DIFFUSION AND HEAT TRANSFER EFFECTS ON EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE

by

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An exact solution of unsteady flow past an exponentially accelerated infinite vertical plate with variable temperature has been presented in the presence of uniform mass diffusion. The plate temperature is raised linearly with time and species concentration level near the plate is made to rise C_w. The dimensionless governing equations are solved using Laplace-transform technique. The velocity profiles fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, a and time. It is observed that the velocity increases with increasing values of a or t.

Key words: *accelerated, vertical plate, exponential, heat transfer, mass diffusion*

Introduction

A few representative fields of interest in which combined heat and mass transfer plays an important role, are filtration processes, the drying of porous materials in textile industries and the saturation of porous materials by chemicals, nuclear reactors, spacecraft design, solar energy collectors, design of chemical processing equipment and pollution of the environment.

A polymer or metal sheet extruded continuously from a die, or a long fiber or filament traveling between a feed roller and a take-up roller, are typical examples of an exponentially accelerated vertical plate. Heat transfer from a heated surface to a quiescent ambient medium occur in many manufacturing processes, the cooling of cylinders, threads or sheets of some polymer materials is of importance in the production line.

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al*. [1].

Kafousias *et al.* [2] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh *et al.* [3].

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The skin friction for accelerated vertical plate has been studied analytically by Hossain *et al.* [4]. Soundalgekar [5] was the first to present an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the usual Laplace-transform technique and the effects of heating or cooling of the plate on the flow-field were discussed through Grashof number.

Soundalgekar [6] studied the problem of the flow past an impulsively started isothermal vertical plate with mass transfer effects. The solution was also derived by the usual Laplace-transform technique. Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Jha *et al.* [7].

It is proposed to study the effects of on flow past an exponentially accelerated vertical plate in the presence of variable temperature and uniform mass diffusion.

The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

Mathematical analysis

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and uniform mass diffusion has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t'\quad 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate raised linearly with time t and the mass is diffused from the plate to the fluid uniformly. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t} \quad g\beta(T \quad T_\infty) \quad g\beta^*(C \quad C_\infty) \quad \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_\mathrm{P}\frac{\partial T}{\partial t} \quad k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C}{\partial t} \quad D\frac{\partial^2 C}{\partial y^2} \tag{3}$$

with the following initial and boundary conditions:

$$\begin{array}{llll} u \quad 0, & T \quad T_\infty, & C \quad C_\infty & \text{for all } y,\ t \quad 0 \\ t \ >0\!: u \quad u_0\exp(a\ t\ ), & T \quad T_\infty \quad (T_\mathrm{w} \quad T_\infty)At & C \quad C_\mathrm{w} & \text{at } y \quad 0 \\ u \quad 0 & T \quad T_\infty & C \quad C_0 & \text{as } y \quad \infty \end{array} \tag{4}$$

where $\mathrm{A} = u_0^2/\nu$.

On introducing the following non-dimensional quantities:

$$U \quad \frac{u}{u_0}, \quad t \quad \frac{t\,u_0^2}{\nu}, \quad Y \quad \frac{yu_0}{\nu}, \quad \theta \quad \frac{T \quad T_\infty}{T_\mathrm{w} \quad T_\infty}$$

$$\mathrm{Gr} \quad \frac{g\beta\nu(T \quad T_\infty)}{u_0^3}, \quad C \quad \frac{C \quad C_\infty}{C_\mathrm{w} \quad C_\infty}, \quad \mathrm{Gc} \quad \frac{\nu g\beta^*(C_\mathrm{w} \quad C_\infty)}{u_0^3} \tag{5}$$

$$\mathrm{Pr} \quad \frac{\mu C_p}{k}, \quad a \quad \frac{a\,\nu}{u_0^2}, \quad \mathrm{Sc} \quad \frac{\nu}{D}$$

in eqs. (1) to (4), leads to:

$$\frac{\partial U}{\partial t} \quad \mathrm{Gr}\theta \quad \mathrm{Gc}C \quad \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} \quad \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} \quad \frac{1}{\mathrm{Sc}}\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are:

$$\begin{aligned} &U \quad 0, \quad \theta \quad 0, \quad C \quad 0, \quad \text{for all } Y, t \quad 0 \\ t>0:\ &U = \exp(at), \quad \theta = t, \quad C \quad 1, \quad \text{at} \quad Y \quad 0 \\ &U \quad 0, \quad \theta \quad 0, \quad C \quad 0 \quad \text{as} \quad Y \quad \infty \end{aligned} \tag{9}$$

The dimensionless governing eqs. (6) to (8) with the initial and boundary conditions (9) are tackled using Laplace transform technique:

$$\theta \quad t\ (1 \quad 2\eta^2\,\mathrm{Pr})\mathrm{erfc}(\eta\sqrt{\mathrm{Pr}}) \quad \frac{2}{\sqrt{\pi}}\eta\sqrt{\mathrm{Pr}}\exp(\quad \eta^2\,\mathrm{Pr}) \tag{10}$$

$$C \quad \mathrm{erfc}(\eta\sqrt{\mathrm{Sc}}) \tag{11}$$

$$\begin{aligned} U \quad &\frac{\exp(at)}{2}[\exp(2\eta\sqrt{at})\mathrm{erfc}(\eta \quad \sqrt{at}) \quad \exp(\quad 2\eta\sqrt{at})\mathrm{erfc}(\eta \quad \sqrt{at})] \\ &\frac{\mathrm{Gr}t^2}{6(\mathrm{Pr} \quad 1)}\ (3 \quad 12\eta^2 \quad 4\eta^4)\mathrm{erfc}(\eta) \quad \frac{\eta}{\sqrt{\pi}}(10 \quad 4\eta^2)\exp(\quad \eta^2) \\ (3 \quad 12\eta^2\,\mathrm{Pr} \quad &4\eta^4(\mathrm{Pr})^2)\mathrm{erfc}(\eta\sqrt{\mathrm{Pr}}) \quad \frac{\eta\sqrt{\mathrm{Pr}}}{\sqrt{\pi}}(10 \quad 4\eta^2\,\mathrm{Pr})\exp(\quad \eta^2\,\mathrm{Pr}) \\ &\frac{\mathrm{Gc}t}{\mathrm{Sc} \quad 1}\ (1 \quad 2\eta^2\mathrm{erfc}(\eta) \quad \frac{2\eta}{\sqrt{\pi}}\exp(\quad \eta^2) \\ &(1 \quad 2\eta^2\mathrm{Sc})\mathrm{erfc}(\eta\sqrt{\mathrm{Sc}}) \quad \frac{2\eta\sqrt{\mathrm{Sc}}}{\sqrt{\pi}}\exp(\quad \eta^2\mathrm{Sc}) \end{aligned} \tag{12}$$

where, $\eta = Y/2\sqrt{t}$.

Results and discussion

For physical understanding of the problem numerical computations are carried out for different physical parameters a, Gr, Gc, Sc, and t upon the nature of the flow and transport. The value of the Schmidt number (Sc) is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number (Pr) is chosen such that they represent air (Pr = 0.71). The numerical values of the velocity are computed for different physical parameters like a, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity profiles for different (a = 0.2, 0.5, 0.8), Gr = Gc = 5, Sc = 0.6, Pr = 0.71, and t = 0.2 and 0.4 are studied and presented in fig. 1. It is observed that the velocity increases with increasing values of a. The trend is also same for varying time t. Figure 2 demonstrates the effects of different thermal Grashof number (Gr = 2, 10) and mass Grashof number (Gc = 2,5) on the velocity when a = 0.5, Pr = 0.71, Sc = 0.6, and t = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

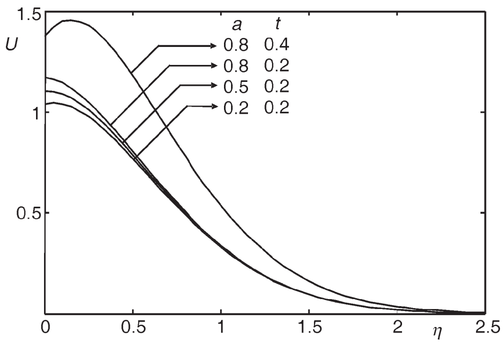

Figure 1. Velocity profiles for different *a* and *t*

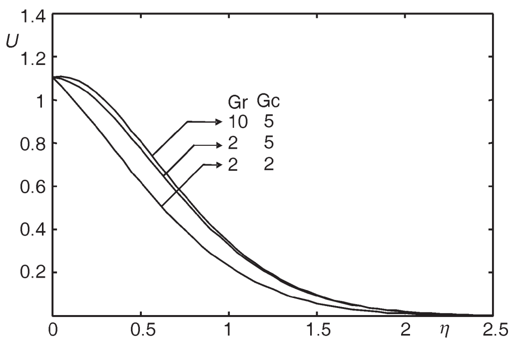

Figure 2. Velocity profiles for different Gr and GC

The effect of velocity for different t values of the Schmidt number (Sc = 0.16, 0.78, 2) and time t = 0.2 are shown in fig. 3. The trend shows that the velocity increases with decreasing Schmidt number or time. It is observed that the relative variation of the velocity with the magnitude of the time and the Schmidt number.

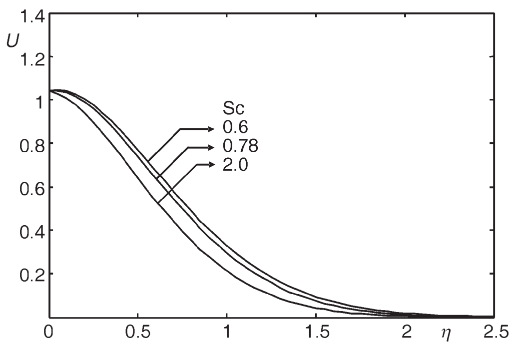

Figure 3. Velocity profiles for different Sc

Conclusions

The theoretical solution of flow past an exponentially accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion have been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number, Schmidt number, *a*, and *t* are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc, *a*, and *t*. But the trend is just reversed with respect to the Schmidt number.

Nomenclature

C' – species concentration in the fluid, [kgm^{-3}]
C – dimensionless concentration
Cp – specific heat at constant pressure, [Jkg^{-1}K]
D – mass diffusion coefficient, [m^{2}s^{-1}]
erfc – complementary error function
Gc – mass Grashof number
Gr – thermal Grashof number
g – acceleration due to gravity, [ms^{-2}]
k – thermal conductivity, [Wm^{-1}K^{-1}]
Pr – Prandtl number, [–]
Sc – Schmidt number, [–]
T – temperature of the fluid near the plate, [K]
t' – time, [s]
U – dimensionless velocity
u – velocity component in x'-direction, [ms^{-1}]
u_0 – velocity of the plate, [ms^{-1}]
Y – dimensionless spatial coordinate normal to the plate
y – coordinate axis normal to the plate, [m]

Greek letters

α – thermal diffusivity, [m^{-2}s^{-1}]
β – volumetric coefficient of thermal expansion, [K^{-1}]
β^* – volumetric coefficient of expansion with concentration, [K^{-1}]
η – similarity parameter
θ – dimensionless temperature
μ – coefficient of viscosity, [Ra s]
ν – kinematic viscosity, [m^{2}s^{-1}]
ρ – density of the fluid, [kgm^{-3}]
τ – dimensionless skin-friction, [kgm^{-1}s^2]

Subscripts

w – conditions on the wall
∞ – free stream conditions

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