cold vertical flat plate in the presence of thermophoresis was solved numerically by Jayaraj [12] and Jayaraj *et al*. [13] for constant and variable properties, respectively. Finally, Chiou [14] analyzed the effect of thermophoresis on submicron particle deposition from a forced laminar boundary layer flow on an isothermal moving plate through similarity solutions and this analysis was extended by Chiou *et al*. [15] convection from a vertical isothermal cylinder. Selim *et al*. [16] discussed the effect of surface mass transfer on mixed-convection flow past a heated vertical permeable flat plate with thermophoresis. Recently, Chamkha *et al*. [17] studied the effect of thermo- phoretic particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium.

Micropolar fluids consist of a suspension of small, rigid, cylindrical elements such as large dumbbell-shaped molecules. The theory of thermomicropolar fluids has been developed by Erigen [2] extending the theory of micropolar fluids. Gorla [18, 19] analyzed the mixed convection in micropolar boundary layer flow on vertical and horizontal plates. Hassanien *et al*. [20, 21] analyzed the natural convection in micropolar boundary layer flow. Nadeem *et al*. [22] have analyzed the effects of variable viscosity, variable thermocapillarity on the flow and heat transfer in a thin film on a horizontal porous shrinking sheet through a porous medium. Bataller [23] study of the flow and heat transfer of an incompressible homogeneous second-grade fluid over a non-isothermal stretching sheet.

Hassanien *et al.* [24] studied an unsteady, two-dimensional, laminar, boundary flow of a micropolar fluid along an isothermal vertical plate immersed in a thermally stratified quiescent fluid.

Motivated by the above investigations and possible applications, it is of interest in the present work to study natural convection flows in micropolar fluids with thermophoresis. Four different boundary conditions, namely, an isothermal wall, a uniform surface heat flux, a plane plume arising from a concentrated horizontal thermal source, and an adiabatic surface with a concentrated energy source at the leading edge. Numerical solutions are presented for the velocity, temperature, concentration, and microrotation of boundary conditions with various parameters.

Analysis

Let us consider a steady, two-dimensional vertical natural convection flows and incorporate the usual Boussinesq and boundary layer assumptions. The fluid properties are assumed to be constant to the plane as [16] and [21]. The governing equations for the problem under consideration can be written as:

$$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} \quad v\frac{\partial u}{\partial y} \quad \frac{\mu \quad K}{\rho}\frac{\partial^2 u}{\partial y^2} \quad g\beta_{\mathrm{T}}(T \quad T_\infty) \quad g\beta_{\mathrm{c}}(C \quad C_\infty) \quad \frac{K}{\rho}\frac{\partial N}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x} \quad v\frac{\partial T}{\partial y} \quad \frac{k}{\rho c_{\mathrm{p}}}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} \quad v\frac{\partial C}{\partial y} \quad \frac{\nu}{\mathrm{Sc}}\frac{\partial^2 C}{\partial y^2} \quad \frac{\partial}{\partial y}(V_{\mathrm{T}} C) \tag{4}$$

$$u\frac{\partial N}{\partial x} \quad v\frac{\partial N}{\partial y} \quad \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} \quad \frac{K}{\rho j} \; 2N \quad \frac{\partial u}{\partial y} \tag{5}$$

where, u and v are the fluid velocity components along the x- and y-axes (which are parallel and normal to the plate, respectively), g is the gravitational force due to acceleration, β – the volumetric coefficient of thermal expansion, β_{c} – the concentration expansion coefficient, T – the temperature of the fluid in the boundary layer, C – the species concentration in the boundary layer, Pr – the Prandtl number, and Sc – the Schmidt number. In eq. (4), the effect of thermophoresis is usually prescribed by means of an average velocity which a particle will acquire when exposed to a temperature gradient. In boundary layer flow, the temperature gradient in the y-direction is very much larger than in the x-direction, and therefore, only the thermophoretic velocity in y-direction is considered. As a consequence, the thermophoretic velocity V_{T}, which appears in eq. (4), may be expressed in the following form:

$$V_{\mathrm{T}} \quad \frac{\kappa\nu}{T}\frac{\partial T}{\partial y} \tag{6}$$

where, T is some reference temperature, the value of $\kappa\nu$ presents the thermophoretic diffusivity, and κ– the thermophoretic coefficient, which ranges in value from 0.2 to 1.2 as observed by Batchelor *et al*. [25] and is defined from the theory of Brock [5] by:

$$\kappa \quad \frac{2C_{\mathrm{s}}\left(\dfrac{\lambda_{\mathrm{g}}}{\lambda_{\mathrm{p}}} \quad C_{\mathrm{t}}\mathrm{Kn}\right)[1 \quad \mathrm{Kn}(C_1 \quad C_2 \mathrm{e}^{\;C_3/\mathrm{Kn}})]}{(1 \quad 3C_{\mathrm{m}}\mathrm{Kn})\left(1 \quad \dfrac{2\lambda_g}{\lambda_{\mathrm{p}}} \quad 2C_{\mathrm{t}}\mathrm{Kn}\right)} \tag{7}$$

where C_1, C_2, C_3, C_{m}, and C_{s} are constants, λ_{g}, and λ_{p} are the thermal conductivities of gas and diffused particles, respectively, and Kn is the Knudsen number, as previously introduced by Mills *et al*. [9] and Tsai [10].

In order to obtain a system of equations applicable to the entire regime of natural convection, we now introduce the following continuous transformations regime:

$$\begin{gathered}
\eta \quad yb(x), \qquad \psi(x, y) \quad \nu c(x) f(\eta, x), \quad c(x) \quad 4x\left(\frac{G_{\mathrm{rx}}}{4}\right)^{1/4}, \quad N \quad \nu c(x)[b(x)]^2 h(\eta, x), \\
\theta \quad \frac{T \quad T_\infty}{(T_0 \quad T_\infty)_0}, \quad (T_0 \quad T_\infty)_0 \quad Mx^n, \qquad \phi \quad \frac{C \quad C_\infty}{(C_0 \quad C_\infty)_0}, \qquad (C_0 \quad C_\infty)_0 \quad M_1 x^n \\
G_{\mathrm{rx,T}} \quad \frac{\mathrm{g}\beta_{\mathrm{T}} x^3 \delta T}{\nu^2}, \; C_{\mathrm{T}} \quad \frac{T_\infty}{\delta T}, \; \delta T \quad (T_0 \quad T_\infty)_0, \quad G_{\mathrm{rx,C}} \quad \frac{\mathrm{g}\beta_{\mathrm{C}} x^3 \delta T}{\nu^2}, \quad \delta C \quad (C_0 \quad C_\infty)_0
\end{gathered} \tag{8}$$

We now define a stream function $\psi(x, y)$ which satisfies the continuity eq. (1) with:

$$u \quad \frac{\partial\psi}{\partial y}, \quad v \quad \frac{\partial\psi}{\partial x} \tag{9}$$

The transformed governing equations for bonundary layer flows become:

$$
\begin{aligned}
&(1 \quad \Delta) f \qquad (n \quad 3) ff \qquad 2(n \quad 1) f^{\ 2} \quad \theta \quad B\phi \quad \Delta h \quad 0 \\
&\frac{1}{\mathrm{Pr}} \theta \qquad (n \quad 3) f\theta \qquad 4nf \ \theta \quad 0 \\
&\frac{1}{\mathrm{Sc}} \phi \qquad (n \quad 3) f\phi \qquad \kappa \ \frac{(\theta \quad C_{\mathrm{T}})\theta \qquad \theta^{\ 2}}{\theta \quad C_{\mathrm{T}}} \ \phi \quad \frac{\theta \ \phi}{\theta + C_{\mathrm{T}}} \qquad 0 \\
&\lambda h \ (n \quad 3) fh \qquad (3n \quad 1) hf \qquad \sigma(2h \quad f \ ) \quad 0
\end{aligned}
\tag{10}
$$

where, $B = \beta_c \delta C / \beta_T \delta T$ is the buoyancy ratio, $\Delta = K/\mu$, $\lambda = \gamma/\mu j$, $\nu = K/\rho$, $\sigma = K/\mu j \beta^2$ are the material constants and the primes indicate differentiation with respect to η.

The transformed boundary conditions are:

(a) Isothermal surface, $n = 0$:

$$
\begin{aligned}
&f(0) \quad 0, \quad f \ (0) \quad 0, \quad \theta(0) \quad 1, \quad \phi(0) \quad 0, \quad h(0) \qquad 0.5 f \ (0), \\
&f \ (\infty) \quad 0, \quad \theta(\infty) \quad 0, \quad \phi(\infty) \quad 1, \quad h(\infty) \quad 0
\end{aligned}
$$

(b) Constant surface heat flux, $n = 1/5$:

$$
\begin{aligned}
&f(0) \quad 0, \quad f \ (0) \quad 0, \quad \theta(0) \quad 1, \quad \phi(0) \quad 0, \quad h(0) \qquad 0.5 f \ (0), \\
&f \ (\infty) \quad 0, \quad \theta(\infty) \quad 0, \quad \phi(\infty) \quad 1, \quad h(\infty) \quad 0
\end{aligned}
$$

(c) Unbounded plane plume rising from a thermal source at $x = 0$, $n = -3/5$:

$$
\begin{aligned}
&f(0) \quad 0, \quad f \ (0) \quad 0, \quad \theta(0) \quad 1, \quad \phi(0) \quad 0, \quad h(0) \qquad 0.5 f \ (0), \\
&f \ (\infty) \quad 0, \quad \theta(\infty) \quad 0, \quad \phi(\infty) \quad 1, \quad h(\infty) \quad 0
\end{aligned}
$$

(d) Adiabatic surface with a concentrated heat source along the leading edge, wall plume, $n = -3/5$:

$$
\begin{aligned}
&f(0) \quad 0, \quad f \ (0) \quad 0, \quad \theta(0) \quad 1, \quad \phi(0) \quad 0, \quad h(0) \qquad f \ (0) \\
&f \ (\infty) \quad 0, \quad \theta(\infty) \quad 0, \quad \phi(\infty) \quad 1, \quad h(\infty) \quad 0
\end{aligned}
\tag{11}
$$

The skin-friction coefficient (C_f), Nusselt number, and the Sherwood number are important physical parameters for this problem. These can be defined as:

$$
C_{\mathrm{f}} \quad \frac{\tau}{\rho u v}; \quad \tau \quad \mu \left.\frac{\partial u}{\partial y}\right|_{y \ 0} \tag{12}
$$

$$
\mathrm{Nu} \quad \frac{q_{\mathrm{w}} v}{(T_{\mathrm{w}} \quad T_{\infty}) k v_{\mathrm{w}}}; \quad q_{\mathrm{w}} \qquad k \left.\frac{\partial T}{\partial y}\right|_{y \ 0} \tag{13}
$$

$$
\mathrm{Sh} \quad \frac{J_{\mathrm{w}} v}{C D v_{\mathrm{w}}}; \quad J_{\mathrm{w}} \qquad D \left.\frac{\partial C}{\partial y}\right|_{y \ 0} \tag{14}
$$

In order to gain physical insight, the system of ordinary differential eqs. (10), along with the boundary conditions (11), are integrated numerically by means of the fourth-order Runge-Kutta method with a shooting technique. The step size $\Delta\eta = 0.05$ is used to obtain the numerical solution with h_{max} and five-decimal accuracy as the criterion for convergency.

Results and discussion

In this section, a comprehensive numerical parametric study is conducted and the results are reported in terms of graphs. This is done to illustrate special features of the solutions. It is clearly seen that the results are given values of the parameters $B, \kappa, \Delta, n, \lambda, \sigma$, Pr, and Sc. Typical the velocity, temperature, concentration and microrotation component profiles are shown in figs. 1-9 for some values of the governing parameters $B, \kappa, \Delta, n, \lambda, \sigma$, Pr, and Sc, and for cases: vertical isothermal surface, vertical surface with uniform heat flux, unbounded plane plume rising from a thermal source at $x = 0$, and adiabatic surface with a concentrated heat source along the leading edge.

We have computed solutions for translational velocity f', dimensionless temperature θ, dimensionless concentration ϕ and microrotation component h for the following general values: $B = 1$, $\kappa = 0.6$, Pr = 0.733, Sc = 1.2, $C_T = 10$, $\lambda = 1$, and $\sigma = 1$. These figures show how the temperature the concentration and microrotation component boundary layer and the wall deposition velocity react to changes with the parameters B, κ, Δ, Pr, and Sc. In addition, the boundary condition η ∞ is approximated by $\eta_{max.} = 6$, which is sufficiently large for the velocity to approach the relevant stream velocity.

In the present analysis we have excluded plots (for conservation of space) for the effects of λ, σ, and C_T which are fixed in the computations.

In all cases when the absence of eq. (4) we get the results of Hassanien *et al.* [20].

The dimensionless velocity, microrotation component, temperature, and concentration profiles for different value of Δ = 0, 1.5, 4.5, 13.5, and 50 are shown in figs. 1 and 2 for all

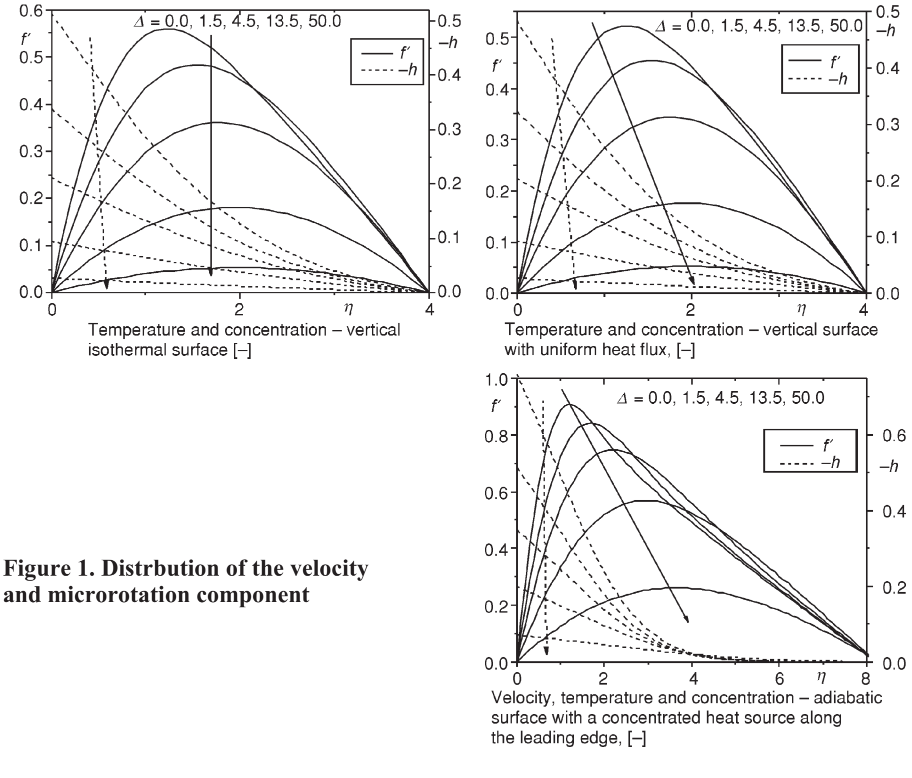

Figure 1. Distrbution of the velocity and microrotation component

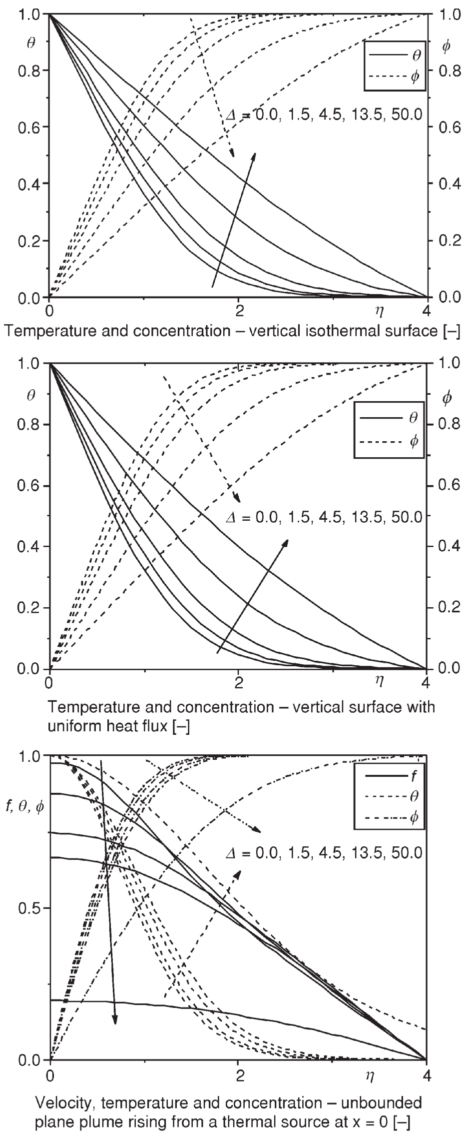

Figure 2. Distribution of the velocity, temperature, and concentration

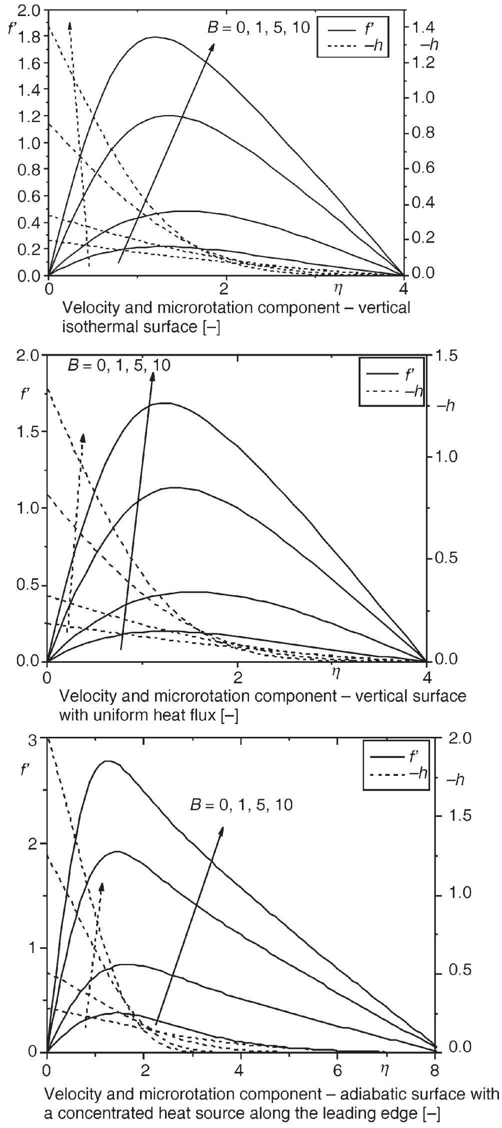

Figure 3. Distribution of the velocity and microrotation component

cases. It is observed that the velocity, microrotation component, and concentration profiles of the fluid decreases with the increase of the value Δ. On the other hand, of the temperature of the fluid increase with the increase of Δ parameter.

The dimensionless velocity, microrotation component, temperature, and concentration profiles for different value of $B = 0, 1, 5, 10$ are shown in figs. 3 and 4 for all cases. It is seen that the velocity, microrotation component, and concentration profiles of the fluid increases with the increase of the value B. But opposed of the temperature of the fluid.

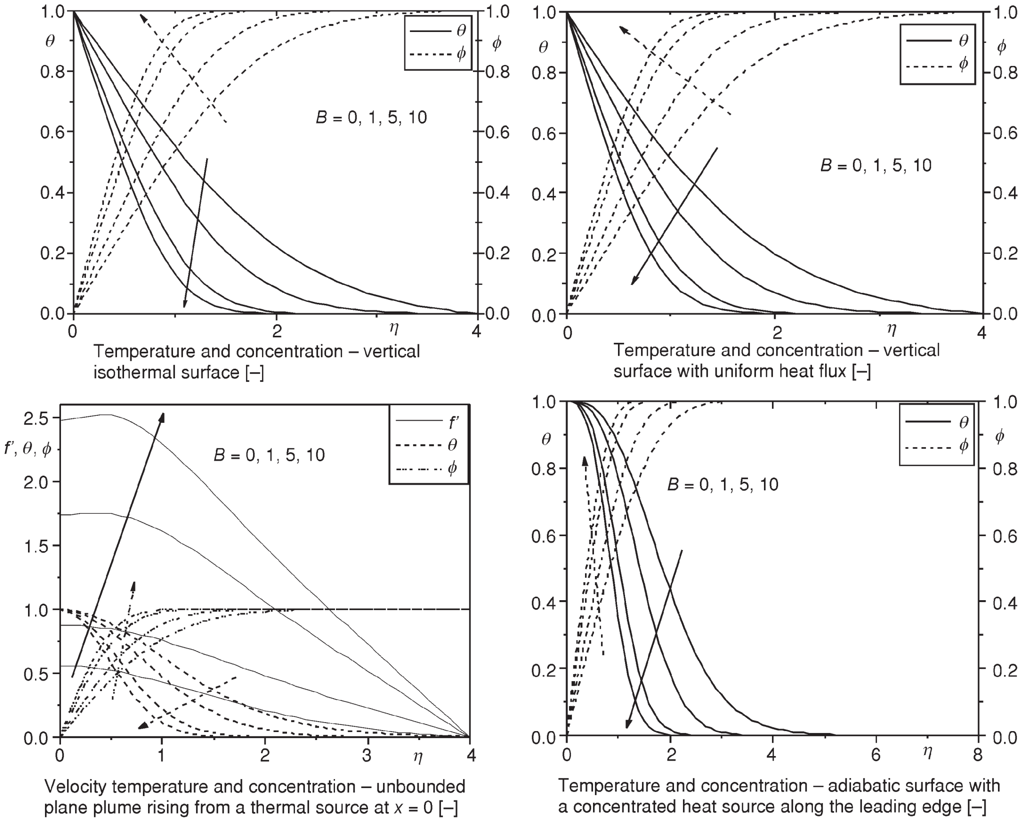

Figure 4. Distribution of the velocity, temperature, and concentration

The dimensionless velocity, microrotation component, temperature, and concentration profiles for different value of Pr = 0.733, 7.0, and 11.6 are shown in figs. 5 and 6 for all

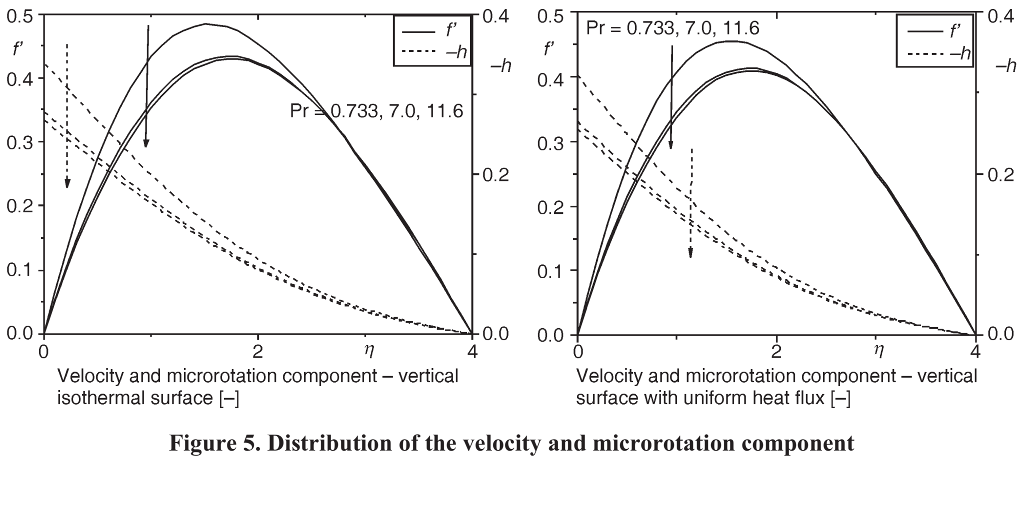

Figure 5. Distribution of the velocity and microrotation component

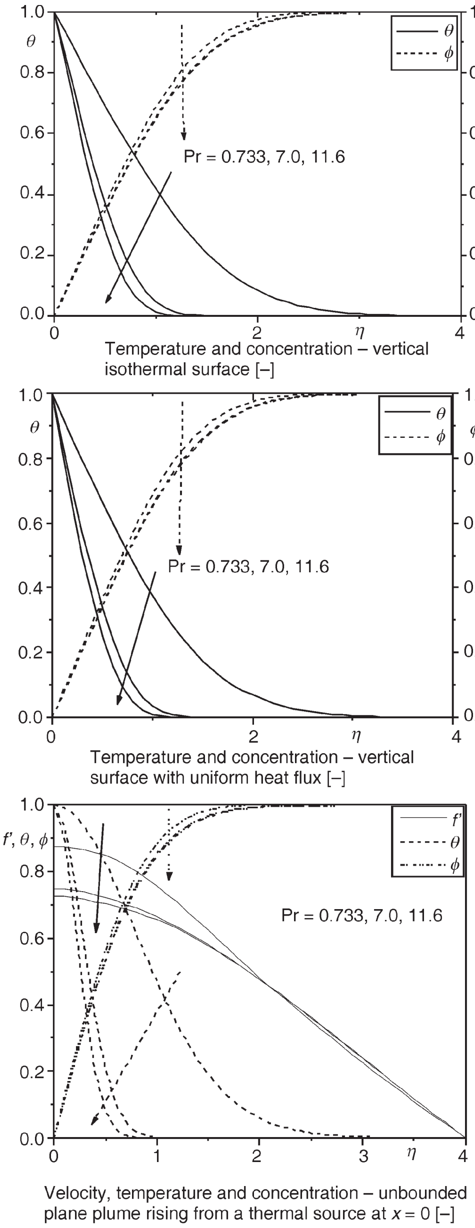

Figure 6. Distribution of the velocity, temperature, and concentration

cases. It is observed that the velocity, microrotation component, temperature, and concentration profiles of the fluid decreases with the increase of the value Pr.

The dimensionless velocity, microrotation component, temperature, and concentration profiles for different value of Sc = 0, 1.2, 3, 5, and 10 are shown in figs. 7 and 8 for all cases. It

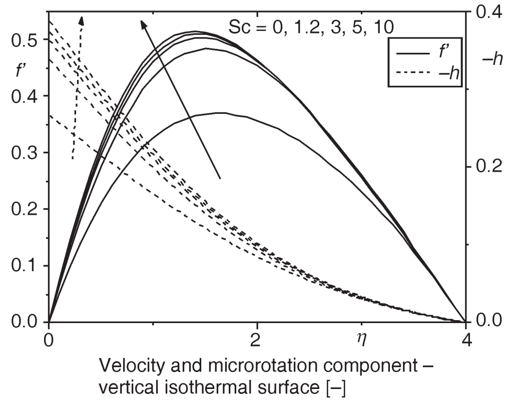

Figure 7. Distribution of the velocity and microrotation component

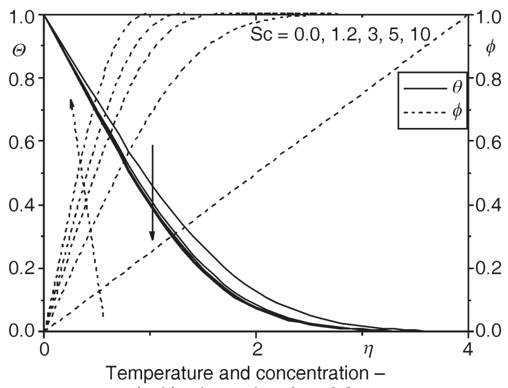

Figure 8. Distribution of the temperature and concentration

is clear that the velocity, microrotation component, and concentration profiles of the fluid increases with the increase of the value Sc.

On the other hand, of the temperature of the fluid decrease with the increase of the Sc parameter.

Figure 9 depicts the dimensionless velocity and concentration profiles for different value of κ = 0, 0.6, 5, 10, and 20. It is observed that the velocity and concentration profiles of the fluid increases with the increase of the value κ.

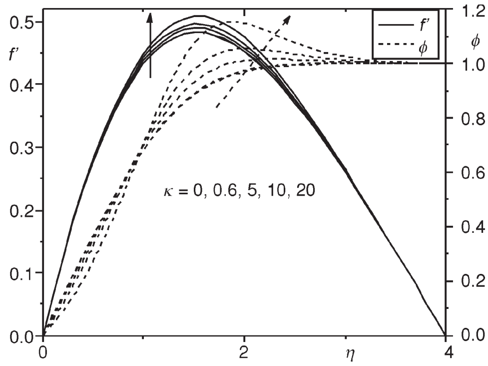

Figure 9. Distribution of the velocity and concentration

Conclusions

In this study, we have presented a steady-state of natural convection flow of micropolar fluid for the case in which the plate is maintained at a given concentration in a thermophoresis while convection arises within the micropolar fluid. Flow of this type represents four different boundary condition problems. The ordinary deferential equations for boundary layer equations are obtained. Numerical solutions are obtained for the fluid and concentration characteristics. The results are given for velocity distributions, micropolar, temperature, and concentration for different values of microrotation parameter, buoyance parameter, the Prandtl number, Schmidt number, and thermophoresis.

The model has been transformed and rendered into dimensionless form. Our numerical results indicate that generally:
- increasing B, Sc, and κ increases the dimensionless parameters f', h, and ϕ,
- as Δ and Pr increase, the dimensionless parameters f', h, and f decrease,
- increasing B, Sc, and κ decreases the dimensionless temperature θ, and
- as Δ increases, the dimensionless temperature θ increases.

The present study is currently being extended to consider the effects of porous media drag forces, unsteadiness and also variable plate conductivity on boundary-layer characteristics and the results of these investigations will be communicated in the near future.

Nomenclature

C – species concentration in the boundary layer, [kgm^{-2}]
C_m, C_s, C_t,
C_1, C_2, C_3 – constants in eq. (7), [–]
C_∞ – species concentration of the ambient fluid, [–]
c_p – specific heat due to constant pressure, [Jkg^{-1}K^{-1}]
D – chemical molecular diffusivity, [–]
f – dimensionless stream function, [–]
g – acceleration due to gravity, [ms^{-2}]
h – dimensionless microrotation component, [–]
J_w – defined in eq. (14), [W, kgm^{-2}s^{-1}]
j – microinertia per unit mass, [m^2]
K – vortex viscosity by eq. (2) and (5), [Nsm^2]
Kn – Knudsen number (= l_g/r_p), [–]
k – thermal conductivity (eq. 3), [Wm^{-2}K^{-1}]
l_g – the mean free path of gas molecules
M, n – defined in eq. (9), [–]
N – angular velosity, [rad^{-1}]
Pr – Prandtl number ($\nu\rho c_p/k$), [–]
r_p – the aerosol particle radius
q_w – defined in eq. (13), [W, kgm^{-2}s^{-1}]
Sc – Schmidt number (ν/D), [–]
T – temperature of the fluid in the boundary layer , [K]
T_w – temperature at the surface, [K]
T_∞ – temperature of the ambient fluid, [K]
u, v – the x- and y-components of the velocity field, [ms^{-1}]
V_T – thermophoretic velocity, [ms^{-1}]
$V(x)$ – transpiration velocity, [ms^{-1}]
x, y – axis in direction along and normal to the plate, [m]

Greek letters

α – thermal diffusivity, [m^2s^{-1}]
β – volumetric expansion coefficient of temperature, [–]
γ – microrotation coupling coefficient, [Ns]
Δ – material constant, [–]
η – non-dimensional pseudo-similarity variable, [–]
θ – dimensionless temperature function, [–]
κ – thermophoretic coefficient defined by eq. (7), [–]
λ – fluid viscosity, [Pa·s]
μ – dynamic viscosity of the fluid, [Pa·s]
ν – kinematic coefficient of viscosity, [–]
ρ – fluid density, [kgm^{-3}]
σ – material constant, [–]
ϕ – dimensionless species concentration, [–]
ψ – stream function, [m^2s^{-1}]

Subscripts

w – condition of the wale
∞ – ambient condition

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