INVESTIGATION OF THE IONIZED GAS FLOW ADJACENT TO POROUS WALL IN THE CASE WHEN ELECTROCONDUCTIVITY IS A FUNCTION OF THE LONGITUDINAL VELOCITY GRADIENT

by

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This paper studies the laminar boundary layer on a body of an arbitrary shape when the ionized gas flow is planar and steady and the wall of the body within the fluid porous. The outer magnetic field is perpendicular to the fluid flow. The inner magnetic and outer electric fields are neglected. The ionized gas electroconductivity is assumed to be a function of the longitudinal velocity gradient. Using transformations, the governing boundary layer equations are brought to a general mathematical model. Based on the obtained numerical solutions in the tabular forms, the behaviour of important non-dimensional quantities and characteristics of the boundary layer is graphically presented. General conclusions about the influence of certain parameters on distribution of the physical quantities in the boundary layer are drawn.

Key words: *boundary layer, ionized gas electroconductivity, porous wall, porosity parameter*

Introduction

The dissociated gas flow have been studied by various investigators like Dorrance [1], Loitsianskii [2, 3], Krivstova [4, 5], Saljnikov [6], and Obrović [7]. They performed a detailed investigation of the dissociated gas flow in the boundary and achieved significant results. Boričić *et al*. [8-10] and Ivanović [11] studied MHD boundary layer on a non-porous and porous contour of the body within the fluid and tried to find the so-called auto-model solution. The ionized gas flow in the boundary layer adjacent to both non-porous body [12, 13] and porous body [14-17] of an arbitrary shape were also studied for different electroconductivity variation laws.

This paper studies a complex ionized gas (air) flow in the boundary layer adjacent to the porous wall in the case when the electroconductivity is a function of the longitudinal velocity gradient.

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Mathematical formulation

At high gas flow velocities (*e. g.* supersonic flight of an aircraft through the Earth's atmosphere), the temperature in the viscous boundary layer increases significantly. At high temperatures ionization of gas (air) occurs together with dissociation. Because of this thermochemical reaction the gas becomes electroconductive. Then the gas (air) consists of positively charged ions, electrons, and atoms of oxygen and nitrogen. If the ionized gas flows in the magnetic field of the power $B_m = B_{my} = B_m(x)$, an electric current is formed in the gas, which causes appearance of the Lorentz force and the Joule's heat. Due to these effects, new terms, not found in the equations for homogenous unionized gas, appear in the equations of the ionized gas boundary layer.

This paper investigates the ionized gas flow when the outer magnetic field is perpendicular to the wall of the body within the fluid. The magnetic Reynolds number is considered very small. The ionized gas of the same physical characteristics as the gas in the main flow, is injected, *i. e.*, ejected perpendicularly to the porous wall with the velocity $v_w(x)$, According to [1], the complete governing equation system with the corresponding boundary conditions takes the following form:

$$\frac{\partial}{\partial x}(\rho u) \quad \frac{\partial}{\partial y}(\rho v) \quad 0 \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} \quad \rho v \frac{\partial u}{\partial y} \quad \frac{\mathrm{d}p}{\mathrm{d}x} \quad \frac{\partial}{\partial y} \quad \mu \frac{\partial u}{\partial y} \quad \sigma B_m^2 u \tag{2}$$

$$\rho u \frac{\partial h}{\partial x} \quad \rho v \frac{\partial h}{\partial y} \quad u \frac{\mathrm{d}p}{\mathrm{d}x} \quad \mu \quad \frac{\partial u}{\partial y}^{2} \quad \frac{\partial}{\partial y} \quad \frac{\mu}{\mathrm{Pr}} \frac{\partial h}{\partial y} \quad \sigma B_m^2 u^2 \tag{3}$$

$$\begin{array}{llll} u \quad 0, & v \quad v_w(x), & h \quad h_w & \text{for } y \quad 0 \\ u \quad u_e(x), & & h \quad h_e(x) & \text{for } y \quad \infty \end{array} \tag{4}$$

The ionized gas electroconductivity σ is assumed to be a function of the longitudinal velocity gradient:

$$\sigma \quad \sigma_0 \frac{v_0}{u_e^2} \frac{\partial u}{\partial y}, \quad (\sigma_0, v_0 \quad \text{const.}) \tag{5}$$

Based on the boundary conditions for the velocity and the density at the outer edge of the boundary layer:

$$u(x, y) \quad u_e(x), \quad \frac{\partial u}{\partial y} \quad 0 \;, \quad \rho \quad \rho_e \tag{6}$$

The pressure is eliminated from eqs. (2) and (3), and the following system is obtained:

$$\frac{\partial}{\partial x}(\rho u) \quad \frac{\partial}{\partial y}(\rho v) \quad 0 \tag{7}$$

$$\rho u \frac{\partial u}{\partial x} \quad \rho v \frac{\partial u}{\partial y} \quad \rho_e u_e \frac{\mathrm{d}u_e}{\mathrm{d}x} \quad \frac{\partial}{\partial y} \quad \mu \frac{\partial u}{\partial y} \quad \sigma B_m^2 u \tag{8}$$

$$\rho u \frac{\partial h}{\partial x} \quad \rho v \frac{\partial h}{\partial y} \quad u \rho_{\mathrm{e}} u_{\mathrm{e}} \frac{\mathrm{d}u_{\mathrm{e}}}{\mathrm{d}x} \quad \mu \quad \frac{\partial u}{\partial y}^{2} \quad \frac{\partial}{\partial y} \frac{\mu}{\mathrm{Pr}} \frac{\partial h}{\partial y} \quad \sigma B_m^2 u^2 \tag{9}$$

The boundary conditions remain unchanged.

Transformation of the variables

In order to apply the general similarity method, instead of physical coordinates x and y, new transformations [3]:

$$s(x) \quad \frac{1}{\rho_0 \mu_0} \, _{0}^{x} \rho_{\mathrm{w}} \mu_{\mathrm{w}} \mathrm{d}x, \quad z(x, y) \quad \frac{1}{\rho_0} \, _{0}^{y} \mathrm{d}y \tag{10}$$

and the stream function $\psi(s, z)$ are introduced:

$$u \quad \frac{\partial \psi}{\partial z}, \quad \tilde{v} \quad \frac{\rho_0 \mu_0}{\rho_{\mathrm{w}} \mu_{\mathrm{w}}} \quad u \frac{\partial z}{\partial x} \quad v \frac{\partial}{\partial_0} \qquad \frac{\partial \psi}{\partial s} \tag{11}$$

The quantities ρ_0 and μ_0 denote the known values of the density and the dynamic viscosity of the ionized gas at a concrete point.

By means of the transformations (10) and (11), the governing equation system together with the boundary conditions comes to:

$$\frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial s \partial z} \quad \frac{\partial^2 \psi}{\partial z} \frac{\partial^2 \psi}{\partial z^2} \quad \frac{\rho_{\mathrm{e}}}{\rho} u_{\mathrm{e}} \frac{\mathrm{d}u_{\mathrm{e}}}{\mathrm{d}s} \quad \nu_0 \frac{\partial}{\partial z} \quad Q \frac{\partial^2 \psi}{\partial z^2} \quad \frac{\rho_0 \mu_0}{\rho_{\mathrm{w}} \mu_{\mathrm{w}}} \frac{\sigma_0 B_m^2}{\rho_0} \frac{\nu_0}{u_e^2} \frac{\partial^2 \psi}{\partial z^2} \frac{\partial \psi}{\partial z} \tag{12}$$

$$\frac{\partial \psi}{\partial z} \frac{\partial h}{\partial s} \quad \frac{\partial \psi}{\partial s} \frac{\partial h}{\partial z} \quad \frac{\rho_{\mathrm{e}}}{\rho} u_{\mathrm{e}} \frac{\mathrm{d}u_{\mathrm{e}}}{\mathrm{d}s} \frac{\partial \psi}{\partial z} \quad \nu_0 Q \quad \frac{\partial^2 \psi}{\partial z^2}^{2} \quad \nu_0 \frac{\partial}{\partial z} \frac{Q}{\mathrm{Pr}} \frac{\partial h}{\partial z}$$
$$+ \frac{\rho_0 \mu_0}{\rho_{\mathrm{w}} \mu_{\mathrm{w}}} \frac{\sigma_0 B_m^2}{\rho_0} \frac{\nu_0}{u_e^2} \frac{\partial^2 \psi}{\partial z^2} \frac{\partial \psi}{\partial z}, \quad Q \quad \frac{\rho \mu}{\rho_{\mathrm{w}} \mu_{\mathrm{w}}} \tag{13}$$

$$\frac{\partial \psi}{\partial z} \quad 0, \qquad \frac{\partial \psi}{\partial} \quad \frac{\mu_0}{\mu_{\mathrm{w}}} v_{\mathrm{w}} \quad \tilde{v}_{\mathrm{w}}, \quad h \quad h_{\mathrm{w}} \quad \text{for} \quad z \quad 0$$
$$\frac{\partial \psi}{\partial z} \quad u_{\mathrm{e}}(s), \qquad h \quad h_{\mathrm{e}}(s) \quad \text{for} \quad z \quad \infty \tag{14}$$

In order to solve the system (12)-(14), the momentum equation is derived:

$$\frac{\mathrm{d}Z^{**}}{\mathrm{d}s} \quad \frac{F_{\mathrm{mp}}}{u_{\mathrm{e}}} \tag{15}$$

While deriving the momentum eq. (15), the usual quantities in the boundary layer theory are introduced:

$$F_{\mathrm{mp}} = 2[\zeta - (2 + H)f] + g - 2\Lambda \tag{16}$$

$$H \quad \frac{\Delta^*}{\Delta^{**}}; \quad \Delta^*(s) \quad _{0}^{\infty} \frac{\rho_{\mathrm{e}}}{\rho} \quad \frac{u}{u_{\mathrm{e}}} \mathrm{d}z, \quad \Delta^{**}(s) \quad _{0}^{\infty} \frac{u}{u_{\mathrm{e}}} \, 1 \quad \frac{u}{u_{\mathrm{e}}} \mathrm{d}z \tag{17}$$

$$f(s) \quad f_1(s) \quad u_{\mathrm{e}} Z^{**}, \quad Z^{**} \quad \frac{\Delta^{**2}}{\nu_0} \tag{18}$$

$$g(s) \quad g_1(s) \quad u_e^{1} N_\sigma \sqrt{\nu_0 Z^{**}} \tag{19}$$

$$N_\sigma \quad \frac{\rho_0 \mu_0}{\rho_{\rm w} \mu_{\rm w}} N, \quad N \quad \frac{\sigma_0 B_m^2}{\rho_0} \tag{20}$$

$$\tau_{\rm w}(s) \quad \mu \frac{\partial u}{\partial y}\bigg|_{y \quad 0} \quad \frac{\rho_{\rm w} \mu_{\rm w}}{\rho_0} \frac{u_{\rm e}}{\Delta^{**}} \zeta, \quad \zeta(s) \quad \frac{\partial \dfrac{u}{u_e}}{\partial \dfrac{z}{\Delta^{**}}}\Bigg|_{z \quad 0} \tag{21}$$

In the momentum equation $\Lambda(s)$ is the porosity parameter, and it is:

$$\Lambda \quad \frac{\mu_0}{\mu_{\rm w}} \frac{v_{\rm w} \Delta^{**}}{\nu_0} \quad \frac{V_{\rm w} \Delta^{**}}{\nu_0} \quad \Lambda(s) \tag{22}$$

where $V_{\rm w}$ is a conditional transversal velocity at the inner edge of the boundary layer of the porous wall of the body within the fluid.

For the used electroconductivity variation law, in order to apply the general similarity method, the boundary conditions and the stream function on the wall of the body within the fluid should remain the same as with the non-porous wall. For that reason, a new stream function is introduced $\psi^*(s, z)$ by the relation:

$$\psi(s, z) = \psi_{\rm w}(s) + \psi^*(s, z), \quad \psi^*(s, 0) = 0 \tag{23}$$

where $\psi(s, 0) = \psi_{\rm w}(s)$ denotes the stream function of the flow adjacent to the wall of the body within the fluid.

Applying the relation (23), the system (12)-(14) is transformed into:

$$\frac{\partial \psi^*}{\partial z} \frac{\partial^2 \psi^*}{\partial s \partial z} \quad \frac{\partial \psi^*}{\partial s} \frac{\partial^2 \psi^*}{\partial z^2} \quad \frac{d\psi_{\rm w}}{ds} \frac{\partial^2 \psi^*}{\partial z^2} \quad \frac{\rho_{\rm e}}{\rho} u_{\rm e} u_e \quad \nu_0 \frac{\partial}{\partial z} \quad Q \frac{\partial^2 \psi^*}{\partial z^2}$$
$$\frac{\sigma_0 B_m^2}{\rho_0} \frac{\nu_0}{u_e^2} \frac{\rho_0 \mu_0}{\rho_{\rm w} \mu_{\rm w}} \frac{\partial^2 \psi^*}{\partial z^2} \frac{\partial \psi^*}{\partial z} \tag{24}$$

$$\frac{\partial \psi^*}{\partial z} \frac{\partial h}{\partial s} \quad \frac{\partial \psi^*}{\partial s} \frac{\partial h}{\partial z} \quad \frac{{\rm d}\psi_w}{{\rm d}s} \frac{\partial h}{\partial z} \quad \frac{\rho_{\rm e}}{\rho} u_{\rm e} u_{\rm e} \frac{\partial \psi^*}{\partial z} \quad \nu_0 Q \quad \frac{\partial^2 \psi^*}{\partial z^2} \quad ^2$$
$$\nu_0 \frac{\partial}{\partial z} \quad \frac{Q}{\Pr} \frac{\partial h}{\partial z} \quad \frac{\sigma_0 B_{\rm m}^2}{\rho_0} \frac{\nu_0}{u_e^2} \frac{\rho_0 \mu_0}{\rho_{\rm w} \mu_{\rm w}} \frac{\partial^2 \psi^*}{\partial z^2} \quad \frac{\partial \psi^*}{\partial z} \quad ^2 \tag{25}$$

$$\psi^* \quad 0 \quad \frac{\partial \psi^*}{\partial z} \quad 0 \qquad h \quad h_{\rm w} \qquad \text{for} \quad z \quad 0$$
$$\frac{\partial \psi^*}{\partial z} \quad u_{\rm e}(s), \quad h \quad h_{\rm e}(s) \quad \text{for} \quad z \quad \infty \tag{26}$$

General mathematical model

In order to derive the generalized boundary layer equations it is necessary to introduce new transformations:

$$s \quad s, \quad \eta(s,z) \quad \frac{\sqrt{u_e^b}}{K(s)} z, \quad \psi^*(s,z) \quad \frac{u_e}{\sqrt{u_e^b}} K(s)\Phi[\eta,\kappa,(f_\mathrm{k}),(g_\mathrm{k}),(\Lambda_\mathrm{k})]$$
$$h(s,z) \quad h_1 \bar{h}[\eta,\kappa,(f_\mathrm{k}),(g_\mathrm{k}),(\Lambda_\mathrm{k})] \tag{27}$$

$$h_\mathrm{e} \quad \frac{u_e^2}{2} \quad h_1 = \text{const.}, \quad K(s) \quad \sqrt{a\nu_0 \int_0^s u_e^{b \; 1} \mathrm{d}s}, \quad a, b \quad \text{const.} \tag{28}$$

where $\eta(s, z)$ is the newly introduced transversal variable, Φ – the newly introduced stream function, and $\bar{h}$ – the non-dimensional enthalpy.

Some important quantities and characteristics of the boundary layer (16)-(21) can be written in the form of more suitable relations:

$$u \quad u_\mathrm{e} \frac{\partial \Phi}{\partial \eta} \tag{29}$$

$$\Delta^{**}(s) \quad \frac{K(s)}{\sqrt{u_e^b}} B(s), \quad B(s) \quad \int_0^\infty \frac{\partial \Phi}{\partial \eta} \; 1 \quad \frac{\partial \Phi}{\partial \eta} \; \mathrm{d}\eta \tag{30}$$

$$\frac{\Delta^*(s)}{\Delta^{**}(s)} \quad H \quad \frac{A(s)}{B(s)}, \quad A(s) \quad \int_0^\infty \frac{\rho_\mathrm{e}}{\rho} \quad \frac{\partial \Phi}{\partial \eta} \; \mathrm{d}\eta \tag{31}$$

$$\zeta \quad B \; \frac{\partial^2 \Phi}{\partial \eta^2}_{\eta \; 0} \tag{32}$$

$$\frac{f}{B^2} \quad \frac{a u_e}{u_e^b} \int_0^s u_e^{b \; 1} \mathrm{d}s \tag{33}$$

In the general similarity transformations (27), with the non-dimensional functions Φ and $\bar{h}$, a local parameter of the ionized gas compressibility $\kappa = f_0$, a set of the form parameters f_k [3], a set of magnetic parameters g_k, and a set of porosity parameters Λ_k [18] are introduced:

$$\kappa \quad f_0(s) \quad \frac{u_e^2}{2h_1} \tag{34}$$

$$f_\mathrm{k}(s) \quad u_e^{k \; 1} u_e^{(k)} Z^{**k}, \quad (k \quad 1, 2, 3, \ldots) \tag{35}$$

$$g_\mathrm{k}(s) \quad u_e^{k \; 2} N_\sigma^{(k \; 1)} \sqrt{\frac{\nu_0}{Z^{**}} Z^{**k}} \tag{36}$$

$$\Lambda_\mathrm{k}(s) \quad u_e^{k \; 1} \; \frac{V_w}{\sqrt{\nu_0}}^{(k \; 1)} \; \frac{Z^{**k}}{\sqrt{Z^{**}}} \tag{37}$$

They represent independent variables instead of the longitudinal variable s.

The local compressibility parameter $\kappa = f_0$ and the sets of parameters satisfy the following corresponding simple recurrent differential equations:

$$\frac{u_\mathrm{e}}{u_e} f_1 \frac{\mathrm{d}\kappa}{\mathrm{d}s} \quad 2\kappa f_1 \quad \theta_0 \tag{38}$$

$$\frac{u_e'}{u_e} f_1 \frac{\mathrm{d}f_k}{\mathrm{d}s} = [(k-1)f_1 + kF_{mp}]f_k + f_{k+1} \equiv \theta_k, \quad (k = 1,2,3,\ldots) \tag{39}$$

$$\frac{u_e'}{u_e} f_1 \frac{\mathrm{d}g_k}{\mathrm{d}s} = \left[(k-2)f_1 + \left(k - \frac{1}{2}\right) F_{mp}\right] g_k + g_{k+1} \equiv \gamma_k \tag{40}$$

$$\frac{u_e'}{u_e} f_1 \frac{\mathrm{d}\Lambda_k}{\mathrm{d}s} = \left[(k-1)f_1 + \frac{2k-1}{2} F_{mp}\right] \Lambda_k + \Lambda_{k+1} \equiv \chi_k \tag{41}$$

Applying the similarity transformations (27) and (34)-(37) the system (24)-(26) can be written as:

$$\frac{\partial}{\partial \eta}\left(Q \frac{\partial^2 \Phi}{\partial \eta^2}\right) + \frac{aB^2 + (2-b)f_1}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial \Phi}{\partial \eta}\right)^2\right] + \frac{g_1}{B^2} \frac{\partial^2 \Phi}{\partial \eta^2} \frac{\partial \Phi}{\partial \eta} -$$
$$- \frac{\Lambda_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} = \frac{1}{B^2}\left[\sum_{k=0}^{\infty} \theta_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial^2 \Phi}{\partial \eta^2}\right) + \sum_{k=1}^{\infty} \gamma_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial g_k} - \frac{\partial \Phi}{\partial g_k} \frac{\partial^2 \Phi}{\partial \eta^2}\right) +\right.$$
$$\left. + \sum_{k=1}^{\infty} \chi_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial \Lambda_k} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial^2 \Phi}{\partial \eta^2}\right)\right] \tag{42}$$

$$\frac{\partial}{\partial \eta}\left(\frac{Q}{\Pr} \frac{\partial \bar{h}}{\partial \eta}\right) + \frac{aB^2 + (2-b)f_1}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \frac{2\kappa f_1}{B^2} \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} + 2\kappa Q \left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)^2 + \frac{2\kappa g_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} \left(\frac{\partial \Phi}{\partial \eta}\right)^2 -$$
$$- \frac{\Lambda_1}{B} \frac{\partial \bar{h}}{\partial \eta} = \frac{1}{B^2}\left[\sum_{k=0}^{\infty} \theta_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial \bar{h}}{\partial \eta}\right) +\right.$$
$$\left. + \sum_{k=1}^{\infty} \gamma_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial g_k} - \frac{\partial \Phi}{\partial g_k} \frac{\partial \bar{h}}{\partial \eta}\right) + \sum_{k=1}^{\infty} \chi_k \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial \Lambda_k} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial \bar{h}}{\partial \eta}\right)\right] \tag{43}$$

The transformed boundary conditions are:

$$\Phi = \frac{\partial \Phi}{\partial \eta} = 0, \quad \bar{h} = \bar{h}_w = \text{const.} \quad \text{for} \quad \eta = 0$$
$$\frac{\partial \Phi}{\partial \eta} \to 1, \quad \bar{h} \to \bar{h}_e = 1 - \kappa \quad \text{for} \quad \eta \to \infty \tag{44}$$

Neither eqs. (42) and (43) nor the boundary conditions (44) contain the outer velocity of the boundary layer. Therefore, this equation system is generalized and it represents a general mathematical model of the ionized gas flow adjacent to the porous wall of the body within the fluid for the assumed electroconductivity variation law (5).

Numerical solution

When the generalized equation system (42)-(43) with the boundary conditions (44) is numerically solved, a finite number of parameters is adopted and the solution is obtained in *n*-parametric approximation. Due to many difficulties in solution of this equation system, it can be solved only with a relatively small number of parameters. If it is assumed that:

$$\kappa \quad f_0 \quad 0 \quad f_1 \quad f \quad 0, \quad g_1 \quad g \quad 0, \quad \Lambda_1 \quad \Lambda \quad 0 \tag{45}$$

$$f_2 \quad f_3 \quad \ldots \quad 0, \quad g_2 \quad g_3 \quad \ldots \quad 0, \quad \Lambda_2 \quad \Lambda_3 \quad \ldots \quad 0 \tag{46}$$

the obtained equation system is significantly simplified. Furthermore, when the general similarity method is applied, the so-called localization is performed. If we neglect derivatives per the compressibility, magnetic, and porosity parameters ($\kappa \quad / \; g = 0, \quad \Lambda$, the equation system (42)-(43) is significantly simplified, and in the four-parametric three times localized approximation, it has the following form:

$$\frac{\partial}{\partial\eta}\, Q\frac{\partial^2\Phi}{\partial\eta^2} \quad \frac{aB^2 \quad (2 \quad b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2} \quad \frac{f}{B^2}\,\frac{\rho_e}{\rho} \quad \frac{\partial\Phi}{\partial\eta}^{2} \quad \frac{g}{B^2}\frac{\partial^2\Phi}{\partial\eta^2}\frac{\partial\Phi}{\partial\eta}$$
$$\frac{\Lambda}{B}\frac{\partial^2\Phi}{\partial\eta^2} \quad \frac{F_{mp}f}{B^2} \quad \frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f} \quad \frac{\partial\Phi}{\partial f}\frac{\partial^2\Phi}{\partial\eta^2} \tag{47}$$

$$\frac{\partial}{\partial\eta} \quad \frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta} \quad \frac{aB^2 \quad (2 \quad b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta} \quad \frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta} \quad 2\kappa Q \quad \frac{\partial^2\Phi}{\partial\eta^2}^{2}$$
$$\frac{2\kappa g}{B}\frac{\partial^2\Phi}{\partial\eta^2} \quad \frac{\partial\Phi}{\partial\eta}^{2} \quad \frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta} \quad \frac{F_{mp}f}{B^2} \quad \frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f} \quad \frac{\partial\Phi}{\partial f}\frac{\partial\bar{h}}{\partial\eta} \tag{48}$$

The boundary conditions (44) remain unchanged.

In the equations of the system (47)-(48) the subscript 1 is left out in some parameters. Each of the equations contains a term that characterizes the porous wall of the body within the fluid.

For the numerical integration of the obtained system of differential partial equations of the third order, it is necessary to decrease the order of the differential equations. Using [6]:

$$\frac{u}{u_e} \quad \frac{\partial\Phi}{\partial\eta} \quad \varphi \quad \varphi(\eta, \kappa, f, g, \Lambda) \tag{49}$$

the order of the differential equations is decreased, so the system together with the boundary conditions comes to:

$$\frac{\partial}{\partial\eta}\, Q\frac{\partial\varphi}{\partial\eta} \quad \frac{aB^2 \quad (2 \quad b)f}{2B^2}\Phi\frac{\partial\Phi}{\partial\eta} \quad \frac{f}{B^2}\,\frac{\rho_e}{\rho} \quad \varphi^2$$
$$\frac{g}{B}\frac{\partial\varphi}{\partial\eta}\varphi \quad \frac{\Lambda}{B}\frac{\partial\varphi}{\partial\eta} \quad \frac{F_{mp}f}{B^2} \quad \varphi\frac{\partial\varphi}{\partial f} \quad \frac{\partial\Phi}{\partial f}\frac{\partial}{\partial\eta} \tag{50}$$

$$\frac{\partial}{\partial\eta} \quad \frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta} \quad \frac{aB^2 \quad (2 \quad b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta} \quad \frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\varphi \quad 2\kappa Q \quad \frac{\partial\varphi}{\partial\eta}^{2} \quad \frac{2\kappa g}{B}\frac{\partial\varphi}{\partial\eta}\varphi^2 \quad \frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta}$$
$$\frac{F_{mp}f}{B^2} \quad \varphi\frac{\partial\bar{h}}{\partial f} \quad \frac{\partial\Phi}{\partial f}\frac{\partial\bar{h}}{\partial\eta} \tag{51}$$

$$\begin{aligned} &\Phi \quad \varphi \quad 0, \quad \bar{h} \quad \bar{h}_w \quad \text{const.} \quad \text{for} \quad \eta \quad 0 \\ &\varphi \quad 1, \quad \bar{h} \quad \bar{h}_e \quad 1 \quad \kappa \quad \text{for} \quad \eta \quad \infty \end{aligned} \tag{52}$$

In order to solve the obtained system (50)-(52), it is necessary to determine the analytic forms of distribution of certain physical quantities that are themselves part of the equations. For the non-dimensional function Q [15] and the density ratio ρ_e/ρ [4], the following expressions are adopted:

$$Q \quad Q(\bar{h}) \quad \sqrt[3]{\frac{\bar{h}_w}{h}}, \quad \frac{\rho_e}{\rho} \quad \frac{\bar{h}}{1 \quad \kappa} \tag{53}$$

A concrete numerical solution of the obtained system of non-linear and conjugated differential partial equations (50)-(52) is performed using finite differences method, *i. e.*, "passage method" or TDA method. Based on the scheme of the plane integration grid [6], the system (50)-(52) is brought to the following system of linear algebraic equations:

$$a^i_{M,K\ 1}\varphi^i_{M\ 1,K\ 1} \quad 2b^i_{M,K\ 1}\varphi^i_{M,K\ 1} \quad c^i_{M,K\ 1}\varphi^i_{M\ 1,K\ 1} \quad g^i_{M,K\ 1} \tag{54}$$

$$a^j_{M,K\ 1}\bar{h}^j_{M\ 1,K\ 1} \quad 2b^j_{M,K\ 1}\bar{h}^j_{M,K\ 1} \quad c^j_{M,K\ 1}\bar{h}^j_{M\ 1,K\ 1} \quad g^j_{M,K\ 1} \tag{55}$$

$$M = 2, 3, ..., N-1; \quad K = 0, 1, 2, ... , \quad i, j = 0, 1, 2,...$$

$$\begin{array}{llll} \Phi^i_{1,K\ 1} \quad \varphi^i_{1,K\ 1} \quad 0, & \bar{h}^j_{1,K\ 1} \quad \bar{h}_w \quad \text{const.} & \text{for} & M \quad 1 \\ \varphi^i_{N,K+1} \quad 1, & \bar{h}^j_{N,K\ 1} \quad 1 \quad \kappa & \text{for} & M \quad N \end{array} \tag{56}$$

The coefficients $a^i_{M,K\ 1}$, $b^i_{M,K\ 1}$, $c^i_{M,K\ 1}$, and $g^i_{M,K\ 1}$ of the dynamic equation are determined with the expressions:

$$a^i_{M.K\ 1} \quad Q^{j\ 1}_{M,K\ 1} \quad \frac{1}{4}(Q^{j\ 1}_{M,K\ 1} \quad Q^{j\ 1}_{M\ 1,K\ 1}) \quad \frac{\Delta\eta}{2(B^{i\ 1}_{K\ 1})^2} \quad a(B^{i\ 1}_{K\ 1})^2 \quad (2 \quad b)f_{K\ 1} \quad \frac{\Phi^{i\ 1}_{M,K\ 1}}{2}$$
$$F^{i\ 1}_{mp,K\ 1}f_{K\ 1}\frac{\Phi^{i\ 1}_{M,K\ 1} \quad \Phi_{M,K}}{\Delta f} \quad \frac{\Delta\eta}{2}\frac{g}{B^{i\ 1}_{K\ 1}}\varphi^{i\ 1}_{M,K\ 1} \quad \frac{\Delta\eta}{2}\frac{\Lambda}{B^{i\ 1}_{K\ 1}} \tag{57}$$

$$b^i_{M,K\ 1} \quad Q^{j\ 1}_{M,K\ 1} \quad \frac{(\Delta\eta)^2}{2(B^{i\ 1}_{K\ 1})^2} \quad f_{K\ 1}\varphi^{i\ 1}_{M,K\ 1} \quad 1 \quad \frac{F^{i\ 1}_{mp,K\ 1}}{\Delta f} \tag{58}$$

$$c^i_{M,K\ 1} \quad Q^{j\ 1}_{M,K\ 1} \quad \frac{1}{4}(Q^{j\ 1}_{M,K\ 1} \quad Q^{j\ 1}_{M,K\ 1}) \quad \frac{\Delta\eta}{2(B^{i\ 1}_{K\ 1})^2} \quad a(B^{i\ 1}_{K\ 1})^2 \quad (2 \quad b)f_{K\ 1} \quad \frac{\Phi^{i\ 1}_{M,K\ 1}}{2}$$
$$F^{i\ 1}_{mp,K\ 1}f_{K\ 1}\frac{\Phi^{i\ 1}_{M,K\ 1} \quad \Phi_{M,K}}{\Delta f} \quad \frac{\Delta\eta}{2}\frac{g}{B^{i\ 1}_{K\ 1}}\varphi^{i\ 1}_{M,K\ 1} \quad \frac{\Delta\eta}{2}\frac{\Lambda}{B^{i\ 1}_{K\ 1}} \tag{59}$$

$$g^i_{M,K\ 1} \quad \frac{(\Delta\eta)^2}{(B^{i\ 1}_{K\ 1})} \quad f_{K\ 1}\frac{\bar{h}^j_{M,K\ 1}}{1 \quad \kappa} \quad F^{i\ 1}_{mp,K\ 1}f_{K\ 1}\varphi^{i\ 1}_{M,K\ 1}\frac{\varphi_{M,K}}{\Delta f} \tag{60}$$

For the thermodynamic equation, these coefficients are:

$$a^{j}_{M,K\ 1} \quad \frac{Q^{j\ 1}_{M,K\ 1}}{\Pr} \quad \frac{1}{4\Pr}(Q^{j\ 1}_{M\ 1,K\ 1} \quad Q^{j\ 1}_{M\ 1,K\ 1}) \quad \frac{\Delta\eta}{2(B^{i\ 1}_{K\ 1})^{2}}$$
$$[a(B^{i\ 1}_{K\ 1})^{2} \quad (2 \quad b)f_{K\ 1}]\frac{\Phi^{i\ 1}_{M,K\ 1}}{2}F^{i\ 1}_{mp,K\ 1}f_{K\ 1}\frac{\Phi^{i\ 1}_{M,K\ 1} \quad \Phi_{M,K}}{\Delta f} \quad \frac{\Delta\eta}{2}\frac{\Lambda}{B^{i\ 1}_{K\ 1}} \tag{61}$$

$$b^{j}_{M,K\ 1} \quad \frac{Q^{j\ 1}_{M,K\ 1}}{\Pr} \quad \frac{(\Delta\eta)^{2}}{2(B^{i\ 1}_{K\ 1})^{2}} \; f_{K\ 1}\varphi^{i\ 1}_{M,K\ 1} \; \frac{2\kappa}{1 \quad \kappa} \; \frac{F^{i\ 1}_{mp,K\ 1}}{\Delta f} \tag{62}$$

$$c^{j}_{M,K\ 1} \quad \frac{Q^{j\ 1}_{M,K\ 1}}{\Pr} \quad \frac{1}{4\Pr}(Q^{j\ 1}_{M\ 1,K\ 1} \quad Q^{j\ 1}_{M\ 1,K\ 1}) \quad \frac{\Delta\eta}{2(B^{i\ 1}_{K\ 1})^{2}}$$
$$[a(B^{i\ 1}_{K\ 1})^{2} \quad (2 \quad b)f_{K\ 1}]\frac{\Phi^{i\ 1}_{M,K\ 1}}{2}F^{i\ 1}_{mp,K\ 1}f_{K\ 1}\frac{\Phi^{i\ 1}_{M,K\ 1} \quad \Phi_{M,K}}{\Delta f} \quad \frac{\Delta\eta}{2}\frac{\Lambda}{B^{i\ 1}_{K\ 1}} \tag{63}$$

$$g^{j}_{M,K\ 1} \quad \frac{(\Delta\eta)^{2}}{(B^{i\ 1}_{K\ 1})^{2}}F^{i\ 1}_{mp,K\ 1}f_{K\ 1}\varphi^{i\ 1}_{M,K\ 1}\frac{\overline{h}_{M,K}}{\Delta f} \quad \frac{\kappa}{2}Q^{j\ 1}_{M,K\ 1}(\varphi^{i}_{M\ 1,K\ 1} \quad \varphi^{i}_{M\ 1,K\ 1})^{2}$$
$$\frac{\Delta\eta}{B^{i\ 1}_{K\ 1}}\kappa g(\varphi^{i}_{M\ 1,K\ 1} \quad \varphi^{i}_{M\ 1,K\ 1})(\varphi^{i\ 1}_{M,K\ 1})^{2} \tag{64}$$

From the algebraic eqs. (54)-(56), the following formulae are obtained :

$$\varphi^{i}_{N,K\ 1} \quad 1 \tag{65}$$

$$\varphi^{i}_{M,K\ 1} \quad K^{i}_{M,K\ 1} \quad L^{i}_{M,K\ 1}\varphi^{i}_{M\ 1,K\ 1} \tag{66}$$

$$\varphi^{i}_{1,K\ 1} \quad 0 \tag{67}$$

$$\overline{h}^{j}_{N,K\ 1} \quad 1 \quad \kappa \tag{68}$$

$$\overline{h}^{j}_{M,K\ 1} \quad K^{j}_{M,K\ 1} \quad L^{j}_{M,K\ 1}\overline{h}^{j}_{M\ 1,K\ 1} \tag{69}$$

$$\overline{h}^{j}_{1,K\ 1} \quad \overline{h}_{\mathrm{w}} \quad \text{const.}$$
$$M \quad N \quad 1, \quad N \quad 2,\ldots,3,2; \quad i,j \quad 1,2,3,\ldots \tag{70}$$

and they are used to calculate the values of the functions φ and $\overline{h}$ at discrete points in the direction of decrease of the subscript M.

In the formulae (65)-(70), the passage coefficients for the dynamic equations are:

$$K^{i}_{M,K\ 1} \quad \frac{a^{i}_{M,K\ 1}K^{i}_{M\ 1,K\ 1} \quad g^{i}_{M,K\ 1}}{2b^{i}_{M,K\ 1} \quad a^{i}_{M,K\ 1}L^{i}_{M\ 1,K\ 1}}, \quad K^{i}_{1,K\ 1} \quad \varphi^{i}_{1,K\ 1} \quad 0 \tag{71}$$

$$L^{i}_{M,K\ 1} \quad \frac{c^{i}_{M,K\ 1}}{2b^{i}_{M,K\ 1} \quad a^{i}_{M,K\ 1}L^{i}_{M\ 1,K\ 1}}, \quad L^{i}_{1,K\ 1} \quad 0 \tag{72}$$

These coefficients for the thermodynamic equation have the same form but they are essentially different:

$$K^{j}_{M,K\;\;1}\quad\frac{a^{j}_{M,K\;\;1}K^{j}_{M\;\;1,K\;\;1}\quad g^{j}_{M,K\;\;1}}{2b^{j}_{M,K\;\;1}\quad a^{j}_{M,K\;\;1}L^{j}_{M\;\;1,K\;\;1}},\quad K^{j}_{1,K\;\;1}\quad \bar{h}^{j}_{1,K\;\;1}\quad \bar{h}_{\mathrm{w}}\quad \text{const.} \tag{73}$$

$$L^{j}_{M,K\;\;1}\quad\frac{c^{j}_{M,K\;\;1}}{2b^{j}_{M,K\;\;1}\quad a^{j}_{M,K\;\;1}L^{j}_{M\;\;1,K\;\;1}},\quad L^{j}_{1,K\;\;1}\quad 0 \tag{74}$$

$$M\quad 2,3,\ldots,\;N\quad 2,N\quad 1$$

Based on the recurrent formulae (71)-(74), the passage coefficients in the direction of the increase of the subscript M are calculated. After all the discrete points of the calculating layer have been gone through twice, the solutions of the functions φ and $\bar{h}$ that correspond to that layer are calculated. The procedure is then repeated for all the calculating layers of the plane integration grid until the integration is performed in the whole range of the possible change of the parameter of the form f. Based on [13], the number of nodes is determined for each calculating layer as $N = 401$.

Prandtl number depends little on the temperature, therefore in this paper its value is considered to be constant and for air it is Pr = 0.712. According to [6], the optimal values for the constants a and b are: $a = 0.4408$, $b = 5.7140$.

Results

For the numerical solution of the equation system (50)-(52), a program in FORTRAN program language has been written. As the first derivative is neglected due to localization per the compressibility, porosity, and magnetic parameters, the program is designed to enable the solution of the equations for in advance given values of these now simple parameters. Numerical solutions are obtained in the output database in the tabular form.

The following results have been obtained.

Regardless of the fact whether the ionized gas is injected into the main flow or ejected from it, at different cross-sections of the boundary layer, the non-dimensional velocity u/u_{e} very quickly converges towards unity (fig. 1).

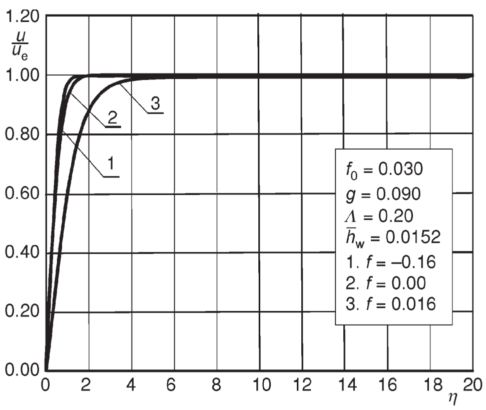

Figure 1. Diagram of the non-dimensional velocity u/u_{e}

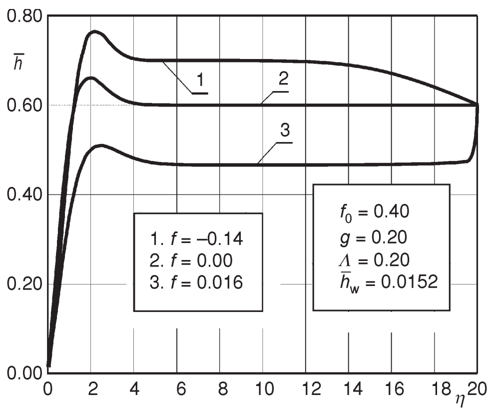

Figure 2. Distribution of the non-dimensional enthalpy h

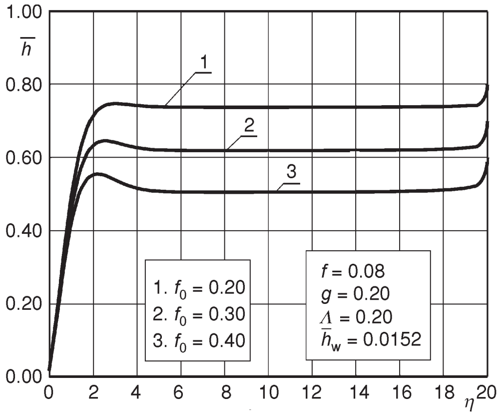

Figure 3. Distribution of the non-dimensional enthalpy for different values of the parameter f_0

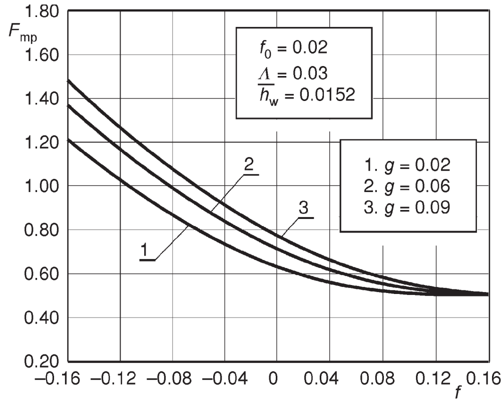

Figure 4. Distribution of the characteristic function F_{mp}

The compressibility parameter $\kappa = f_0$ has little influence on the corresponding distributions of the non-dimensional velocity.

In the presented (figs. 2 and 3) and other diagrams for distribution of the non-dimensional enthalpy we notice a great change of the enthalpy near the wall of the body within the fluid and near the outer edge of the boundary layer.

The change of the porosity parameter has a great influence on the distribution of the non-dimensional enthalpy $\bar{h}$ in the ionized gas boundary layer (fig. 3).

The magnetic field has a great influence on the characteristic of the boundary layer F_{mp} (fig. 4) and the non-dimensional friction function ζ. By increasing the values of the magnetic parameter, the separation of the boundary layer is postponed (fig. 5).

Based on the diagrams that are not presented here, it can be concluded that variation of the porosity parameter has little influence on the profiles of the non-dimensional velocities u/u_e.

The porosity parameter Λ has a great influence on the non-dimensional friction function ζ (fig. 6). Consequently, it also has a great influence on the boundary layer separation point.

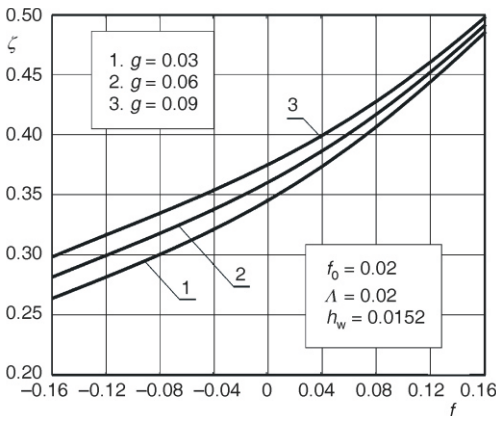

Figure 5. Distribution of the non-dimensional friction function $\zeta(g)$

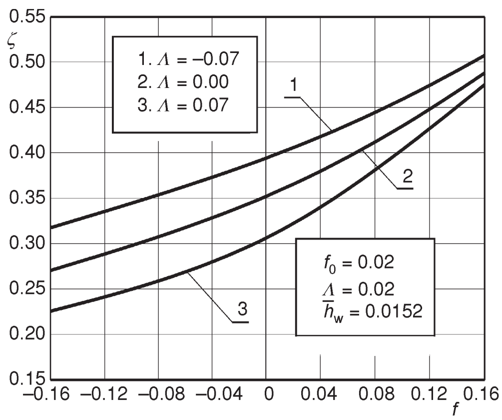

Figure 6. Distribution of the non-dimensional friction function $\zeta(\Lambda)$

It is noted that the injection of air postpones the separation of the ionized gas boundary layer because the separation point moves down the flow.

This parameter has a significant influence on the characteristic function of the boundary layer on the porous wall F_{mp}.

Conclusions

This paper studies the ionized gas planar steady flow in the boundary layer adjacent to the porous wall. The ionized gas of the same characteristics as the gas in the main current is injected *i. e.*, ejected perpendicularly to the wall. The outer magnetic field is perpendicular to the contour of the body. The gas electroconductivity is assumed a function of the longitudinal velocity gradient.

The aim of the investigation is to apply the general similarity method to the studied problem and solve the obtained equations. The governing equation system is transformed, brought to a general form, and then numerically solved by application of the finite differences method. However, the numerical solution is fraught with difficulties, mainly of mathematical nature, although there are some difficulties related to thermochemical and physical processes of the gas flow.

Complex fluid flow problems can be successfully solved using general similarity method. Distributions of the solutions of the ionized gas boundary layer equations for the used electroconductivity variation law are shown to be same as with other similar compressible fluid flow problems. Some new facts about the influence of the magnetic field and the porosity on the boundary layer separation have also been discovered. Important quality results here obtained enable an insight in the distribution of physical and characteristic quantities at different cross-sections of the boundary layer.

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Nomenclature

A, B – boundary layer characteristics, [–]
B_m – induction of outer magnetic field [$= B_m(x)$] [Vsm^{-2}]
a, b – constants, [–]
c_p – specific heat of ionized gas at constant pressure, [Jkg^{-1}K^{-1}]
F_{mp} – characteristic boundary layer function, [–]
f_1 – first form parameter ($= f$), [–]
f_k – set of form parameters, [–]
g_1 – first magnetic parameter ($= g$), [–]
g_k – set of magnetic parameters, [–]
H – boundary layer characteristic, [–]
h – enthalpy, [Jkg^{-1}]
$\bar{h}$ – non-dimensional enthalpy, [–]
h_e – enthalpy at the outer edge of the boundary layer, [Jkg^{-1}]
h_w – enthalpy at the wall of the body within the fluid, [Jkg^{-1}]
h_1 – enthalpy at the front stagnation point of the body within the fluid, [Jkg^{-1}]
i, j – iteration number, [–]
M – discrete point, [–]
Pr – Prandtl number ($= \mu c_p/\lambda$), [–]
p – pressure, [Pa]
Q – non-dimensional function, [–]
s – new longitudinal variable, [m]
u – longitudinal projection of velocity in the boundary layer, [ms^{-1}]
u_e – velocity at the boundary layer outer edge, [ms^{-1}]
V_w – conditional transversal velocity, [ms^{-1}]

v – transversal projection of velocity in the boundary layer, [ms^{-1}]
v_w – velocity of injection (or ejection) of the fluid, [ms^{-1}]
x, y – longitudinal and transversal coordinate, [m]
Z^{**} – function, [s]
z – new transversal variable, [m]

Greek symbols

Δ^* – conditional displacement thicknesses, [m]
Δ^{**} – conditional momentum loss thickness, [m]
ζ – non-dimensional friction function, [–]
η – non-dimensional transversal coordinate, [–]
κ – local compressibility parameter, (= f_0) [–]
Λ_1 – first porosity parameter (= Λ), [–]
Λ_k – set of porosity parameters, [–]
λ – thermal conductivity coefficient, [$Wm^{-1}K^{-1}$]
μ – dynamic viscosity, [Pa·s]
μ_0 – known values of dynamic viscosity of the ionized gas, [Pa·s]
μ_w – given distributions of dynamic viscosity at the wall of the body within the fluid, [Pa·s]
ν_0 – kinematic viscosity at a concrete point of the boundary layer, [m^2s^{-1}]
ρ – density of ionized gas, [kgm^{-3}]
ρ_e – ionized gas density at the outer edge of the boundary layer, [kgm^{-3}]
ρ_w – given distributions of density at the wall of the body within the fluid, [kgm^{-3}]
ρ_0 – known values of density of the ionized gas, [kgm^{-3}]
σ – electroconductivity, [$Nm^3V^{-2}s^{-1}$]
τ_w – shear stress at the wall of the body within the fluid, [Nm^{-2}]
Φ – non-dimensional stream function, [–]
ψ – stream function, [m^2s^{-1}]
ψ^* – new stream function, [m^2s^{-1}]

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