

**MEASURABLE VALUES, NUMBERS AND FUNDAMENTAL
PHYSICAL CONSTANTS:
IS THE BOLTZMANN CONSTANT k_B A FUNDAMENTAL
PHYSICAL CONSTANT?**

by

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The status of fundamental physical constants is discussed. The nature of fundamental physical constants is cleared up, based on the analysis of the Boltzmann constant. A new definition of measurable values, “mathematical” and “physical” numbers and fundamental physical constants is proposed. Mathematical numbers are defined as values insensitive to the choice of both units and frames of reference, whereas “physical numbers” are dimensionless values, insensitive to transformations of units and sensitive to the transformations of the frames of reference. Fundamental constants are classified as values sensitive to transformations of the units and insensitive to transformations of the frames of reference. It is supposed that a fundamental physical constant necessarily allows diminishing the number of independent etalons in a system of units.

Key words: *the Boltzmann constant, entropy, fundamental constant, dimensional value, dimensionless value, lex parsimoniae, invariant relationships*

**Introduction: measurable values, numbers, and
fundamental physical constants**

It is customary to divide physical notions into dimensional quantities, dimensionless quantities (numbers), and fundamental physical constants. Let us take as an example Newton’s famous Law of Gravitation: $F = G(Mm/r^2)$, where F is the force exerted on the object, M and m are point (or spherically symmetric) gravitational masses, r is the distance between the masses (or their centers), and G is the gravitational constant, $G = 6.67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$. It is easily seen that Newton’s Law of Gravitation involves each kind of physical quantity: F , M , m , and r are dimensional quantities, G is a fundamental physical constant, and the exponent 2 is dimensionless. The difference between dimensional and dimensionless parameters (numbers) is well-accepted as fundamental. Scientific thinking presumes that numbers are products of the human mind; in contrast, measurable quantities exist objectively, whatever we mean by the adverb “objectively”. Fundamental physical constants remain separate, in spite of the fact that they are measurable; their status in physics is unique. The existence of fundamental physical constants makes possible physical laws, that is invariant relationships between physical quantities.

We may naturally ask for a criterion enabling us to distinguish between dimensional quantities, dimensionless quantities, and fundamental physical constants. Are these concepts separated by impenetrable barriers? Our analysis of the Boltzmann constant and entropy will

show that the situation is complicated and needs clearing up. Moreover, we will demonstrate that the difference between dimensional and dimensionless quantities is not as definite as it usually assumed. We also will propose the “*lex parsimoniae*” inspired criteria allowing distinction between the main kinds of physical values.

Is the Boltzmann constant a fundamental physical constant?

We will start from a detailed analysis of the physical and philosophical natures of the Boltzmann constant. There are two reasons to commence here. First of all, the status of the Boltzmann constant in thermal physics is unclear. A clarification of this status will allow us to formulate the general definition of a fundamental physical constant. Secondly, the Boltzmann constant is connected with entropy, which is numerable but is not a measurable physical quantity.

Let us start from the well-accepted in physics and chemistry definition of the Boltzmann constant. According to the IUPAC Compendium of Chemical Terminology, the Boltzmann constant $k_B = 1.380\,658\,(12) \cdot 10^{-23}$ J/K is a *fundamental* physical constant relating temperature to energy [1]. As a rule, the Boltzmann constant is included in the general list of fundamental physical constants [2-5]; in the standard college and university textbooks by Resnik, Holliday, Krane, and Sears, Zemansky it is called the *universal* constant [6, 7].

However, the classical textbook of statistical physics by Landau and Lifshitz displays a different attitude towards the Boltzmann constant [8]. According to Landau and Lifshitz, as well as other respectable sources, the Boltzmann constant plays a much more modest role in physics. It is no more than a numerical coefficient transforming one unit of energy into another, similar to the numerical coefficient transforming yards to meters. The same attitude towards the Boltzmann constant is expressed in the Feynman and Berkeley Lectures of Physics [9, 10].

Yet another possible approach to the Boltzmann constant was demonstrated by Kittel [11, 12]. He introduced first the *statistical* temperature τ as defined by:

$$\frac{1}{\tau} = \frac{\partial S}{\partial U} \quad (1)$$

where S , U , and N are the entropy, the energy of the system and the number of particles, respectively. The reciprocal *thermodynamic* temperature $1/T$ is introduced by Kittel as the integration factor for the heat value dQ . For the reversible processes $dS = dQ/T$ is a full differential. According to Kittel, this statement constitutes the second law of thermodynamics. The existence of a proportionality between the statistical and thermodynamic temperatures, given by the equation:

$$\tau = k_B T \quad (2)$$

is recognized by Kittel as an *experimental* fact. This suggests that Kittel is inclined to consider k_B a fundamental constant. Thus, we conclude that the attitude of the scientific community towards the Boltzmann constant is at the very least incoherent.

The problem has a long and honorable history. The Boltzmann constant was originally introduced in 1900 by Plank in his analysis of blackbody radiation. Plank assumed that the Boltzmann constant is a fundamental constant, and that when it is established for molecular motions, it will be the same for radiation phenomena [13]. However, as was pointed out above, this approach is not generally accepted today [8-10], and the situation calls for additional insights.

What is the physical meaning of the Boltzmann constant?

The discussion about the true nature of the Boltzmann constant is not merely semantic or scholastic. The problem touches the deepest foundations of thermodynamics and the understanding of the nature of fundamental constants. One of today's most debated topics in thermodynamics is whether it is possible to reduce thermodynamics to statistical mechanics [14-16]. It is generally assumed that statistical mechanics explains the fundamental nature of thermal behaviors, whereas thermodynamics can describe them only phenomenologically [8, 14-16]. We want to emphasize that k_B plays the key role in this reduction, connecting the statistical and thermodynamic temperatures according to eq. (2). Indeed, accepting that k_B is no more than a numerical coefficient transforming one temperature to another supports the idea that the total reduction of thermodynamics to statistical mechanics is possible; on the contrary, if k_B is a universal constant the situation is much more complicated.

As a matter of fact, we do not have a precise definition of what makes a constant universal or fundamental. The Encyclopedia Britannica calls the gravitational constant G a physical constant and the Planck constant \hbar a universal constant, however, the exact difference remains unclear. The Encyclopedia Britannica, as well as the highly authoritative "Physical Encyclopedia," states that fundamental constants are physical values allowing invariant relationships between measurable physical values. Note that we have already demonstrated that the gravitational constant G allows such a relationship between forces, distances, and masses [17].

It is also latently accepted that fundamental constants allow invariant relationships between physical values *different in kind*. Therein lies a basic difference between the universal constants and the numerical coefficients that transform one type of measurement into another, since the latter only relate values of the same physical nature. However, accepting this criterion of "fundamentality" leaves many things unclear; for instance, why do the charge and mass of an electron deserve places on the list of fundamental physical constants? Thus we conclude that the physical and philosophical status of fundamental constants is at least vague and calls for elucidation. It has to be emphasized that leading physicists even do not agree in the question: how many fundamental constants does physics need? Duff, Okun, and Veneziano in their recent "Dialogue on the number of fundamental constants" uphold just incompatible opinions, *i. e.* Okun argues in favor of three fundamental constants: \hbar , c , and G , according to Veneziano there are two fundamental dimensionful constants, and Duff asserts that physics at all does not need the notion of the fundamental constants [18]. According to Duff " \hbar , c , and G are nothing but conversion factors *e. g.* mass to length, energy to frequency and energy to mass". And they are "no different from Boltzmann's constant" [18].

In the same paper Okun states that: "physics consists of measurements, formulas, and words" [18]. We will focus in our paper on words which form the field common for physics and philosophy. We will propose a criterion inspired by Occam's razor that will allow us to redefine fundamental physical constants.

Fundamental physical constants and systems of units

The procedure of measurement, that is, of comparing the measured value with a standard unit constitutes the basis of empirical science. Such comparisons require a comprehensive system of units to be used for reference. It is reasonable to inquire how many standard units such a system should include.

The conventional MLT system of units (such as CGS system) is based on three standard units: the unit of mass M , the unit of distance L , and the unit of time T . Thus, the dimension of any physical value A can be expressed as:

$$[A] = M^\alpha L^\beta T^\gamma \quad (3)$$

The principle of Occam's razor known also as "parsimony law" implies that the optimal system of units would include exactly one standard unit. The existence of fundamental constants, which provide invariant relationships between measurable values, allows us to decrease the number of independent units; for example, in a quantum field theory it is convenient to use the system of units in which $c = \hbar = 1$. The only standard unit in this system is the fundamental unit L , thus the dimension of A is given by:

$$[A] = L^\alpha c^\beta \hbar^\gamma \quad (4)$$

It is apparent that the desire to decrease the number of standard units has roots outside the realm of physics. It is also clear that it will be impossible to construct a system of physical units based on L , c , and k_B , L , M , and k_B , or L , \hbar , and k_B . The Boltzmann constant does not allow us to decrease the number of standard units that comprise our system. However, using the mass and charge of an electron allows us to make such a reduction. This is realized in the Hartree system of units. Thus, our analysis permits us to redefine a fundamental or universal constant. Such a constant must allow an invariant relationship between physical quantities of different kind, and furthermore, must necessarily decrease the number of independent units in a system of units. We believe no distinction between universal and fundamental constants is necessary. Finally, let us show that one more important property of fundamental constants has to be taken into account.

Fundamental physical constants, numbers and measurable values in their relation to the transformation of frames of reference and basic units

We already concluded that the Boltzmann constant *is not* a true fundamental physical constant. Thus, the famous Boltzmann equation $S = k_B \ln \Omega$, relating the entropy of a system S and Ω , the number of distinct microscopic states available to the system, could be rewritten as $S = \ln \Omega$. This could be done by an appropriate choice of a system of units, in which the temperature is measured in the units of energy, and $k_B = 1$ (see ref. 8).

Let us continue our analysis. It can be seen that the entropy of a system defined in such a way becomes not only *dimensionless* but also *numerable* value, in contrast to all other known physical values which are *measurable*. Hence entropy becomes an object of the same nature as the number π , the exponent in the Law of Gravitation, and the fine structure constant $\alpha = e^2/\hbar c$. Indeed, entropy, defined as $S = \ln \Omega$, is not sensitive either to transformations of the units or the frames of reference; in fact, the entropy of the system is invariant in the Theory of Relativity [19]. It is noteworthy that there exist dimensionless values insensitive to transformations of units and sensitive to the transformations of the frames of reference such as the relativistic multiplier $\gamma = (1 - v^2/c^2)^{1/2}$.

Numbers, according to philosophical tradition, are very different from measurable physical values. We suggest that actually the distinction between numbers and measurable values could be revealed relatively to transformations of units and frames of reference. Moreover, we propose to *define* true or mathematical numbers as the values invariant under transforma-

tions of both the units and the frames of reference. Thus all known quantities could be classified as: (a) dimensional quantities, sensitive to transformations of units and the frames of reference, (b) dimensionless quantities, insensitive to transformations of units and sensitive to the transformations of the frames of reference; these magnitudes could be called “physical numbers”, (c) fundamental physical constants, sensitive to transformations of the units and insensitive to transformations of the frames of reference, and (d) numbers (including entropy!) which are insensitive to the choice of both units and frames of reference; these quantities could be called “mathematical numbers”.

Continuing our analysis of entropy, it is reasonable to suggest that the number 1 becomes a fundamental unit for mathematical numbers (including entropy), and all aforementioned values could be defined as *measurable*. Both dimensional and dimensionless values obtain their meanings when measured, that is, when compared to their reference units: dimensional values, when compared to their reference unit and dimensionless values, when compared to the number 1.

Duff and Veneziano agree in their recent paper that “physics is always dealing, in the end, with dimensionless quantities” and “all matters are pure numbers” [18]. If it is so the number 1 definitely becomes the most important fundamental constant. Duff when trying to develop the criterion allowing clear distinction between fundamental constants and other physical quantities, introduces alien and asks “whether there are any experiments that can be performed which would tell us whether the alien’s universe has the same or different constants of nature as ours. If the answer is yes, we shall define these constants to be fundamental otherwise not” [18]. We suggest that the number 1 will be the fundamental constant for all possible universes.

Actually difference between dimension and dimensionless quantities then becomes apparent only with respect to transformations of frames of reference or basic units. We conclude that fundamental physical constants could be defined as dimensional physical values allowing invariant relationships between physical values different in kind and insensitive to transformations of the frames of reference (a similar approach was developed recently by Wilczek [20]). Their distinctive feature is that they allow us to decrease the number of independent standard units in our system of units.

Conclusions

The status of the Boltzmann constant is clarified using a new approach to classification of physical quantities. The distinction between “mathematical” and “physical” numbers is proposed. The distinction is based on the relation to transformations of units and frames of reference. Mathematical numbers are defined as values insensitive to the choice of both units and frames of reference, whereas “physical numbers” are dimensionless values, insensitive to transformations of units and sensitive to the transformations of the frames of reference. Fundamental constants are classified as values sensitive to transformations of the units and insensitive to transformations of the frames of reference. *Lex parsimoniae* based approach suggests that a fundamental physical constant necessarily allows diminishing the number of independent etalons in a system of units. We conclude that the Boltzmann constant *is not* a true fundamental physical constant.

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