NATURAL CONVECTION HEAT TRANSFER IN A PARTIALLY DIVIDED TRAPEZOIDAL ENCLOSURE

by

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Natural convection heat transfer in a partially divided trapezoidal enclosure is studied numerically using the control volume method. Summer and winter conditions are separately examined by imposing regarding thermal boundary conditions. A horizontal divider included and its two different placements are considered. It is shown that heat transfer results are not significantly altered by the presence of the divider for summer condition. For winter condition, on the other hand, decrement in heat loss and effect on the flow and the temperature fields by the presence and the placement of the divider are observed, respectively. As a horizontal divider is placed to oppose buoyancy, the flow strength becomes weaker and formation of two separate levels of temperature uniformity occurs. As the divider is placed to assist buoyancy, the flow gets stronger and tends to form relatively uniform temperature field within whole enclosure.

Key words: *natural convection, trapezoidal enclosure, partial divider*

Introduction

Natural convection in enclosures has been receiving a good deal of research interest because of its broad application fields covering air gaps in building insulation materials, air motion between the absorber and the transparent cover in solar collectors, flow arising in rooms due to thermal energy sources, cooling of heat generation component in the electrical and nuclear industries and so on. Many of the studies performed in the existing literature considered regular geometries like square, rectangular, and triangular. However, many enclosures encountered in practice are complex geometries.

A classification on the natural convection within the rectangular enclosures reported by Ostrach [1] separates the phenomena basically in two parts: conventional convection, due to vertically imposed temperature difference and unstable convection, due to horizontally imposed temperature difference. Resulting density gradient is normal to the gravity vector for the first one, and is parallel but opposed to it for the later. Because of the wide variety of reported works on the subject, only the related papers are discussed here. A solution for the conventional convection case performed by De Vahl Davis [2] describes the development and the characteristics of a bench mark solution in greater detail. Unstable convection problem under several thermal boundary conditions at sidewalls studied by Corcione [3] comparatively presents the results of the configurations through dimensionless correlation-equations. Besides these types of two basic conditions, some works on rather complex boundary conditions such as imposing the temperature differences between two adjacent walls have been also found in the literature. Studies on rectangular enclosures, heated from below and cooled along one side by November *et al*. [4], heated from one side and cooled from the ceiling by Aydin *et al*. [5] and heated from below and symmetrically cooled from the sides by Ganzarolli *et al*. [6] are the examples for differentially heated adjacent wall cases. Study on right triangular enclosure was performed by Akinsete *et al*. [7] for cooled bottom wall and heated along the inclined wall case. Natural convection in isosceles triangular enclosures heated from below and cooled from the inclined walls were studied by Salmun [8] and by Asan *et al*. [9]. Natural convection in a partitioned trapezoidal enclosure reported by Moukalled *et al.* [10] undertakes the effect of the summertime and the wintertime boundary conditions. Moukalled *et al*. [11] dealt with the same geometry of reference [10] and employed the conventional type boundary conditions to construct the buoyancy-assisting and the buoyancy-opposing arrangements. The effects of heated short vertical wall, the buoyancy-assisting arrangement, and of cooled short vertical wall, the buoyancy-opposing arrangement, are analyzed in the work.

In the present work, natural convection in a partially divided trapezoidal enclosure is studied. It is aimed to analyse the effect of the presence and the placements of a horizontal divider on natural convection heat transfer under summer condition, cooled bottom wall, and under winter condition, heated bottom wall. The divided enclosure representing two different configurations for a duplex house-like physical model is considered for a practical point of view. A non-divided trapezoidal enclosure is also taken into consideration for the analysis to have a comparable assessment of the results.

Analysis

The physical configuration and related computational domain considered for the present problem is illustrated in fig. 1. The natural convection is considered to be two-dimensional, steady, and laminar. The fluid is assumed to be incompressible, with constant physical properties and negligible viscous dissipation. The buoyancy effects upon momentum transfer are taken into account through the Boussinesq approximation.

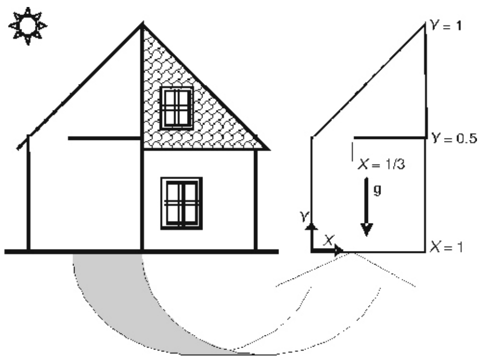

Figure 1. Sketch for the physical condition and the corresponding computational domain

Employing the above assumptions into the conservation equations of mass, momentum, and energy, the following set of governing equations in dimensionless form is obtained:

$$\frac{\partial U}{\partial X} \quad \frac{\partial V}{\partial Y} \quad 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} \quad V\frac{\partial U}{\partial Y} \quad \frac{\partial P}{\partial X} \quad \Pr \quad \frac{\partial^2 U}{\partial X^2} \quad \frac{\partial^2 U}{\partial Y^2} \tag{2}$$

$$U\frac{\partial U}{\partial X} \quad V\frac{\partial V}{\partial Y} \quad \frac{\partial P}{\partial Y} \quad \mathrm{Ra}\Pr\theta \quad \Pr \quad \frac{\partial^2 V}{\partial X^2} \quad \frac{\partial^2 V}{\partial Y^2} \tag{3}$$

$$U\frac{\partial \theta}{\partial X} \quad V\frac{\partial \theta}{\partial Y} \quad \frac{\partial^2 \theta}{\partial X^2} \quad \frac{\partial^2 \theta}{\partial Y^2} \tag{4}$$

Two different sets of thermal boundary conditions are considered namely the cold bottom wall and the hot bottom wall may represent summer and winter conditions, respectively. For both cases, the hot wall corresponds to $\theta = 1$ and the cold wall corresponds to $\theta = 0$. The no-slip condition $U = V = 0$ for the velocities is assumed along the boundary wall and the divider surfaces. The physical configuration is assumed to be geometrically symmetric along the higher vertical wall as represented in fig. 1. Therefore, the only half of the configuration is considered for the analysis. The wall of the symmetry is set to the insulation boundary condition while the bottom wall and short vertical wall together with the adjacent inclined walls are set according to the summer and the winter considerations. The divider is located on the imaginary line that divides the whole region in two subregions: a square region at lower domain and a right triangular region at upper domain. It is considered as a passive divider that is not actively participating in heat transfer mechanism rather obstructs the momentum fluxes only. Thermal boundary conditions expressed above may be presented mathematically as:

$$\begin{array}{ll} \text{at } X = 0 \text{ (short vertical wall) and} & \theta = 1, \text{for summer} \\ \text{at } Y = 0.5 + X\sqrt{2} \text{ for every } X \text{ (inclined wall)} & \theta = 0, \text{for winter} \end{array}$$

$$\text{at } Y = 0 \quad \begin{array}{l} \theta = 0, \text{ for summer} \\ \theta = 1, \text{ for winter} \end{array}$$

$$\text{at } X = 1, \frac{\partial \theta}{\partial X} = 0$$

The finite control volume discretization practice is applied to eqs. (1)-(4) and the discretized equations are implicitly solved through the SIMPLE algorithm, as in reference [12]. The convection heat transfer within the enclosure is obtained from the numerical solutions of the governing differential equations. Heat transfer results are presented in terms of average $\overline{\text{Nu}}$ number. The local Nu number along the base defined for the cooled or heated surface is given as follows:

$$\text{Nu}_{\text{h/c}} = -\left.\frac{\partial \theta}{\partial Y}\right|_{Y=0} \tag{5}$$

The corresponding average Nu number for the horizontal bottom wall is given by:

$$\overline{\text{Nu}} = \int_0^1 \text{Nu}_{\text{h/c}} \text{d}X \tag{6}$$

The normalized average Nusselt number is given as:

$$\text{Nu}^* = \frac{\overline{\text{Nu}}}{\overline{\text{Nu}}_{\text{o,h/c}}} \tag{7}$$

where $\overline{\text{Nu}}_{\text{o,h/c}}$ is the average Nu number calculated for the case of a pure conduction exist within the enclosure.

A mash refinement study is performed but the results are not presented here. The test shows that using a total of 60×120 uniform control volumes is enough for acceptable accuracy of the results. Increasing mesh from 60×120 to higher value does not alter numerical results of the dependent variable fields. The solution is considered to be converged when the maximum absolute value of the mass source and the percent change of the dependent variable field from iteration to iteration are smaller than a prescribed value, *e. g.*, 10^{-5}.

Results and discussions

The present work has the objective to investigate the effect of placement of a horizontal divider on natural convection heat transfer in a trapezoidal enclosure. The results of the divided enclosures are presented together with the results of non-divided enclosure to have a comparable assessment of natural convection phenomena occurring within the enclosure. To assess the validity of the solution procedure, the normalized average Nu number obtained in the present work is compared with the result of Moukalled *et al*. [10] (fig. 2). General trends of the average Nu number variations with Ra number seem quite similar. Discrepancies are assumed to be acceptable in numerical point of view. The results of present work are illustrated as variation of average Nu number with Ra number, streamline and isotherm contour patterns for three different configurations and for four different Ra numbers. In addition to these illustrations, the maximum stream function values are listed in tab. 1 to observe the effect of the presence and the placement of divider on the flow strength.

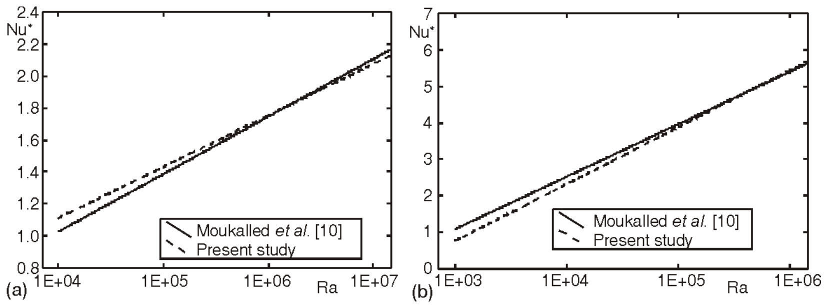

Figure 2. Comparative results of the present predictions and of reference [10] for normalized average Nusselt number –(a) summer condition, (b) winter condition

Variation of the average Nu number with Ra number is presented in fig. 3. As comparing the summer and winter conditions, the quantitative level of average Nu numbers for the summer condition seems well below than that of the winter condition. As shown in fig. 3(a), the average Nu number is almost independent from the problem configuration. For the winter condition, average Nu number is depending on the problem configuration – fig. 3(b). Dependence of the problem configuration is getting significant with increasing Ra number.

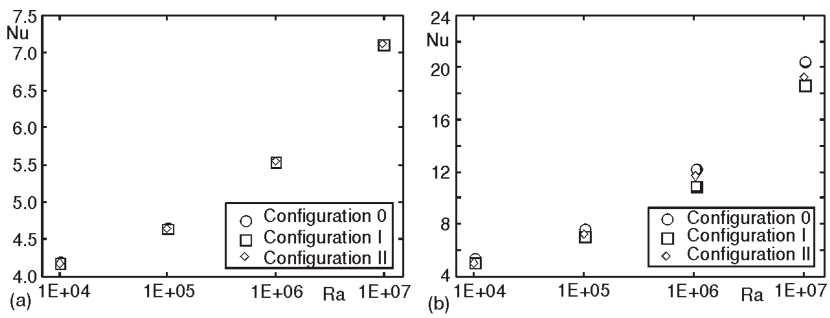

Figure 3. Comparative results of average Nusselt number for different configurations – (a) summer condition, (b) winter condition

Streamline and isotherm contour plots relevant to the basic geometry in fig. 1 are shown for summer and winter conditions in figs. 4 and 5, respectively. The contour plots located at left panel represent the streamlines while the contour plots located at right panel represent the isotherms. Non-divided enclosure is indicated as Configuration 0. The enclosures having the divider attached to insulated vertical wall and to point of intersection between vertical and inclined walls are indicted as Configuration I and Configuration II, respectively. Streamlines and isotherms are presented for above prescribed configurations and for four different Ra numbers from 10^4 to 10^7.

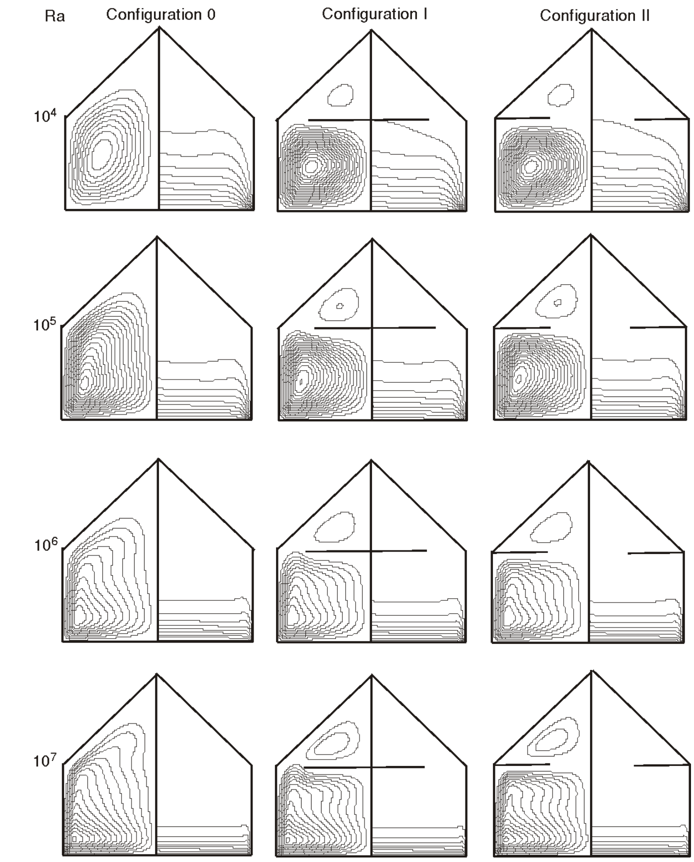

Figure 4. Comparative results of streamlines and isotherms for summer condition

As shown for Configuration 0 of summer condition, the flow consists of a single recirculation cell within the prescribed Ra number interval. The center of the cell moves closer toward the left-bottom corner and streamlines are tending to persist at all along the domain with increasing Ra number. Corresponding isotherms show that the warmer fluid occupies the upper region of the enclosure and this region extends toward cold bottom wall with increasing Ra number. For Configuration I, formation of streamline contours at the lower domain are quite similar to that of Configuration 0, with the exception of formation of a weaker recirculation cell at the upper domain. Isotherms presented for Configuration I shows the same pattern as Configuration 0. Change of divider placement, as in Configuration II of summer condition, does not alter the formation of streamlines and isotherms. Therefore, fig. 4 concludes that presence of divider separates the flow field in two domains and causes forming an independent recirculation cell for each domain, but cannot create a noticeable effect on the temperature field.

Winter condition as presented in fig. 5, produces quite different flow and temperature fields than that of summer condition. Even for Configuration 0, Ra number variation has noticeable effect on the formation of streamlines and isotherms. Increasing Ra number compresses the streamlines near to walls and causes forming a boundary layer type flow along the boundaries. In addition to this fact, formation of secondary small cell at upper region, and near top corner are detected for Ra = 10^5 and Ra = 10^7, respectively. Unlike summer condition, isotherm patterns for winter condition of Configuration 0 clearly reflects the buoyant effect especially for high Ra numbers and this effect leads high temperature gradient near the cold boundaries. Two inde-

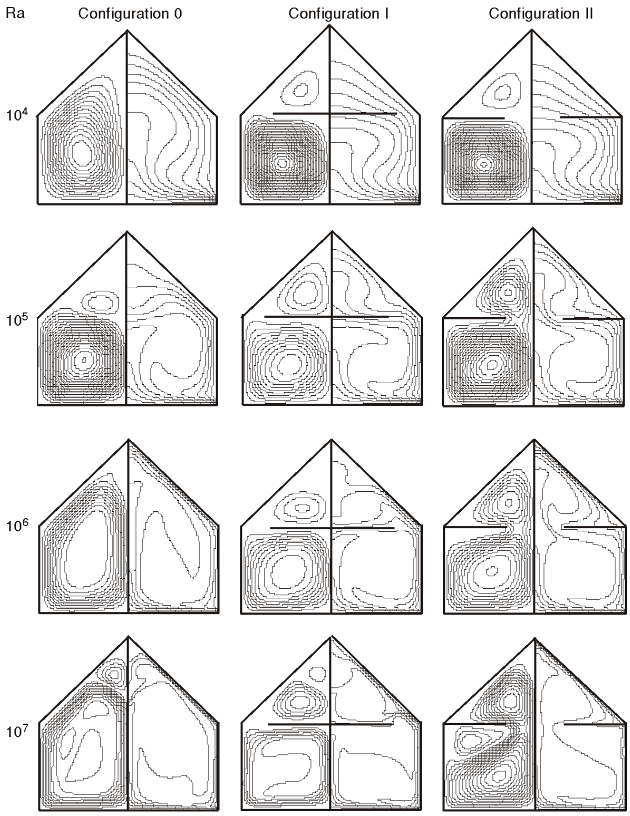

Figure 5. Comparative results of streamlines and isotherms for winter condition

pendent recirculation cells occur for Configuration I and formation of streamlines at the lower domain are quite similar to that of Configuration 0. Upper domain recirculation cells are stronger than corresponding summer condition cells of fig. 4. As in Configuration 0, a small cell near top corner of the upper domain is also detected for Ra = 10^7. Although the recirculation cells seem to be stronger than summer case application, flow in lower domain cannot penetrate into the upper domain because of the buoyancy-opposing effect of the divider. By examining the corresponding isotherm contours, the buoyancy-opposing effect can be easily observed. The isotherms concentrated along the cold vertical wall in lower domain pass throughout the gap between domains and spread out upper domain for high Ra numbers. Therefore, the placement of divider for Configuration I opposes the buoyancy and results in preventing penetration of hot fluid to upper domain via convective transport. On the other hand, streamline and temperature contours for Configuration II as presented in the last column of fig. 5 show some distinguish features which are worth to point out. For Ra =10^4, both streamline and isotherm plots are quite similar to that of Configuration I. As Ra number increases, the placement of divider allows the fluid in lower domain to penetrate into upper domain through the gap between the domains. The increased Ra number leads to a stronger penetration effect and forming a secondary recirculation cell at upper domain. A third recirculation cell between top of the lower domain and the divider is detected for Ra = 10^7. Unlike to Configuration I, buoyant assisting effect is observed for Configuration II. The isotherms originated from heated bottom wall cross across the lower domain, pass through the gap between domains and spread out upper domain.

Above discussion on streamline and isotherm contours concludes that presence of divider significantly affects the flow and temperature fields for winter condition. Configuration I divides the flow field in two parts and forms two different levels of temperature uniformity within the enclosure. Namely the upper domain is filled with relatively cold fluids and the lower one is filled with warmer fluid. Configuration II, on the other hand, tends to preserve whole temperature uniformity within the enclosure.

Table 1 indicates that winter condition creates higher flow strength than summer condition in general. An additional general trend may extract from the table that the presence of divider slightly increases the flow strength for summer condition, and decreases it for winter condition up to Ra = 10^6. Both the presence and placement of the divider is becoming less effective on the flow strength with increasing Ra numbers for the summer condition. However, the placement of divider becomes significant with increasing Ra number for winter condition. The general trend in decrement of flow strength is violated for higher Ra number values, *e. g.* for Ra = 10^6 and Ra = 10^7. Comparing the placements of the divider for winter condition, Configuration II is found to be in charge of creating higher flow strength for higher Ra number values.

Table 1. Maximum stream function values

Arrangement	$\Psi_{max.}$							
	Summer condition				Winter condition			
	Ra = 10^4	Ra = 10^5	Ra = 10^6	Ra = 10^7	Ra = 10^4	Ra = 10^5	Ra = 10^6	Ra = 10^7
Configuration 0	2.001	3.658	6.023	8.977	7.637	24.483	60.603	114.604
Configuration I	2.007	3.752	6.017	8.979	5.816	20.744	49.904	78.842
Configuration II	2.010	3.765	6.018	8.979	5.73	20.704	56.658	149.889

Conclusions

The performed analysis dealt with a partially divided trapezoidal enclosure representing a single space living environment like a duplex house-like configuration. The analysis is completed for two different placements of a divider together with a non-divider enclosure. The procedure is tested comparing with the results of the published work of a trapezoidal enclosure and found a satisfactorily predictive tool to employ it to the presented configurations.

For the summer condition, neither the presence nor the placement of the divider has a meaningful effect on the flow and the temperature fields and on the heat gain. For the winter condition, on the other hand, the flow and the temperature fields and resultant convective heat loss are influenced by the presence and the placements of the divider. The divider placed to oppose buoyancy, the Configuration I of winter condition, reduces the flow strength, forms two independent recirculation cells and results in two different levels of temperature uniformity. The divider placed to assist buoyancy, the Configuration II of winter condition, tends to increase the flow strength with increasing Ra number, forms two dependent recirculation cells and creates relatively uniform temperature distribution within whole enclosure.

Finally, the Configuration I is found to be the best choice to reduce the heat loss, while the Configuration II is preferable in terms of sustaining uniform temperature distribution within enclosure.

Nomenclature

g – gravitational acceleration, [ms^{-2}]
L – length of horizontal wall, [m]
Nu – Nusselt number ($= -d\theta/dY_{Y=0}$), [–]
Ra – Rayleigh number [$= g\beta(T_h - T_c)L^3Pr/\nu^2$], [–]
T – temperature, [K]
U – dimensionless velocity component (uL/α)
u – horizontal velocity component, [ms^{-1}]
V – dimensionless velocity component (vL/α)
v – vertical velocity component, [ms^{-1}]
X – dimensionless coordinate (x/L)

x – horizontal coordinate, [m]
Y – dimensionless coordinate (y/L)
y – vertical coordinate, [m]

Greek symbols

θ – dimensionless temperature $[=(T-T_c)/(T_h-T_c)]$

Subscripts

c – cold
h – hot

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