

OPTIMAL TEMPERATURES AND MAXIMUM POWER OUTPUT OF A COMPLEX SYSTEM WITH LINEAR PHENOMENOLOGICAL HEAT TRANSFER LAW

by

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A complex system including several heat reservoirs, finite thermal capacity subsystems with different temperatures and a transformer (heat engine or refrigerator) with linear phenomenological heat transfer law $[q = \Delta(T^{-1})]$ is studied by using finite time thermodynamics. The optimal temperatures of the subsystems and the transformer and the maximum power output (or the minimum power needed) of the system are obtained.

Key words: *linear phenomenological heat transfer law, complex system, finite time thermodynamics, thermodynamic optimization*

Introduction

Since finite time thermodynamics and entropy generation minimization has been advanced, much work has been carried out on the performance analysis and optimization of finite time processes or finite size devices [1-16]. Amelkin *et al.* [17, 18] discussed the maximum power processes of multi-heat-reservoir heat engine with stationary temperature reservoirs, found that some reservoirs were not used in heat transfer in order to achieve an optimal performance of the system, and further found that independent of the number of reservoirs the working fluid used only two isotherms and two adiabatics. Tsirlin *et al.* [19] analyzed the thermodynamic process with given rate and obtained the general conditions that the minimal dissipation should obey. Tsirlin *et al.* [20-23] investigated the optimal process and optimal performance of the open controllable macrosystems and applied the method to microeconomic systems, obtained many important results. Huleihil *et al.* [24] studied the optimal piston trajectories for adiabatic processes in the presence of friction. Tsirlin *et al.* [25] further studied the optimal temperature and maximum power out of a complex system, which includes a transformer, several heat reservoirs and finite capacity subsystems with different temperatures, under Newtonian heat transfer law.

In general, heat transfer is not necessarily Newtonian and also obeys other laws, and heat transfer laws have the significant influences on the performance of thermodynamic cycles [26-34]. In this paper, based on ref. [25], the optimal temperatures and maximum power output of the complex system are obtained by using another linear heat transfer law, *i. e.* linear phenomenological heat transfer law $[q = \Delta(T^{-1})]$, in the heat transfer processes inside of the system. The linear phenomenological heat transfer law is used in irreversible thermodynamics, the heat conductivities in this case are the so-called kinetic coefficients by Callen [35].

System model

The stationary state of a thermodynamic system is shown in fig 1.

The system includes heat reservoirs with constant temperatures, finite capacity subsystems (reservoir with constant temperature, finite capacity subsystem with a temperature that depends on its extensive variables, the heat/mechanical energy transformer) with different temperatures and a transformer. We assume the quantity of all reservoirs and finite-capacity

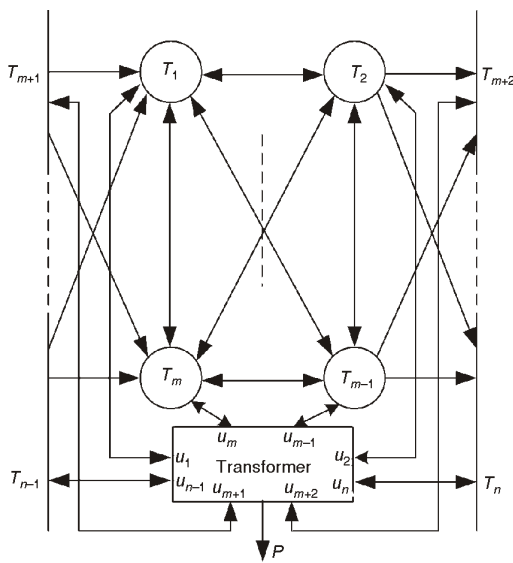


Figure 1. Complex thermodynamic system model

subsystems is n and the quantity of the subsystems is m . We divide m finite-capacity subsystems into two categories – the subsystems with fixed and free temperatures, the quantity of the subsystems with free temperatures is r . The transformer generates power and if the maximum power is negative then it corresponds to the minimum of the external power consumed by the system. Assuming each subsystem is in internal equilibrium and all irreversibility arises at the boundaries of subsystems. There are heat transfers between subsystems each other and heat reservoirs. Denote the subsystem's (or heat reservoir's) temperature as T_i , the heat flow from the i^{th} to the j^{th} subsystems as $q_{ij}(T_i, T_j)$, the temperature of working fluid of the transformer when it contacts the i^{th} subsystem (or heat reservoir) as u_i , the heat transfer between the i^{th} subsystem (or heat reservoir) and the transformer as $q_i(T_i, u_i)$, and the power output of the transformer as P .

The objective is to find the optimal temperatures $u_{i\text{opt}}$ for contact between the transformer and each of the subsystems and the heat reservoirs such that the power P is maximal.

Defining the heat flow entering each subsystem is positive. When T_i increase, q_{ij} decrease monotonically, and when T_j increase, q_{ij} increase monotonically. If $T_i = T_j$ then $q_{ij} = 0$. If there is no contact between subsystems then $q_{ij} = 0$. Assuming the functions $q_{ij}(T_i, T_j)$ are continuously differentiable and the transformer is endoreversible and the entropy generation in it is equal to zero. If the heat transfers between subsystems, heat reservoirs and the transformer obey linear phenomenological heat transfer law, then:

$$q_i \propto \alpha_i (u_i^{-1} - T_i^{-1}), \quad q_{ij} \propto \alpha_{ij} (T_i^{-1} - T_j^{-1}) \quad (1)$$

where α_i and α_{ij} are the heat conductivities. The maximal power problem takes the form:

$$P = \sum_{i=1}^n \alpha_i (u_i^{-1} - T_i^{-1}) \quad \max_{u_i > 0} \quad (2)$$

From the entropy balance of the working fluid of the transformer, one can obtain

$$\sum_{i=1}^n \frac{\alpha_i (u_i^{-1} - T_i^{-1})}{u_i} = 0 \quad (3)$$

From the energy balance of the subsystem, one can obtain:

$$\prod_{j=1}^n \alpha_{ij} (T_i^{-1} - T_j^{-1}) = \alpha_i (u_i^{-1} - T_i^{-1}), \quad i = 1, \dots, m \quad (4)$$

The m finite capacity subsystems can be divided into two categories; one is the subsystems with free temperatures $T_i (i = 1, \dots, r)$, which can be controlled jointly with u_i to maximize P , and another is the subsystems with fixed temperatures ($i = r + 1, \dots, m$).

Optimal solutions

Defining the Lagrange function:

$$L = \prod_{i=1}^n \alpha_i (u_i^{-1} - T_i^{-1}) - \beta \prod_{i=1}^m \frac{\alpha_i (u_i^{-1} - T_i^{-1})}{u_i} - \beta \prod_{i=m+1}^n \frac{\alpha_i (u_i^{-1} - T_i^{-1})}{u_i} + \sum_{i=1}^m \lambda_i \left[\prod_{j=1}^n \alpha_{ij} (T_i^{-1} - T_j^{-1}) - \alpha_i (u_i^{-1} - T_i^{-1}) \right] \quad (5)$$

where β and λ_i are Lagrange multipliers. From $L = L(u_i, T_i)$ and $\partial L / \partial T_i = 0$, one can obtain:

$$\frac{\partial L}{\partial u_i} = 0 \implies u_{i \text{opt}} = \frac{2\beta T_i}{\beta - T_i(1 - \lambda_i)}, \quad i = 1, \dots, n \quad (6)$$

$$\frac{\partial L}{\partial T_i} = 0 \implies \alpha_i^{-1} - \frac{\beta}{u_i} - \lambda_i \sum_{j=1}^n (\lambda_j - \lambda_i) \alpha_{ij} = 0, \quad i = 1, \dots, r \quad (7)$$

$\lambda_i = 0$ when $j > m$. The stationary condition for Lagrange function for heat reservoirs yields:

$$\frac{\partial L}{\partial u_i} = 0 \implies u_{i \text{opt}} = \frac{2\beta T_i}{\beta - T_i}, \quad i = m + 1, \dots, n \quad (8)$$

The stationary condition for Lagrange function for subsystems yields:

$$\frac{\partial L}{\partial u_i} = 0 \implies u_{i \text{opt}} = \frac{2\beta T_i}{\beta - T_i(1 - \lambda_i)}, \quad i = 1, \dots, m \quad (9)$$

Systems consisting of reservoirs and transformer

If the system consists of n reservoirs ($m = 0$) and the transformer, one can obtain from eqs. (3) and (8):

$$u_{i \text{opt}} = \frac{2T_i \sqrt{\prod_{i=1}^n \alpha_i / \prod_{i=1}^n \alpha_i T_i^{-2}}}{\sqrt{\prod_{i=1}^n \alpha_i / \prod_{i=1}^n \alpha_i T_i^{-2}} - T_i}, \quad i = 1, \dots, n \quad (10)$$

The maximal power output is:

$$P^* = \prod_{i=1}^n \alpha_i \frac{1}{2 \sqrt{\prod_{j=1}^n \alpha_j / \prod_{i=1}^n \alpha_j T_j^{-2}}} = \frac{1}{2T_i}, \quad i, j = 1, \dots, n \quad (11)$$

If the system only has two heat reservoirs ($n = 2$), eqs. (10) and (11) become the results in ref. [27].

Systems consisting of subsystems, reservoirs, and transformer

Consider a system where the temperatures of all subsystems are free ($r = m$). The maximal power output problem can be decomposed into three sub-problems.

- (1) To maximize the power $P_r^*(\sigma_r)$ derived from the contact between the transformer and reservoirs for the given entropy flow from the reservoirs to the transformer σ_r . That is, under the constraints:

$$\sum_{i=1}^n \frac{q_i}{u_i} = \sigma_r \quad (12)$$

to maximize the eq. (13):

$$P_r^*(\sigma_r) = \sum_{i=1}^n q_i \quad (13)$$

Solving eqs. (1), (8), and (12), one can obtain the optimal temperatures of the reservoirs:

$$u_{i\text{opt}} = \frac{2T_i \sqrt{\sum_{i=1}^n \alpha_i / \sum_{i=1}^n \alpha_i T_i^2 - 4\sigma_r}}{\sqrt{\sum_{i=1}^n \alpha_i / \sum_{i=1}^n \alpha_i T_i^2 - 4\sigma_i} T_i}, \quad i = 1, \dots, n \quad (14)$$

Then the maximal power generated from contacts with reservoirs is:

$$P_r^*(\sigma_r) = \sum_{i=1}^n \frac{\alpha_i}{2} \frac{\sqrt{\sum_{j=1}^n \alpha_j}}{\sqrt{\sum_{j=1}^n \alpha_j T_j^2 - 4\sigma_r} T_i^{-1}}, \quad i, j = 1, \dots, n \quad (15)$$

- (2) To maximize the power $P_s^*(\sigma_s)$ generated from contact between the transformer and finite capacity subsystems subject to the given flow of entropy from the subsystems to the transformer, working fluid σ_s . That is, under the constraints:

$$\sum_{i=1}^m \frac{q_i}{u_i} = \sigma_s \quad (16)$$

and eq. (4), to maximize eq. (17):

$$P_s^*(\sigma_s) = \sum_{i=1}^m q_i \quad (17)$$

From eq. (1), the subsystems, temperatures u_i can be expressed in terms of q_i and T_i :

$$\frac{1}{u_i} = \frac{q_i}{\alpha_i} + \frac{1}{T_i} \quad (18)$$

Equation (4) can be rewritten as the set of linear equations with respect to the temperatures $T_i (i = 1, \dots, m)$:

$$\frac{1}{T_i} - \sum_{j=1}^n \alpha_{ij} = \sum_{j=1}^m \frac{\alpha_{ij}}{T_j} q_j, \quad i = 1, \dots, m \quad (19)$$

or in matrix form:

$$A(\alpha) \cdot T = C(q) \quad (20)$$

where

$$A(\alpha) = \begin{pmatrix} \alpha_{11} & \bar{\alpha}_1 & \alpha_{12} & \cdots & \alpha_{1m} \\ \alpha_{21} & \alpha_{22} & \bar{\alpha}_2 & \cdots & \alpha_{2m} \\ \cdots & \cdots & \ddots & \cdots & \cdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mm} & \bar{\alpha}_m \end{pmatrix}, \quad \bar{\alpha}_i = \sum_{j=1}^n \alpha_{ij}$$

$$T = \begin{pmatrix} T_1^{-1} & & & & \\ & T_2^{-1} & & & \\ & & \vdots & & \\ & & & & T_m^{-1} \end{pmatrix}, \quad C(q) = \begin{pmatrix} q_1 & R_1 \\ q_2 & R_2 \\ \vdots & \vdots \\ q_m & R_m \end{pmatrix}, \quad R_i = \sum_{j=m+1}^n \frac{\alpha_{ij}}{T_j}$$

Assuming the matrix A is reversible and the temperatures, reciprocal of the systems can be expressed in terms of the heat flows q_i and the fixed reservoir temperatures

$$T = A^{-1}C(q) \tag{21}$$

$$\frac{1}{T_i(q)} = b_{i1}(q_1, R_1) + b_{i2}(q_2, R_2) + \cdots + b_{im}(q_m, R_m) \tag{22}$$

where $b_{ij}(\alpha)$ are the components of its inverse matrix A^{-1} . Substituting eq. (22) into eq. (18), one can obtain the $u_i(q_i)$ with respect to q_i . Equation (16) can be expressed as:

$$\sum_{i=1}^m q_i T_i^{-1} - \frac{q_i}{\alpha_i} - \sigma_s \tag{23}$$

The Lagrange function for the problem (17) is:

$$L = \sum_{i=1}^m q_i \left[1 - \lambda_s T_i^{-1} - \frac{\lambda_s q_i}{\alpha_i} \right] - \lambda_s \sigma_s \tag{24}$$

Its stationary condition on q_i is:

$$\frac{\partial L}{\partial q_j} = 0 \Rightarrow \sum_{i=1}^m q_i \lambda_s b_{ij} - 1 - \frac{\lambda_s}{T_j} - \frac{2\lambda_s q_j}{\alpha_j} = 0, \quad j = 1, \dots, m \tag{25}$$

Solving the set of $m + 1$ equations – eqs. (23) and (25), gives the optimal $q_i^*(\sigma_s)$ and λ_s . Substituting the results into eqs. (17), (18), and (22), one can obtain the optimal $T_i^*(\sigma_s)$, $u_{i\text{opt}}(\sigma_s)$, and $P_s^*(\sigma_s)$.

- (3) To find the overall maximum power subject to the entropy balance for the working fluid of the transformer. That is, under the condition:

$$\sigma_r + \sigma_s = 0 \tag{26}$$

to maximize the eq. (27)

$$P(q_i) = P_r^*(\sigma_r) = P_s^*(\sigma_s) \tag{27}$$

The Lagrange function of the problem (27) is

$$L = P_r^*(\sigma_r) - P_s^*(\sigma_s) - \lambda(\sigma_r - \sigma_s) \tag{28}$$

Its stationary condition on σ_r and σ_s yields

$$\frac{\partial P_s^*}{\partial \sigma_s} = \frac{\partial P_r^*}{\partial \sigma_r} \tag{29}$$

From eq. (26), one can obtain the power $P(q_i)$ is maximal when:

$$\sigma_r = -\sigma_s \quad (30)$$

Numerical example

Consider a complex system which consists of two heat reservoirs, two finite capacity subsystems and a transformer. The structure of the system is shown in fig 2. The matrix of heat conductivities has the form

$$\{\alpha_{ij}\} \begin{matrix} 0 & 2 \cdot 10^7 & 4 \cdot 10^7 & 0 \\ 2 \cdot 10^7 & 0 & 0 & 2 \cdot 10^7 \\ 4 \cdot 10^7 & 0 & 0 & 0 \\ 0 & 2 \cdot 10^7 & 0 & 0 \end{matrix}$$

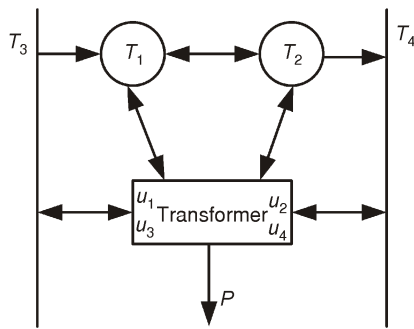


Figure 2. Two reservoir thermodynamic system

where $i, j = 1, 2$ correspond to the interaction between subsystems and $i > 2$ or $j > 2$ to the interaction between the subsystems and the reservoirs. The heat conductivities for transformer-subsystems and transformer-reservoirs interaction are $\alpha_1 = 10^7$ WK, $\alpha_2 = 2 \cdot 10^7$ WK, $\alpha_3 = 4 \cdot 10^7$ WK, $\alpha_4 = 0.9 \cdot 10^7$ WK, and the reservoir temperatures are $T_3 = 1000$ K and $T_4 = 300$ K.

First, one can obtain $u_3(\sigma_r)$, $u_4(\sigma_r)$, and $P_r^*(\sigma_r)$ with respect to σ_r by substituting the parameters above into eqs. (14) and (15) and obtain $q_1(\sigma_s)$, $q_2(\sigma_s)$, and $P_s^*(\sigma_s)$ with respect to σ_s by using eqs. (22), (23), and (25). Then, one can obtain the values of σ_r and σ_s by using eqs. (29) and (30). Finally, substituting σ_r and σ_s into equations above, one can obtain the optimal temperatures of the subsystems and the transformer.

The calculation yields the following optimal temperatures for the subsystems and transformer: $T_1 = 682.98$ K, $T_2 = 434.37$ K, $u_{1,opt} = 608.69$ K, $u_{2,opt} = 474.07$ K, $u_{3,opt} = 711.36$ K, and $u_{4,opt} = 388.74$ K. The maximum power output is $P^* = 7321.60$ W.

Conclusions

This paper gives a method to calculate the optimal temperature and maximum power output of complex system which consists of several heat reservoirs, finite thermal capacity subsystems with different temperatures, and a transformer, and obtains the analytical expressions of the temperatures and the power based on ref. [25]. The heat transfer is assumed to be linear phenomenological heat transfer law. In general, the results obtained in this paper are different from those obtained in ref. [25]. The method presents herein provide a means for improving evaluation of the temperature distribution and the mechanical energy limits in practical complex system.

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Nomenclature

A	– matrix
b	– components of inverse matrix
C	– matrix
L	– Lagrange function
m	– quantity of subsystems, [–]
n	– quantity of all reservoirs and subsystems, [–]
P	– power output, [W]
q	– heat flow, [W]
R	– sum of heat flows, [W]
r	– quantity of subsystems with free temperatures, [–]
T	– matrix
T	– temperature of reservoirs or subsystems, [K]
u	– temperature of working fluid of the transformer, [K]

Greek letters

α	– heat conductivity, [WK]
$\bar{\alpha}$	– sum of heat conductivities, [WK]
β	– Lagrange multiplier, [–]
λ	– Lagrange multiplier, [–]
σ	– entropy flow, [WK ⁻¹]

Subscripts and superscripts

i	– i^{th}
j	– j^{th}
*	– maximal
m	– m^{th}
r	– reservoir
s	– subsystem
opt	– optimal

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