

MHD FREE CONVECTION-RADIATION INTERACTION ALONG A VERTICAL SURFACE EMBEDDED IN DARCIAN POROUS MEDIUM IN PRESENCE OF SORET AND DUFOUR'S EFFECTS

by

***Shyam Sunder TAK^{a*}, Rajeev MATHUR^b,
Rohit Kumar GEHLOT^c, and Aiyub KHAN^a***

^a Department of Mathematics and Statistics, J. N. V. University, Jodhpur, Raj., India

^b Lachoo Memorial College of Science and Technology, Shastri Nagar, Jodhpur, Raj., India

^c Marwar Engineering College and Research Centre, Kharda-Bhandu, Narnadi, Jodhpur, Raj., India

Original scientific paper
UDC: 536.25:517.957:537.63
DOI: 10.2298/TSCI1001137T

The effects of thermal radiation and magnetic field on heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated Darcian porous medium has been investigated taking into account the Soret and Dufour effects. The Rosseland approximation for the radiative heat flux is used in the energy equation. It is found that the similarity solution exists in the present case. The resulting set of coupled non-linear ordinary differential equations is solved numerically using shooting technique. Dimensionless velocity, temperature, and concentration profiles are presented graphically for various values of radiation parameter and the Nusselt and Sherwood numbers are tabulated for different values of the involved parameters. It is found that the Nusselt number increases and Sherwood number decreases as the radiation parameter increases but both the Nusselt number and Sherwood number decrease as the magnetic field parameter increases.

Key words: *free convection, radiation, magnetic field, Soret and Dufour's effects, porous media*

Introduction

The study of coupled heat and mass transfer by natural convection from uniform surfaces embedded in a saturated porous medium has drawn considerable attention over the last few years, due to many important engineering and geophysical applications. A comprehensive account of the available information in this field is provided in recent books by Nield *et al.* [1], Ingham *et al.* [2, 3], and Vafai [4].

There has been considerable interest in studying flows of electrically conducting fluids over surfaces in the presence of magnetic field (Chamkha [5], Cheng [6]). The effect of magnetic field on heat and mass transfer by natural convection from vertical surface in porous media

* Corresponding author; e-mail: sstak@rediffmail.com

has been studied by Cheng [6] using an integral approach. Cheng neglected Soret and Dufour effects.

Double diffusion in heat and mass transfer refers to Soret and Dufour effects. Soret effect [thermal-diffusion] refers to mass flux produced by a temperature gradient and the Dufour effect [diffusion-thermo] refers to heat flux produced by a concentration gradient. These effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. There are however, exceptions. Eckert *et al.* [7] present several cases when the Dufour effect cannot be neglected. The latest developments concerning Soret and Dufour effects in natural convection in porous media may be found in the book by Nield *et al.* [1] and Vafai [4]. Alam *et al.* [8] studied Soret and Dufour's effect in absence of magnetic effect whereas Postelnicu [9], Alam *et al.* [10], and Chamkha *et al.* [11] have studied the Soret and Dufour effects in the presence of magnetic field on heat and mass transfer by natural convection from a vertical surface in porous media. Postelnicu [12] studied the influence of chemical reactions along with Soret and Doufer's effect in the absence of magnetic field on free convection.

The interaction of radiation with free convection in porous media has also been studied by many researchers. Whitaker [13] studied the radiant energy transport in porous media. Chandrasekhara *et al.* [14] considered the composite heat transfer in the case of flow past a horizontal surface embedded in a saturated porous medium. Raptis [15], using Rosseland approximation for radiative heat flux, studied the free convection flow through porous medium. Hossain *et al.* [16] studied the effect of radiation on free convection from an inclined surface placed in Darcian porous media. Chamkha [17] studied solar radiation assisted free convection from a vertical plate in a porous medium with a more general Darcy-Forchheimer-Brinkman flow. Chamkha *et al.* [18] extended the results obtained by Chamkha [17] to a variable porosity medium and observed that the boundary friction and Nusselt number are decreased as the porous medium parameter value is increased. In these studies Chamkha, used an exponential type of approximation for incident solar radiation flux. Bakier [19] studied the thermal radiation effect on mixed convection from vertical surfaces in saturated Darcian porous media. The same problem in a non-Darcian porous medium [Forchheimer flow model] with suction-injection velocity at vertical wall has been considered by Murthy *et al.* [20]. Recently, the interaction of radiation with free convection under different physical conditions has been studied by many authors [21-25]. In all the above studies the Soret and Dufour effects were neglected.

Recently, Postelnicu [9, 12] considered the Soret and Dufour effects and influence of magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media. Postelnicu neglected the radiative heat flux in the energy equation whereas Chamkha *et al.* [11] have included it in the same problem with Forchheimer model. Chamkha *et al.* [11] used Rosseland approximation for radiative heat transfer, which was further approximated by expanding temperature function T^4 by Taylor's series valid near T_∞ , the temperature of the ambient medium. Thus this approximation is valid in the free convection region near thermal boundary layer edge. In the present work, we have extended the work of Postelnicu [9, 12] and Chamkha *et al.* [11] to include the effect of thermal radiation, using Rosseland approximation for radiative heat flux without further approximation to it making it valid for whole region of free convection flow.

Governing equations and mathematical analysis

Consider the natural convection heat and mass transfer from an impermeable vertical wall in a fluid saturated porous medium in presence of uniform transverse magnetic field. The

fluid is considered to be a gray, absorbing-emitting radiation but non-scattering and the Rosseland approximation for the radiative heat flux is used in the energy equation. The wall is maintained at constant wall temperature T_w and constant wall concentration C_w . The temperature and mass concentration of the ambient medium are assumed to be T_∞ and C_∞ , respectively, where $T_w > T_\infty$ and $C_w > C_\infty$. The x -co-ordinate is measured along the plate from its leading edge, and the y -co-ordinate normal to it. It is assumed that the porous medium is in thermal equilibrium with the fluid and is isotropic and homogeneous. Also the properties of the fluid and porous medium are constant, the Darcy law, the Boussinesq approximation and the boundary layer approximation are applicable. Then the governing equations for the boundary layer flow, heat and mass transfer from the wall $y=0$ into the fluid saturated porous medium $x > 0$ and $y > 0$ are given by:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u = \frac{K\sigma\mu_e^2 H_0^2}{\mu} - \frac{gK[\beta_T(T - T_\infty) - \beta_C(C - C_\infty)]}{\nu} \quad (2)$$

$$u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial T}{\partial y} - \frac{1}{\rho C_p} q_r - \frac{D_m}{C_s} \frac{k_T}{C_p} \frac{\partial^2 C}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - \frac{D_m}{T_m} k_T \frac{\partial^2 T}{\partial y^2} \quad (4)$$

where α_m and D_m are the thermal diffusivity and mass diffusivity, respectively, ρ is the density, C_p and C_s are the specific heat at constant pressure and concentration susceptibility, respectively, k_T is the thermal diffusion ratio, σ , μ_e , and H_0 are electrical conductivity, magnetic permeability, and magnetic field intensity, respectively, u and v are Darcian velocities in the x and y -direction. K is the Darcy permeability, μ – the viscosity, ν – the kinematic viscosity, g – the gravity, and T_m – the mean fluid temperature. β_T and β_C are the coefficients of thermal expansion, respectively, and concentration expansion, and q_r – the radiative heat flux.

The boundary conditions of the problem are:

$$\begin{aligned} y = 0 : v = 0, T = T_w, C = C_w \\ y \rightarrow \infty : u = 0, T = T_\infty, C = C_\infty \end{aligned} \quad (5)$$

We assume the Rosseland approximation [26] for radiative heat flux, which leads to:

$$q_r = \frac{4\sigma_B}{3\kappa^*} \frac{\partial T^4}{\partial y} \quad (6)$$

where σ_B is the Stefan-Boltzmann constant and κ^* is the mean absorption coefficient.

In view of eqs. (6), eq. (3) now becomes:

$$u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial T}{\partial y} - \frac{16\sigma_B}{3\kappa^*\rho C_p} T^3 - \frac{D_m}{C_s} \frac{k_T}{C_p} \frac{\partial^2 C}{\partial y^2} \quad (7)$$

We now use the similarity variables proposed by Cheng *et al.* [27] as:

$$\begin{aligned}\eta & \frac{y\sqrt{\text{Ra}_x}}{x}, \\ \psi & \alpha_m \sqrt{\text{Ra}_x} f(\eta), \\ \theta & \frac{T - T_\infty}{T_w - T_\infty}, \\ \phi & \frac{C - C_\infty}{C_w - C_\infty},\end{aligned}\tag{8}$$

where the stream function ψ is defined as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}\tag{9}$$

and $\text{Ra}_x = K\beta_T(T_w - T_\infty)x/\nu\alpha_m$ is the local Rayleigh number. The governing eqs. (1), (2), (7), and (4), using eqs. (8) and (9), become:

$$(1 + M)f' = \theta + N\phi\tag{10}$$

$$1 - \frac{4(\theta - \beta)^3}{3R} \theta - \frac{1}{2}f\theta - D_f\phi\tag{11}$$

$$\phi - \text{Le} \cdot \frac{1}{2}f\phi - S_r\theta\tag{12}$$

with boundary conditions

$$\begin{aligned}\eta = 0: & f = 0, \theta = 1, \phi = 1 \\ \eta = \infty: & f = 0, \theta = 1, \phi = 0\end{aligned}\tag{13}$$

where a prime denotes differentiation $dt/d\eta$ and

$$\beta = \frac{T_\infty}{T_w - T_\infty} \quad (\text{temperature ratio})$$

$$M = \frac{K\sigma\mu_e^2 H_0^2}{\mu} \quad (\text{magnetic field parameter}),$$

$$\text{Le} = \frac{\alpha_m}{D_m} \quad (\text{Lewis number}),$$

$$D_f = \frac{D_m k_T}{\alpha_m C_s C_p} \frac{C_w - C_\infty}{T_w - T_\infty} \quad [\text{Dufour number}],$$

$$S_r = \frac{D_m k_T}{\alpha_m T_m} \frac{T_w - T_\infty}{C_w - C_\infty} \quad (\text{Soret number}),$$

$$R = \frac{\kappa^* k_T}{4\sigma_B (T_w - T_\infty)^3} \quad (\text{radiation parameter}),$$

$$N = \frac{\beta_c}{\beta_T} \frac{C_w - C_\infty}{T_w - T_\infty} \quad (\text{buoyancy parameter}).$$

It may be noted that in the absence of radiative heat flux, the radiation parameter $R = \infty$ and in this case, the set of eqs. (10) to (12) with boundary conditions (13) reduces to that obtained by Postelnicu [9] for non-radiating fluids.

Numerical analysis and discussion

The eqs. (10) to (12) with boundary conditions (13) have been solved numerically using Newton shooting method developed by Keller [28], the details of which may be found in Cebeci *et al.* [29]. For numerical integration the Runge Kutta fourth order scheme was used with a step size of 0.005. The boundary condition as $\eta \rightarrow \infty$ was chosen to satisfy at a suitable large value of $\eta = 20$. Numerical solutions, obtained by the present method have been compared with those obtained by Postelnicu [9] in the particular case where no thermal radiation is present (*i.e.* $R = \infty$) and it is found that both the results are in good agreement, up to the order of 10^{-5} .

The parameters involved in the present problem are: R – the radiation parameter, M – the magnetic field parameter, D_f – the Dufour number, S_r – the Soret number, and N – the buoy-

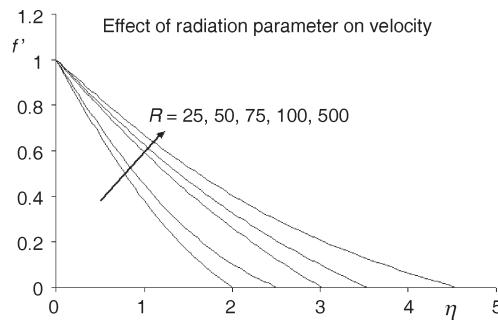


Figure 1. Velocity function f' plotted against η for various values of R taking $M = 1$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

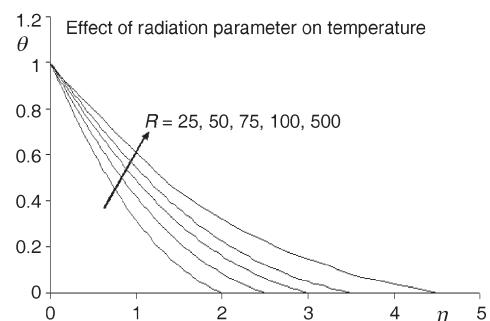


Figure 2. Temperature plotted against η for various values of R taking $M = 1$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

ancy parameter. To observe the effect of thermal radiation and magnetic field on the free convection we have plotted velocity function f' , temperature function θ and concentration function ϕ against η for various values of M and R keeping D_f , S_r , and N fixed.

The effect of thermal radiation parameter R on the velocity, temperature, and concentration are shown in figs. 1 to 3. It is noted from these figures that velocity, temperature, and concentration increases as R increases, keeping other parameters fixed. Hence the presence of ther-

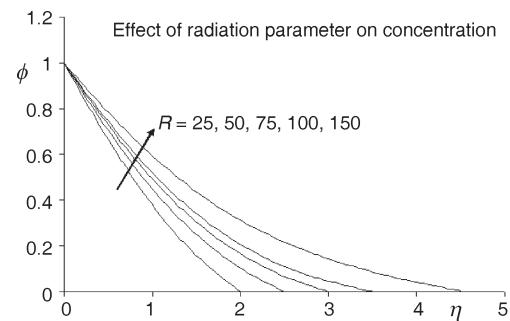


Figure 3. Concentration plotted against η for various values of R taking $M = 1$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

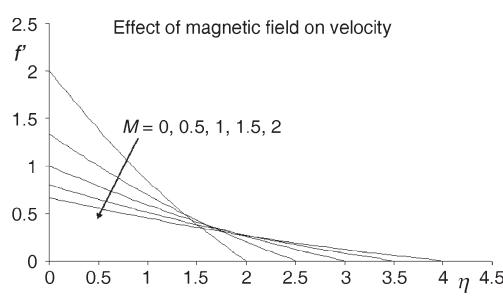


Figure 4. Velocity plotted against η for various values of M taking $R = 100$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

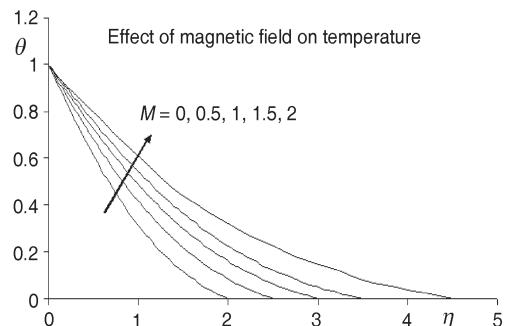


Figure 5. Temperature plotted against η for various values of M taking $R = 100$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

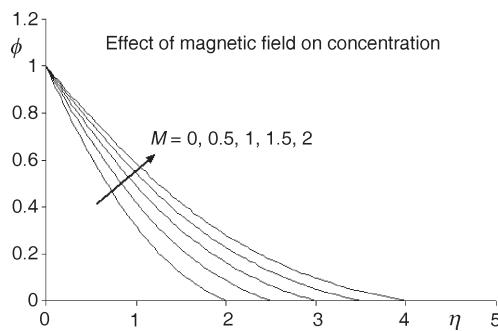


Figure 6. Concentration plotted against η for various values of M taking $R = 100$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

The present problem are the local Nusselt number and local Sherwood number, which are given by the expressions:

Table 1. Numerical values of local Nusselt and Sherwood numbers for various values of magnetic field parameter M taking parameters: $R = 100$, $Le = 1$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

M	$Nu_x / Ra_x^{1/2}$	$Sh_x / Ra_x^{1/2}$
0	-0.24041	-0.68907
0.5	-0.23516	-0.58777
1	-0.23168	-0.51281
1.5	-0.22957	-0.47084
2	-0.22792	-0.44154

mal radiation enhances the thermal state of the fluid causing its temperature and concentration to increase.

The effect of transverse magnetic field parameter M on the velocity, temperature, and concentration is displayed in figs. 4 to 6. It is observed from these figures that as M increases the thickness of velocity, thermal and concentration boundary layers increases. This result is in conformity with that of Postelnicu [9]. It may also be noted from fig. 4 that near the plate the velocity decreases with the increase in magnetic field parameter M whereas opposite phenomenon occurs near the boundary layer edge.

The parameters of engineering interest for

Tables 1 and 2 present the local Nusselt number and local Sherwood number calculated for different values of R and M , respectively, keeping other parameters fixed.

We observe from tab. 1 that the local Nusselt number and local Sherwood number decreases as magnetic field parameter M increases (if negative values are encountered absolute values are considered) which supports the results of earlier workers Cheng [6], Postelnicu [9], Chamka [11], and Chen [30].

It is observed from tab. 2 that the local Nusselt number increases and local Sherwood number decreases as radiation parameter R increases (if negative values are encountered absolute values are considered) confirming to Chamka [11], Prasad *et al.* [23], and Mohamed *et al.* [25].

To observe the effect of temperature ratio β on the local Nusselt number, numerical values are presented for various radiation parameter values in table 3. It is noted from tab. 3 that the local Nusselt number increases as β increases for a fixed R confirming to Mohamed *et al.* [25].

Figure 7 depicts the variation of the local Nusselt number with temperature ratio β for different values of R . The observation is obviously same as drawn from tab. 3.

The effect of Buoyancy parameter N has already been studied by Cheng [6], Postelnicu [9], and Chamka [11]. They observed that the local Nusselt number and local Sherwood number increases as the Boyuancy parameter N increases.

It may be finally concluded from the above numerical discussion that the velocity, thermal and concentration boundary layers thicknesses increase as R or M increases keeping

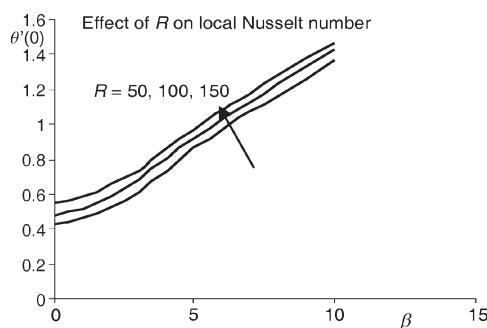


Figure 7. The local Nusselt numbers plotted against temperature ratio β for various values of radiation parameter R and $N = 1$, $D_f = 0.05$,

other parameters fixed. When M increases local Nusselt number and local Sherwood number decreases whereas when R increases local Nusselt number increases and local Sherwood number decreases other parameters being fixed. The local Nusselt number also increases when the temperature ratio β increases for fixed value of R .

Table 2. Numerical values of local Nusselt and Sherwood numbers for various values of radiation parameter R taking parameters: $M = 1.0$, $Le = 1$, $D_f = 0.05$, $N = 1$, and $S_r = 1.2$

R	$Nu_x/Ra_x^{1/2}$	$Sh_x/Ra_x^{1/2}$
25	-0.251161	-0.538404
50	-0.254346	-0.534746
75	-0.257391	-0.531063
100	-0.260016	-0.526963
500	-0.336588	-0.511413

Table 3. Numerical values of local Nusselt numbers for various values of R radiation parameter, β and $Le = 1$, $N = 1$, $D_f = 0.05$, and $S_r = 1.2$

Temperature ratio	$R = 50$	$R = 100$	$R = 150$
$\beta = 0$	0.425765	0.480617	0.556031
$\beta = 1$	0.465871	0.516440	0.578491
$\beta = 2$	0.524914	0.587326	0.662762
$\beta = 3$	0.611667	0.686399	0.737122
$\beta = 4$	0.733965	0.813123	0.862122
$\beta = 5$	0.861311	0.921311	0.966024
$\beta = 6$	0.980001	1.040044	1.093094
$\beta = 7$	1.075523	1.126336	1.182261
$\beta = 8$	1.163922	1.228992	1.292559
$\beta = 9$	1.267583	1.338294	1.383975
$\beta = 10$	1.366319	1.428948	1.471172

Nomenclature

C	- concentration, [molm^{-3}]	T_m	- mean fluid temperature, [K]
C_p	- specific heat at constant pressure, [JK^{-1}]	u, v	- Darcian velocities in the x, y -directions, respectively, [ms^{-1}]
C_s	- concentration susceptibility, [molm^{-3}]	x, y	- Cartesian co-ordinates, [m]
D_f	- Dufour number [= $(D_m k_T / \alpha_m C_s C_p)(C_w - C_\infty)(T_w - T_\infty)$], [-]		
D_m	- mass diffusivity, [m^2s^{-1}]		
f'	- velocity function, [-]		
g	- gravitational acceleration, [ms^{-2}]		
H_0	- magnetic field intensity, [Am^{-1}]		
K	- Darcy permeability, [Hm^{-1}]		
k_T	- thermal conductivity, [$\text{Wm}^{-1}\text{K}^{-1}$]		
Le	- Lewis number (= α_m/D_m), [-]		
M	- Magnetic field parameter (= $K\sigma\mu_e^2 H_0^2/\mu$), [-]		
N	- buoyancy parameter [= $(\beta_c/\beta_T)(C_w - C_\infty)(T_w - T_\infty)$], [-]		
Nu_x	- local Nusselt number {[= $-x/(T_w - T_\infty)$] ($y_{y=0}$)}, [-]		
q_r	- radiative heat flux, [mT^{-3}]		
R	- radiation parameter		
	[= $\kappa^* k_r / 4\sigma_B (T_w - T_\infty)^3$], [-]		
Ra_x	- local Rayleigh number [$Kg\beta_T(T_w - T_\infty)x/\nu\alpha_m$], [-]		
Sh_x	- local Sherwood number {[= $-x(C_w - C_\infty)(\partial C/\partial y)$] $_{y=0}$ }, [-]		
S_r	- Soret number [= $(D_m k_T / \alpha_m T_m) / (T_w - T_\infty) / (C_w - C_\infty)$], [-]		
T	- temperature of ambient medium, [K]		

References

- [1] Nield, D. A., Bejan, A., Convection in Porous Media, 2nd ed., Springer, New York, USA, 2006
 - [2] Ingham, D. , Pop, I., Transport Phenomena in Porous Media I, Pergamon press, Oxford, UK, 1998
 - [3] Ingham, D. , Pop, I., Transport Phenomena in Porous Media II, Pergamon press, Oxford, UK, 2002
 - [4] Vafai, K, Handbook of Porous Media, Taylor and Francis, Boca Raton, Fla., USA, 2005
 - [5] Chamkha, A. J., Similarity Solutions for Buoyancy Induced Flow of a Power-Law Fluid over a Horizontal Surface Immersed in a Porous Medium, *Int. Comm. Heat Mass Transfer*, 24 (1997), 6, pp. 805-814
 - [6] Cheng, C. Y., Effect of a Magnetic Field on Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media – An Integral Approach, *Int. Comm. Heat Mass Transfer*, 26 (1999), 7, pp. 935-943
 - [7] Eckert, F. R. G., Drake, R. M., Analysis of Heat and Mass Transfer, McGraw Hill, New York, USA, 1972
 - [8] Alam, M. S., *et al.*, Dufour and Soret Effects on Steady Free Convection and Mass Transfer Flow Past a Semi Infinite Vertical Porous Plate in a Porous Medium, *Int. J. of Applied Mechanics and Engineering*, 11 (2006), 3, pp. 535-545
 - [9] Postelnicu, A., Influence of a Magnetic Field on Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media Considering Soret and Dufour Effects, *Int. J. of Heat and Mass Transfer*, 47 (2004), 6-7, pp. 1467-1472
 - [10] Alam, M. S., Rehman, M. M. , Dufour and Soret Effects on Mixed Convection Flow Past a Vertical Porous Flat Plate with Variable Suction, *Nonlinear Analysis: Modelling and Control*, 11 (2006), 1, pp. 3-12

- [11] Chamkha, A. J., Ben-Nakhi, A., MHD Mixed Convection – Radiation Interaction Along a Permeable Surface Immersed in a Porous Medium in the Presence of Soret and Dufour’s Effects, *Heat and Mass Transfer*, 44 (2008), 7, pp. 845-856
- [12] Postelnicu, A., Influence of Chemical Reaction on Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media Considering Soret and Dufour’s Effects, *Heat and Mass Transfer*, 43 (2007), 6, pp. 595-602
- [13] Whitaker, S., Radiant Energy Transport in Porous Media, *Int. Engng. Chem. Fund.*, 19 (1980), 2, pp. 210-218
- [14] Chandrasekhara, B. C., Nagaraju, P., Composite Heat Transfer in the Case of a Steady Laminar Flow of a Gray Fluid with Small Optical Density Past a Horizontal Plate Embedded in a Saturated Porous Media, *Wärme-Stoffübertrag.*, 23 (1988), 6, pp. 343-352
- [15] Raptis, A., Radiation and Free Convection Flow through a Porous Medium, *Int. Comm Heat Mass Transfer*, 25 (1998), 2, pp. 289-295
- [16] Hossain, M. A., Pop, I., Radiation Effect on Darcy Free Convection Flow Along on Inclined Surface Placed in Porous Media, *Heat and Mass Transfer*, 32 (1997), 4, pp. 223-227
- [17] Chamkha, A. J., Solar Radiation Assisted Convection in Uniform Porous Medium Supported by a Vertical Flat Plate, *Trans. ASME. J. Heat Transfer*, 119 (1997a), 1, pp. 89-96
- [18] Chamkha, A. J., Issa, C., Khanafer, K., Natural Convection Due to Solar Radiation from a Vertical Plate Embedded in a Porous Medium with Variable Porosity, *J. Porous Media*, 4 (2001), 1, pp. 69-77
- [19] Bakier, A. Y., Thermal Radiation Effect on Mixed Convection from Vertical Surfaces in Saturated Porous Media, *Int. Comm. Heat Mass Transfer*, 28 (2001), 1, pp. 119-126
- [20] Murthy, P. V. S. N., Mukherjee, S., Srinivasacharya, D., Krishna, P. V. S. S. R., Combined Radiation and Mixed Convection from a Vertical Wall with Suction/Injection in a Non-Darcy Porous Medium, *Acta Mechanica*, 168 (2004a), 3-4, pp. 145-156
- [21] Grosan, T., Pop, I., A Note on the Effect of Radiation on Free Convection over a Vertical Flat Embedded in a Non-Newtonian Fluid Saturated Porous Medium, *Int. J. of Applied Mechanics and Engineering*, 11 (2006), 3, pp. 715-722
- [22] Muthucumaraswamy, R., Chandrakala, P., Antony Raj, S., Radiative Heat and Mass Transfer Effects on Moving Isothermal Vertical Plate in the Presence of Chemical Reaction, *Int. J. of Applied Mechanics and Engineering*, 11 (2006), 3, pp. 639-646
- [23] Ramachandra Prasad, V., Bhaskar Reddy, N., Muthucumaraswamy, R., Transient Radiative Hydromagnetic Free Convection Flow Past an Impulsively Started Vertical Plate with Uniform Heat and Mass Flux, *Theoret. Appl. Mech.*, 33 (2006), 1, pp. 31-63
- [24] Ramachandra Prasad, V., Bhaskar Reddy, N., Radiation and Mass Transfer Effects on an Unsteady MHD Free Convection Flow Past a Heated Vertical Plate in a Porous Medium with Viscous Dissipation, *Theoret. Appl. Mech.*, 34 (2007), 2, pp. 135-160
- [25] Mohamed, R. A., Mahdy, A., Hady, F. M., Combined Radiation and Free Convection from a Vertical Wavy Surface Embedded in Porous Media, *Int. J. of Applied Math and Mech.*, 4 (2008), 1, pp. 49-58
- [26] Brewster, M. Q., Thermal Radiative Transfer and Properties , John Wiley and Sons. Inc., New York, USA, 1992
- [27] Cheng, P., Mynkowycz, W. J., Free Convection about a Vertical Flat Plate Embedded in a Saturated Porous Medium with Application to Heat Transfer From a Dike, *J. Geophys. Res.*, 82 (1977), pp. 2040-2044
- [28] Keller, H. B., Numerical Methods for Two-Point Boundary Value Problems, Ginn-Blaisdell, Waltham, Mass., USA, 1968
- [29] Cebeci, T., Bradshaw, P., Momentum Transfer in Boundary Layer, Hemisphere Publishing Corporation, New York, USA, 1977, pp. 62-65
- [30] Chen, C. H., Heat and Mass Transfer in MHD Flow by Natural Convection from a Permeable, Inclined Surface with Variable Wall Temperature and Concentration, *Acta Mechanica*, 172 (2004), 3-4, pp. 219-235