# MAXIMUM WORK OUTPUT OF MULTISTAGE CONTINUOUS CARNOT HEAT ENGINE SYSTEM WITH FINITE RESERVOIRS OF THERMAL CAPACITY AND RADIATION BETWEEN HEAT SOURCE AND WORKING FLUID

by

***Jun LI, Lingen CHEN*, and Fengrui SUN***

Postgraduate School, Naval University of Engineering, Wuhan, P. R. China

Original scientific paper
UDC: 536.33:66.011
DOI: 10.2298/TSCI1001001L

*Optimal temperature profile for maximum work output of multistage continuous Carnot heat engine system with two reservoirs of finite thermal capacity is determined. The heat transfer between heat source and the working fluid obeys radiation law and the heat transfer between heat sink and the working fluid obeys linear law. The solution is obtained by using optimal control theory and pseudo-Newtonian heat transfer model. It is shown that the temperature of driven fluid monotonically decreases with respect to flow velocity and process duration. The maximum work is obtained. The obtained results are compared with those obtained with infinite low temperature heat sink.*

Key words: *radiation heat transfer, finite reservoir, multistage continuous Carnot heat engine, maximum work output, optimal control*

## Introduction

Since finite time thermodynamics and entropy generation minimization have been advanced, much work has been carried out on the performance analysis and optimization of finite time processes and finite size devices [1-14]. Rubin [15, 16] analyzed the optimal configuration of endoreversible heat engines with Newton heat transfer law [$q \quad \Delta(T)$] and different constraints, and derived the optimal configuration of the engines. Badescu [17] studied the optimal heating paths with Newton and radiative heat transfer laws by taking the minimum entropy generation and minimum lost available work as the objectives. The obtained analytical expressions of the optimal paths were presented in dimensionless forms. The similarities and differences between various heating strategies under the two heat transfer laws were compared. Amelkin *et al.* [18, 19] discussed the maximum power processes of multi-heat-reservoir heat engine with stationary temperature reservoirs, found that some reservoirs were not used in heat transfer in order to achieve an optimal performance of the system, and further found that independent of the number of reservoirs the working fluid used only two isotherms and two adiabatics. Song *et al.* [20]

* Corresponding author; e-mail: lgchenna@yahoo.com, lingenchen@hotmail.com

obtained the optimal configurations of endoreversible heat engine for maximum power and maximum efficiency with linear phenomenological heat transfer law. Song *et al.* [21] further obtained the optimal configuration of endoreversible heat engine for maximum power with radiative heat transfer. Sieniutycz *et al*. [22] obtained the extremal work and optimal temperature profile of multistage endoreversible continuous heat engine and heat pump systems with one reservoir of finite thermal capacity. Sieniutycz [23] further obtained those of multistage endoreversible discrete heat engine and heat pump systems with one reservoir of finite thermal capacity. Sieniutycz *et al.* [24] and Kuran [25] established seudo-Newtonian heat transfer model by using optimal control theory, and obtained the optimal temperature profile and extremal work of multistage continuous irreversible Carnot heat engine and heat pump systems with one reservoir of finite thermal capacity, in which the heat transfer between heat source and the working fluid obeys radiation law and the heat transfer between working fluid and the heat sink obeys linear law. Sieniutycz [26, 27] further given Hamilton-Jacobi-Bellman equations for calculating extremal work with non-Newtonian heat transfer. In this paper, the maximum work and optimal temperature profile of continuous multistage Carnot heat engine system with two reservoirs of finite thermal capacity, in which the heat transfer between heat source and the working fluid obeys radiation law and the heat transfer between the working fluid and heat sink obeys linear law, are obtained by using optimal control theory based on refs. [24-27].

## System model

The system model is shown in fig. 1.

The first fluid (driving fluid) and second fluid flow along the x-axis, the infinitesimal Carnot heat engines are located continuously between two separated boundary layers of the fluids. Each infinitesimal Carnot heat engine is the same. The driving fluid supplies the pure heat to the infinitesimal Carnot engine at a high temperature $T = T_1$ and releases the pure heat to the second fluid at a low temperature $T = T_2$. The cumulative work is delivered at the last stage in a finite time. The volume flux of the first and second fluids are $\dot{V}_1$ and $\dot{V}_2$, respectively. The heat transfer between the first fluid and the infinitesimal Carnot engine obeys radiation law and the heat transfer between the second fluid and the infinitesimal Carnot engine obeys linear law. $T_{1C}$ and $T_{2C}$ are upper and lower temperatures of the working fluid circulating in each infinitesimal Carnot engine.

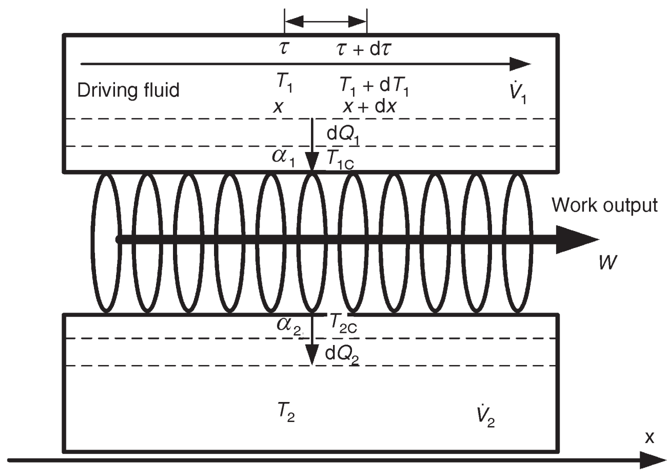


**Figure 1. Work production model of multi-stage continuous heat engine system**

Based on heat transfer theory [28] and pseudo--Newtonian heat transfer model [24-27], the heat flux between the first fluid and each engine is:

$$\mathrm{d}Q_1 = \mathrm{d}\gamma_1(T_1 - T_{1\mathrm{C}}) \tag{1}$$

where $\mathrm{d}\gamma_1 = \alpha_1(T_1^3)\mathrm{d}A_1$ is the heat conductivity between engines and the first fluid, $\alpha_1(T_1^3)$ – the heat transfer coefficient, and $\mathrm{d}A_1$ – the corresponding exchange surface area. The heat flux between the second fluid and each engine is:

$$\mathrm{d}Q_2 = \mathrm{d}\gamma_2(T_{2\mathrm{C}} - T_2) \tag{2}$$

where $\mathrm{d}\gamma_2 = \alpha_2\mathrm{d}A_2$ is the heat conductivity between engines and the second fluid, $\alpha_2$ – the heat transfer coefficient, and $\mathrm{d}A_2$ – the corresponding exchange surface area. From eq. (1), one can obtain:

$$T_{1\mathrm{C}} \quad T_1 \quad \frac{\mathrm{d}Q_1}{\mathrm{d}\gamma_1} \tag{3}$$

From the entropy balance of the working fluid, one can obtain:

$$\frac{\mathrm{d}Q_1}{T_{1\mathrm{C}}} \quad \frac{\mathrm{d}Q_2}{T_{2\mathrm{C}}} \tag{4}$$

Substituting eqs. (1)-(3) into eq. (4) yields:

$$T_{2\mathrm{C}} \quad \frac{T_2 \;\; T_1 \quad \dfrac{\mathrm{d}Q_1}{\mathrm{d}\gamma_1}}{T_1 \quad \dfrac{\mathrm{d}Q_1}{\mathrm{d}\gamma_1} \quad \dfrac{\mathrm{d}Q_1}{\mathrm{d}\gamma_2}} \tag{5}$$

Substituting eqs. (3) and (5) into $\eta = 1 - (T_{2\mathrm{C}}/T_{1\mathrm{C}})$ yields:

$$\eta \quad 1 \quad \frac{T_2}{T_1 \quad \dfrac{\mathrm{d}Q_1}{\mathrm{d}\gamma_1} \quad \dfrac{\mathrm{d}Q_1}{\mathrm{d}\gamma_2}} \tag{6}$$

Defining the overall heat conductivity as:

$$\mathrm{d}\gamma \quad \frac{\mathrm{d}\gamma_1\mathrm{d}\gamma_2}{\mathrm{d}\gamma_1 \quad \mathrm{d}\gamma_2} \quad \frac{\alpha_1 k\mathrm{d}A\alpha_2(1 \quad k)\mathrm{d}A}{\alpha_1 k\mathrm{d}A \quad \alpha_2(1 \quad k)\mathrm{d}A} \quad \frac{\alpha_1\alpha_2}{\alpha_1 \quad \alpha_2}\mathrm{d}A \quad \alpha\,(T_1)\mathrm{d}A \tag{7}$$

where $\alpha_1 = k\alpha_1$, $\alpha_2 = (1 - k)\alpha_2$, $A = A_1 + A_2$, and $k = A_1/A$. $A_1$ and $A_2$ are cumulative heat exchange surface areas between engines and two heat reservoirs, respectively. Then eq. (6) can be rewritten as:

$$\eta \quad 1 \quad \frac{T_2}{T_1 \quad \dfrac{\mathrm{d}Q_1}{\mathrm{d}\gamma}} \tag{8}$$

According to the conservation of energy $\mathrm{d}\dot{W} = \eta\mathrm{d}Q_1 + \dot{V}_1\mathrm{d}P$, one can obtain:

$$d\dot{W} \quad 1 \quad \frac{T_2}{T_1 \quad \dfrac{\mathrm{d}Q_1}{\mathrm{d}\gamma}} \;\; \mathrm{d}Q_1 \quad \dot{V}_1\mathrm{d}P \tag{9}$$

where $\mathrm{d}\dot{W}$ is the power output of infinitesimal Carnot heat engine, $P = aT_1^4/3$ – the radiation pressure, $a = 4\sigma/\mathrm{c}$ – the universal coefficient related to the Stefan-Boltzmann constant, and c – the light speed. For the first fluid, one has $\mathrm{d}Q_1 = \dot{V}_1 c_v(T_1)\mathrm{d}T_1$, where $c_\mathrm{v}(T_1) = 4aT_1^3$ is the specific heat capacity of constant volume of the first fluid, and $\mathrm{d}T_1$ – the differential temperature change of the driving fluid; the temperature of the driving fluid decreases slightly along its path, *i. e.* $\mathrm{d}T_1 < 0$. For the second fluid, one has $\mathrm{d}Q_2 = \dot{V}_2 c_2 \mathrm{d}T_2$, where $\mathrm{d}T_2$ is the differential temperature change of the second fluid, and $c_2$ – its specific heat capacity. Parameter $c_2$ is assumed to be a constant whenever any integral formulae are derived. Thus eq. (9) becomes:

$$\mathrm{d}\dot{W} \quad \dot{V}_1 c_v(T_1) \; 1 \quad \frac{T_2}{T_1 \quad \frac{\mathrm{d}Q_1}{\mathrm{d}\gamma}} \; \mathrm{d}T_1 \quad \frac{4\mathrm{a}\dot{V}_1 T_1^3 \mathrm{d}T_1}{3} \tag{10}$$

For the convenience of expressing, we define "substitution heat capacity" as [24]:

$$c_\mathrm{h}(T_1) \quad c_\mathrm{v}(T_1) \quad \frac{\mathrm{d}P}{\mathrm{d}T_1} \quad 4\mathrm{a}T_1^3 \quad \frac{4\mathrm{a}T_1^3}{3} \quad \frac{16}{3}\mathrm{a}T_1^3 \tag{11}$$

Thus, eq. (10) can be rewritten as:

$$\mathrm{d}\dot{W} \quad \dot{V}_1 \; c_\mathrm{h}(T_1) \quad c_\mathrm{v}(T_1)\frac{T_2}{T_1 \quad \frac{\mathrm{d}Q_1}{\mathrm{d}\gamma}} \; \mathrm{d}T_1 \tag{12}$$

The cumulative power delivered per unit volume flux of driving fluid is obtained by integration of eq. (12) between an initial temperature $T_{1\mathrm{i}}$ and a final temperature $T_{1\mathrm{f}}$ of the fluid. This integration yields the specific work of the flowing fluid in the form of the functional:

$$W \quad \frac{\dot{W}}{\dot{V}_1} \quad \int_{T_{1i}}^{T_{1f}} (c_\mathrm{h} T_1) \quad c_\mathrm{v}(T_1)\frac{T_2}{T_1 \quad \frac{\mathrm{d}Q_1}{\mathrm{d}\gamma}} \; \mathrm{d}T_1 \tag{13}$$

Defining non-dimensional time as:

$$\tau \quad \frac{\alpha \; a\dot{V}_1 F_1 x}{\dot{V}_1 c_\mathrm{v}} \quad \frac{\alpha \; a\dot{V}_1 F_1 v_1 t_1}{\dot{V}_1 c_\mathrm{v}} \tag{14}$$

where $a\dot{V}_1 = A/\dot{V}_1$ is the total specific exchange area per unit volume of the driving fluid, $F_1$ (assumed to be a constant) – the fluid cross-sectional area perpendicular to x, $v_1$ – the linear velocity of the driving fluid, and $t_1$ – the contact time of this fluid with the heat exchange surface. Therefore, the control variables can be designated here by $u$:

$$u \quad \frac{\mathrm{d}Q_1}{\mathrm{d}\gamma} \quad \frac{\dot{V}_1 c_\mathrm{v}(T_1)\mathrm{d}T_1}{\alpha \; (T_1) a_{\mathrm{V}1} F_1 \mathrm{d}x} \quad \frac{\mathrm{d}T_1}{\mathrm{d}\tau} \quad \dot{T}_1 \tag{15}$$

For the second fluid, one can obtain:

$$\dot{T}_2 \quad \frac{\mathrm{d}T_2}{\mathrm{d}\tau} \quad \frac{\mathrm{d}Q_2}{\dot{V}_2 c_2 \mathrm{d}\tau} \quad \frac{\dot{V}_1 c_\mathrm{v}(T_1)T_2 u}{\dot{V}_2 c_2 (T_1 \quad u)} \quad \frac{4\dot{V}_1 \mathrm{a} T_1^3 T_2 u}{\dot{V}_2 c_2 (T_1 \quad u)} \tag{16}$$

Therefore, eq. (13) becomes:

$$W \quad {}_{\tau_i}^{\tau_f} \quad c_\mathrm{h}(T_1) \quad \frac{c_\mathrm{v}(T_1)T_2}{T_1 \quad u} \quad u\mathrm{d}\tau \tag{17}$$

Substituting $c_\mathrm{h}(T_1) = 16\mathrm{a}T_1^3/3$ and $c_\mathrm{v}(T_1) = 4\mathrm{a}T_1^3$ into eq. (17), one can obtain:

$$W \quad 4\mathrm{a} \quad {}_{\tau_i}^{\tau_f} \quad \frac{4T_1^3}{3} \quad \frac{T_1^3 T_2}{T_1 \quad u} \quad u\mathrm{d}\tau \tag{18}$$

where $\tau_\mathrm{i}$ and $\tau_\mathrm{f}$ are the initial and final non-dimensional time of the process, respectively. We will solve the problem by using the optimal control theory.

## Application of the optimal control theory

We define the Hamiltonian function as:

$$H \quad 4\mathrm{a} \quad \frac{4T_1^3}{3} \quad \frac{T_1^3 T_2}{T_1 \quad u} \quad u \quad \lambda_1 u \quad \frac{4\mathrm{a}\lambda_2 \psi T_1^3 T_2 u}{T_1 \quad u} \tag{19}$$

where $\lambda_1$ and $\lambda_2$ are adjoint variables and $\psi = \dot{V}_1/\dot{V}_2 c_2$ is constant positive for a fixed system. Therefore the control equation is:

$$\frac{\partial H}{\partial u} \quad 0 \quad \lambda_1 \quad \frac{16\mathrm{a}T_1^3}{3} \quad 4\mathrm{a}T_1^4 T_2 \frac{1 \quad \lambda_2\psi}{(T_1 \quad u)^2} \quad 0 \tag{20}$$

The adjoint equations are:

$$\dot{\lambda}_1 \quad \frac{\partial H}{\partial T_1} 16\mathrm{a}T_1^2 u \quad \frac{4\mathrm{a}T_1^2 T_2 u(2T_1 \quad 3u)(1 \quad \lambda_2\psi)}{(T_1 \quad u)^2} \tag{21}$$

$$\dot{\lambda}_2 \quad \frac{\partial H}{\partial T_2} \quad \frac{4\mathrm{a}(\lambda_2\psi \quad 1)T_1^3 u}{T_1 \quad u} \tag{22}$$

The derivative of both sides of eq. (20) with respect to the non-dimensional time $\tau$ is:

$$\dot{\lambda}_1 \quad \frac{4\mathrm{a}T_1^4 \dot{T}_2(1 \quad \lambda_2\psi)}{(T_1 \quad u)^2} \quad 16\mathrm{a}T_1^2 u \quad \frac{16\mathrm{a}T_1^3 T_2 u(1 \quad \lambda_2\psi)}{(T_1 \quad u)^2}$$
$$\frac{8\mathrm{a}T_1^4 T_2(1 \quad \lambda_2\psi)(u \quad \dot{u})}{(T_1 \quad u)^3} \quad 4\mathrm{a}\frac{\psi T_1^4 T_2 \dot{\lambda}_2}{(T_1 \quad u)^2} \quad 0 \tag{23}$$

Substituting eqs. (16), (21), and (22) into eq. (23) yields:

$$2T_1^2 \dot{u} \quad T_1 u^2 \quad 3u^3 \quad 0 \tag{24}$$

The solution of eq. (24) is represented by two expressions:

$$\frac{2}{3}\xi(\sqrt{T_1^3} \quad \sqrt{T_\mathrm{1i}^3}) \quad \ln\frac{T_1}{T_\mathrm{1i}} \quad \tau \quad \tau_\mathrm{i} \tag{25}$$

$$u \quad \frac{T_1}{\xi\sqrt{T_1^3} \quad 1} \tag{26}$$

where $\xi$ is an arbitrary constant which is not equal to zero. Equation (25) is the optimal temperature profile of the driving fluid. Substituting eqs. (25) and (26) into eq. (16) yields:

$$\ln\frac{T_2}{T_{2i}} \quad \frac{8a\psi}{3\psi}(\sqrt{T_1^3} \quad \sqrt{T_{1i}^3}) \quad \frac{4a\psi}{3}(T_1^3 \quad T_{1i}^3) \tag{27}$$

where $T_{2i}$ is the initial temperature of the second fluid. Equation (27) is the temperature profile of the second fluid. It is easy to prove that the curve of eq. (25) is the maximum curve by using Legendre condition. Therefore, for fixed initial and final states of the system ($T_{1i} = T_{10}$, $T_{2i} = T_e$, and $T_{1f} = T_f$, $T_{2f}$ is unknown, $T_{1f} \quad T_{2f}$), process duration ($\tau_i = 0$, $\tau = \tau_f$) and with the help of eq. (25), one can obtain:

$$\xi \quad \frac{3 \quad \tau_f \quad \ln\dfrac{T_f}{T_{10}}}{2(\sqrt{T_f^3} \quad \sqrt{T_{10}^3})} \tag{28}$$

Substituting eqs. (25)-(27) into eq. (18) yields the maximum cumulative work output per unit volume driving fluid:

$$W_{max.} \quad 4a\int_{T_{1i}}^{T_{1f}}\frac{4T_1^3}{3dT_1} \quad 4aT_{2i}\int_{T_{1i}}^{T_{1f}}(\frac{1}{\xi}\sqrt{T_1} \quad T_1^2)\exp\left(8a\psi\frac{\sqrt{T_1^3} \quad \sqrt{T_{1i}^3}}{3\psi} \quad 4a\psi\frac{T_1^3 \quad T_{1i}^3}{3}\right)dT_1$$

$$4a\frac{T_{1f}^4 \quad T_{1i}^4}{3} \quad T_{2i}\frac{\exp\left(8a\psi\dfrac{\sqrt{T_{1f}^3} \quad \sqrt{T_{1i}^3}}{3\psi} \quad 4a\psi\dfrac{T_{1f}^3 \quad T_{1i}^3}{3}\right) \quad 1}{\psi} \tag{29}$$

When the second fluid is infinite heat reservoir ($T_2 \quad T_e$, $\psi \quad 0$) and $T_{1f} = T_e$, one can obtain:

$$W_{\psi \quad 0,\,max.} \quad 4a\lim_{\psi \quad 0}\int_{T_{1i}}^{T_{1f}}\frac{4T_1^3}{3dT_1} \quad T_{2i}\int_{T_{1i}}^{T_{1f}}\frac{1}{\xi}\sqrt{T_1} \quad T_1^2 \quad \exp\left(8a\psi\frac{\sqrt{T_1^3} \quad \sqrt{T_{1i}^3}}{3\xi}\right.$$

$$\left.4a\psi\frac{T_1^3 \quad T_{1i}^3}{3}\right)dT_1$$

$$4a\lim_{\psi \quad 0}\frac{T_{1f}^4 \quad T_{1i}^4}{3} \quad T_{2i}\frac{\exp\left(8a\psi\dfrac{\sqrt{T_{1f}^3} \quad \sqrt{T_{1i}^3}}{3\xi} \quad 4a\psi\dfrac{T_{1f}^3 \quad T_{1i}^3}{3}\right) \quad 1}{4a\psi}$$

$$4a\frac{T_e^4 \quad T_{1i}^4}{3} \quad 4aT_e\left(2\frac{\sqrt{T_e^3} \quad \sqrt{T_{1i}^3}}{3\xi} \quad \frac{T_e^3 \quad T_{1i}^3}{3}\right) \tag{30}$$

Equation (30) is another form of the result in refs. [24-26] when internal irreversibility factor $\Phi = 1$.

## Numerical example

In the numerical calculation for the performance characteristics of the system, $T_{1i} = 5800$ K, $T_{1f} = 1000$ K, $T_{2i} = 300$ K, $a = 7.6 \cdot 10^{-19}$ Ws/m$^3$K$^4$, $\tau = 150$, and $\psi = 6.075 \cdot 10^6$ m$^3$K/J are set. Substituting these data into eqs. (25)-(29), one can obtain $T_{2f} = 1000$ K and $\xi = -0.000542228$. The maximum cumulative work output per unit volume driving fluid is $W = 0.10$ J. The temperature profiles of the first and second fluid are shown in fig. 2. From fig. 2 it can be seen that the optimal temperature profile of the first fluid is a monotonic decreasing function of the non-dimensional time.

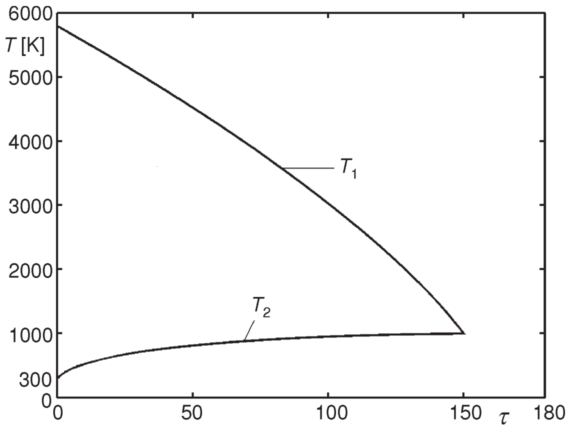


**Figure 2. Optimal temperature profiles**

## Conclusions

Based on finite time thermodynamics, the model of multistage continuous Carnot heat engine system with two reservoirs of finite thermal capacity is established. The heat transfer between heat source and the working fluid obeys radiation law and the heat transfer between heat sink and the working fluid obeys linear law. It is shown that the optimal temperature of driven fluid for maximum work output monotonically decreases with respect to flow velocity and process duration for fixed initial and final states and process duration. The obtained results can be compared with those of refs. [24-26]. The similarities and differences of the results for two cases are given below: for fixed initial and finial states and process duration, if one only controls the driving fluid, whether the second fluid is finite or not, the optimal temperature profiles and the optimal controls of driving fluid are described by maximums of the same type, eqs. (25) and (26). The maximum work outputs in two process modes are different. In general, the maximum work output of the first mode (second fluid is finite) is not equal to that of second mode (second fluid is infinite) for fixed initial and final states. The model and the analysis presented herein provide a way for improving evaluation of the mechanical energy limits in practical systems.

## Acknowledgments

This paper is supported by the Program for New Century Excellent Talents in University of P. R. China (Project No. 20041006) and The Foundation for the Author of National Excellent Doctoral Dissertation of P. R. China (Project No. 200136). The authors wish to thank the reviewer for his careful, unbiased, and constructive suggestions, which led to this revised manuscript.

## Nomenclature

*A* – cumulative heat exchange surface areas between engines and heat reservoir, [$m^2$]
A – total heat exchange surface area of the first and second fluids, [$m^2$]
*a* – total specific exchange area per unit volume, [$sm^{-1}$]
a – universal coefficient related to the Stefan-Boltzmann constant (= $4\sigma$/c), [$Wsm^{-3}K^{-4}$]
*c* – specific heat capacity, [$Jm^{-3}K^{-1}$]
c – light speed, [$ms^{-1}$]
*F* – cross-sectional area, [$m^2$]
*H* – Hamiltonian function, [–]
*k* – ratio of surface area, [–]
*P* – radiation pressure, [$Wsm^{-3}$]
*Q* – cumulative heat, [J]
*T* – variable temperature of fluid, [K]
*t* – time, [s]
*u* – rate of temperature change as the control variable, [K]
$\dot{V}$ – volume flux of the fluid, [$m^3s^{-1}$]
*v* – linear velocity of fluid, [$m^3s^{-1}$]
*W*, $\dot{W}$ – work and power, [J, W]
x – transfer area coordinate, [–]

*Greek letters*

$\alpha$ – heat transfer coefficients of fluid, [$Wm^{-2}K^{-1}$]
$\alpha$ – substitution heat transfer coefficient, [$Wm^{-2}K^{-1}$]
$\gamma$ – heat conductivity, [$WK^{-1}$]
$\eta$ – first-law efficiency, [–]
$\lambda$ – adjoint variable, [–]
$\xi$ – arbitrary constant which is not equal to zero, [–]
$\sigma$ – Stefan-Boltzmann constant, [$Wm^{-2}K^{-4}$]
$\tau$ – non-dimensional time, [–]
$\psi$ – constant positive for a fixed system [$= \dot{V}_1/(\dot{V}_2 c_2)$], [$m^3KJ^{-1}$]

*Subscripts*

C – circulating working fluid
e – environment
f – finial state
h – substitution heat capacity
i – initial state
v – constant volume of the first fluid
1, 2 – first and second fluid

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Paper submitted: November 9, 2008
Paper revised: April 14, 2009
Paper accepted: April 30, 2009