

MATHEMATICAL MODELING AND SIMULATION OF A REED VALVE RECIPROCATING AIR COMPRESSOR

by

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Original scientific paper
UDC: 621.514:66.011
DOI: 10.2298/TSCI0903047G

Mathematical modeling is the process of designing a model of a real system and conducting experiments with it for the purpose of understanding the behaviour of the system. Mathematical simulation is widely used for investigating and designing the compressors. Investigations of the processes of reciprocating compressors using mathematical models is an effective tool by high development of computing technique, which enables complicated problems to be solved with a minimal number of simplifying assumptions. A considerable number of previous works has been done on the mathematical modeling and simulation. The aim of the present work is to construct a model which is easy to understand, easy to detect errors in the process of building a model, and easy to compute a solution. This paper presents a simplified and effective mathematical model for the estimation of reciprocating compressor performance using personal computers that can be easily handled. The effect of operating parameters, speed and discharge pressure on thermodynamic behaviour of compressor in working condition has been analysed. The model has been developed for obtaining cylinder pressure, cylinder volume, cylinder temperature, valve lift and resultant torque at different crank angles and free air delivered and indicated power of the compressor. The model has been validated using experimental results.

Key words: *resultant torque, indicated power, peak pressure, free air delivered, volumetric efficiency*

Introduction

A reciprocating compressor consists of a crankshaft (driven either by a gas engine, electric motor, or turbine) attached to a connecting rod, which transfers the rotary motion of the crankshaft to the piston. The piston performs the reciprocating motion in a cylinder. The piston acting within the cylinder then compresses the air contained within that cylinder. Air enters the cylinder through a suction valve at suction pressure and is compressed to reach the desired discharge pressure. When the air reaches the desired pressure, it is then discharged through a discharge valve. Desired discharge pressure can be reached through utilization of either a single or double acting cylinder. In a double acting cylinder, compression takes place at both the head end and crank end of the cylinder. The cylinder can be designed to accommodate any pressure or capacity, thus making the reciprocating compressor the most popular in the gas industry. Building a mathematical model for any project may be a challenging, yet interesting, task. A thorough understanding of the underlying scientific concepts is necessary. In industry and engineering, it is common practice for a team of people to work together in building a model, with the individual

team members bringing different areas of expertise to the project. Once the model has been developed and applied to the problem, the resulting model solution must be analyzed and checked for accuracy. It may require modifying the model for obtaining reasonable outcome. This refining process should continue until obtaining a model that agrees as closely as possible with the real world observation.

Operation and analysis of actual compressor

Operation and thermodynamic analysis of ideal compressor is very simple, due to the following reasons.

- The suction is assumed to be reversible constant pressure process. This means that the velocity of the air is equal to velocity of piston. This is possible by either of the two ways:
 - (1) the diameter of the cylinder should be equal to the diameter of the suction port, *i. e.*, the flow area is equal to cylinder cross-sectional area, and
 - (2) the compressor speed should be equal to zero.

There is no flow resistance in the suction line and air enters the cylinder at atmospheric temperature and pressure. It is very easy to calculate the mass of air sucked in per cycle or work transfer and other properties using simple thermodynamic equations. The analysis does not depend on the valve dimensions.
- The compression is assumed to follow $pV^n = C$. The exponent of compression is constant and is generally taken as 1.2 to 1.35. At the beginning of compression the pressure of the air is taken as atmospheric pressure. At any crank angle, it is easy to calculate the temperature, pressure, work transfer, heat transfer, *etc.*, using thermodynamic equations.
- Discharge, similar to the suction is assumed to be reversible constant pressure process. There is no flow resistance in the discharge line and the pressure of the discharged air is equal to the reservoir pressure.
- Expansion is similar to the compression process, with constant exponent of expansion.
- The volumetric efficiency of the compressor is calculated based on pressure ratio and clearance ratio. Volumetric efficiency is constant at all the speeds no other effects affect the volumetric efficiency.

But, in actual compressors, the operation and analysis is completely different, due to the following reasons.

- In the actual compressors, the suction or discharge valve plays a vital role. The port diameter will never be equal to cylinder diameter. The flow area will never be equal to the cylinder cross-sectional area. It is not possible to get constant pressure suction. The air enters the cylinder through filter and head. This creates a pressure drop in the flow path. The pressure during suction is affected by the following reasons:
 - (1) the diameter of the port is not equal to the diameter of the cylinder. Therefore, the mass corresponding to the volume displaced by the piston will not be equal to the mass entering the cylinder through the suction port or flow area.
 - (2) the wall temperature will not be equal to atmospheric temperature as in the case of ideal compressors. The movement of the piston with piston rings generates heat and it is dissipated to the air and cylinder wall. This heat addition to the air rises suction temperature and affects the pressure pattern during suction.
 - (3) at earlier stages of compression, the wall temperature will be greater than atmospheric temperature and cylinder air temperature. There will be heat transfer from wall to the air and this will increase the work transfer during compression and therefore the exponent of compression will be greater than adiabatic index of 1.4. At later stages there will be

reverse effect and exponent of compression will be less than 1.4. It is not possible to determine the various properties from simple thermodynamic equations.

- During discharge process, there will be a pressure rise due to the following reasons:
 - (1) the mass corresponding to the volume displaced by the piston discharge process will not be equal to the mass going out of the cylinder,
 - (2) the air is discharged to the reservoir through head and air drier. There will be a pressure drop in discharge line and this will increase the compressor work and pressure in the cylinder, and
 - (3) there may be intermediate opening and closing of valves during discharge. And this will create pressure pulsation during the discharge process.
- Similarly, during expansion process, the exponent of expansion will not be constant due to heat transfer effect as in the case of compression.
- The volumetric efficiency of the compressor can not be calculated based on the pressure ratio and clearance ratio, since the constant pressure ratio in the cycle is ruled out. Therefore it should be calculated from the actual mass delivered per cycle.

Model formation

Modeling is based on the following thermodynamic equations:

Suction

$$mC_v \frac{dT}{dt} - \frac{mRT}{V} \frac{dV}{dt} - \frac{dm_s}{dt} (C_v T - RC_v T_s) - \frac{dQ}{dt} - \frac{\Delta\theta}{\omega} = 0 \quad (1)$$

Compression and re-expansion

$$mC_v \frac{dT}{dt} - \frac{mRT}{V} \frac{dV}{dt} - \frac{dQ}{dt} = 0 \quad (2)$$

Discharge

$$mC_v \frac{dT}{dt} - \frac{mRT}{V} \frac{dV}{dt} - \frac{dm_d}{dt} (RC_v T_d - C_v T) - \frac{dQ}{dt} - \frac{\Delta\theta}{\omega} = 0 \quad (3)$$

The governing equation for determining the instantaneous cylinder pressure is:

$$\frac{pv}{RT} = 1 - \sum_{i=1}^n \text{Bi}(T) \rho_i \quad (4)$$

Note: The second term in eq. (4) is negligible for single stage reciprocating air compressors.

The governing equation for determining the mass flow is:

$$\frac{dm}{d\theta} = \frac{dm_i}{d\theta} - \frac{dm_o}{d\theta} - \frac{dm_{op}}{d\theta} \quad (5)$$

The governing equation for determining the working volume is:

$$\frac{dV}{d\theta} = F \frac{S}{Z} \sin \theta - \frac{n \sin \theta \cos \theta}{\sqrt{1 - n^2 \sin^2 \theta}} \quad (6)$$

Cylinder volume at any crank angle (θ) can be calculated using:

$$V_\theta = V_c + \frac{\pi}{4} D^2 r (1 - \cos \theta) - \frac{\sin^2 \theta}{2} \frac{l_c}{r} \quad (7)$$

Acceleration of piston at any crank angle (a_p) is calculated using:

$$a_p = \omega^2 r \cos \theta \frac{\cos 2\theta}{\frac{r}{l_c}} \quad (8)$$

Resultant force on the piston (F_p) is calculated using:

$$F_p = \frac{\pi}{4} D^2 (p - p_{\text{case}}) - m_{\text{rec}} a_p \quad (9)$$

Resultant force on the crank (F_c) is calculated using:

$$F_c = F_p \sin \theta \frac{\sin 2\theta}{2 \sqrt{\frac{l_c^2}{r^2} \sin^2 \theta}} \quad (10)$$

Resultant torque (T) is calculated using:

$$T = F_p r \sin \theta \frac{\sin 2\theta}{2 \sqrt{\frac{l_c^2}{r^2} \sin^2 \theta}} \quad (11)$$

Indicated power

Since all the processes are not following particular thermodynamic law, it is not advisable to use readily available equation for finding out indicated power (IP) during suction or discharge process. Figure 1 shows the variation of pressure and volume during incremental crank angle and volume during incremental crank angle on p - V diagram used to estimate the indicated power. The following general and effective model is used for estimating IP during any incremental crank angle:

$$IP_\theta = IP_{\theta-1} + (V_{\theta-1} - V_\theta) \frac{P_{\theta-1} - P_\theta}{2} \frac{N}{60} \quad (12)$$

Area of port in the valve plate (A_o) is calculated using:

$$A_o = \frac{\pi}{4} d_o^2 \quad (13)$$

Loss due to back flow is estimated using:

$$m_b = \rho \frac{\pi}{4} d_o^2 SZ \quad (14)$$

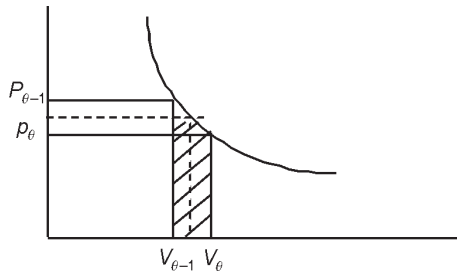


Figure 1. Integration method of estimating indicated power

Compression process

The cylinder pressure at any crank angle (p_θ) can be calculated using:

$$p_{\theta} = p_{\theta 1} \frac{V_{\theta 1}^{n_c}}{V_{\theta}} \quad (15)$$

Temperature of air during compression (T_c) is calculated using,

$$T_{c\theta} = T_{c\theta 1} \frac{V_{\theta 1}^{n_c - 1}}{V_{\theta}} \quad (16)$$

Discharge process

Net force on the discharge valve (F_d) is calculated using:

$$F_d = (p_{\theta} - p_{hd})A_{od}Z_d \quad (17)$$

Initial force on the valve (F_{di}) is calculated using:

$$F_{di} = (p_{ed} - p_{hd})A_{od}Z_d \quad (18)$$

Deflection of delivery reed is calculated from:

$$S_d = \frac{F_d x_d^3 (l_d - x_d)(4l_d - x_d)}{12EI_d l_d^3} \quad (19)$$

The eq. (19) is used for estimating deflection of delivery reed considering the delivery reed to be a propped cantilever beam. The delivery reed valve in closed position and in full open position is shown in fig. 2 and 3.

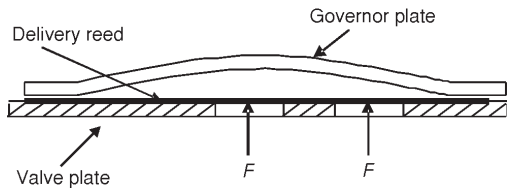


Figure 2. Delivery reed in closed position

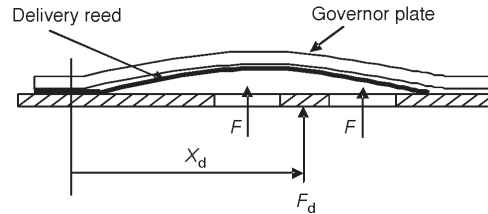


Figure 3. Delivery reed in full open position

Flow area in the I-mode is:

$$A_{fd} = \pi d_{od} S_d \quad (20)$$

Mass of air discharged out during incremental angle (m_{od}) is:

$$m_{od} = \rho_d A_{fd} C_{dd} C_d \sqrt{V_o^2 \frac{2n_d}{n_d - 1} \frac{p_{\theta}}{\rho_{\theta}} \left(1 - \frac{p_d}{p_{\theta}}\right)^{\frac{n_d - 1}{n_d}} \frac{\Delta\theta}{\omega}} \quad (21)$$

$$C_d = \frac{A_{od}}{A_{od}} \frac{A_{fd}}{A_{od}} \frac{A_c}{A_c} \frac{A_{fd}}{A_c} \quad (22)$$

Note: The flow is taking place from cylinder to the reservoir through port, valve, and head. The above correction factor (C_d) should be used which will increase the flow area.

$$V_o = \frac{A_c}{A_{od}} \frac{V_p}{Z_d} \quad (23)$$

Temperature of air in the cylinder is calculated using:

$$T_{\theta} = \frac{m_{rd\theta-1} C_v T_{\theta-1} + m_{od} C_p T_{\theta-1} + \frac{V_p^2}{2}}{m_{rd\theta} C_v} \quad (24)$$

Velocity of piston is calculated using:

$$V_p = \omega r \sin \theta = \frac{r}{l_c} \sin 2\theta \quad (25)$$

Mass remaining in the cylinder at $\theta(m_{rd})$ is calculated from:

$$m_{rd\theta} = m_{rd\theta-1} - m_{od\theta} \quad (26)$$

Pressure at any crank angle (p_{θ}) is calculated from:

$$p_{\theta} = \frac{m_{rd\theta} R T_{d\theta}}{V_{\theta}} \quad (27)$$

Total mass discharged per cycle is:

$$m_{od} = \sum m_{od\theta} - \sum m_{bd} \quad (28)$$

Expansion process

The cylinder pressure at any crank angle (p_{θ}) can be calculated using:

$$p_{\theta} = p_{\theta-1} \left(\frac{V_{\theta-1}}{V_{\theta}} \right)^{n_c} \quad (29)$$

Temperature of air during expansion (T_e) is calculated using:

$$T_{e\theta} = T_{e\theta-1} \left(\frac{V_{\theta-1}}{V_{\theta}} \right)^{n_c - 1} \quad (30)$$

Suction process

Net force on the suction valve (F_s) is calculated using:

$$F_s = (p_{hs} - p_{\theta}) A_{os} Z_s \quad (31)$$

Initial force on the valve (F_{si}) is calculated using:

$$F_{si} = (p_{hs} - p_{es}) A_{os} Z_s \quad (32)$$

Natural frequency of suction valve (ω_{ns}) in I-mode is estimated using:

$$\omega_{ns} = 3.55 \sqrt{\frac{EI_s}{m_s l_s^3}} \quad (33)$$

Natural frequency of valve in II-mode:

$$\omega_{ns} = 22 \sqrt{\frac{EI_s}{m_s l_s^3}} \quad (34)$$

Suction valve stiffness (k_s) is estimated using:

$$k_s = m_s \omega_{ns}^2 \quad (35)$$

The suction reed valve in closed and in full open position is shown in figs. 4 and 5.

Using effective valve dynamics:

$$S_s = \frac{F_s - F_{si}}{k_s J_s} \quad (36)$$

where

$$J_s = \sqrt{1 + \frac{\omega^2}{\omega_{ns}^2} + 2\xi \frac{\omega}{\omega_{ns}} + \frac{\omega^4}{\omega_{ns}^4}} \quad (37)$$

Flow area in I-mode is:

$$A_{fs} = \pi d_{os} S_s \quad (38)$$

Flow area in II-mode is:

$$A_{fs} = \pi d_{os} (S_{s,max} + S_s \text{ in second mode}) \quad (39)$$

$S_{s,max}$ = maximum suction valve lift at the distance x_s and is calculated using:

$$S_{s,max} = \frac{h_s x_s}{l_s} \quad (40)$$

The air enters the port in the valve plate axially, and flows through the gap between the reed valve and valve plate radially. The flow passage between the reed and valve plate can be considered as a nozzle. Considering the flow through the suction reed be a nozzle flow, the following equation can be used to estimate mass of air sucked in through the suction reed.

Mass of air entering during incremental angle (m_{os}) is calculated using:

$$m_{od} = \rho_\theta A_{fd} C_{ds} C_s \sqrt{V_o^2 \frac{2n_s}{n_s - 1} \frac{p_\theta}{\rho_\theta} \left[1 - \frac{p_\theta}{p_{es}} \right]^{\frac{n_s - 1}{n_s}} \frac{\Delta\theta}{\omega}} \quad (41)$$

$$C_s = \frac{A_{os}}{A_{os}} \frac{A_{fs}}{A_c} \frac{A_c}{A_c} \frac{A_{fs}}{A_c} \quad (42)$$

Note: The flow is taking place from cylinder through port, valve, and head. The above correction factor (C_s) should be used which will increase the flow area.

$$V_o = \frac{A_c}{A_{os}} \frac{V_p}{Z_s} \quad (43)$$

Mass remaining in the cylinder at θ (m_{rs}) is calculated from:

$$m_{rs\theta} = m_{rs\theta-1} - m_{os\theta} \quad (44)$$

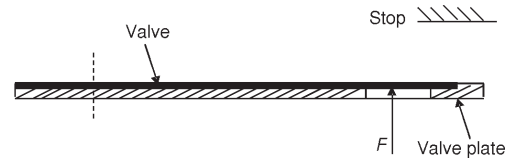


Figure 4. Suction reed in closed position

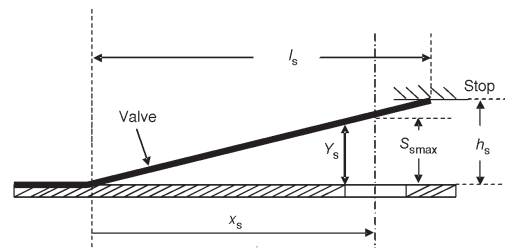


Figure 5. Suction reed in full open position

Total mass drawn in per cycle is:

$$m_{os} = \sum m_{os\theta} \quad (45)$$

Temperature of air in the cylinder is calculated from:

$$T_{s\theta} = \frac{m_{rs\theta} T_{s\theta-1} + m_{os\theta} T_{s\theta}}{m_{rs\theta}} \quad (46)$$

Pressure at any crank angle (p_θ) is calculated from:

$$p_\theta = \frac{m_{rs\theta} R T_{s\theta}}{V_\theta} \quad (47)$$

Total mass sucked-in per cycle is:

$$m_{os} = \sum m_{os\theta} - \sum m_{bs} \quad (48)$$

Free air delivered (FAD) by the compressor is calculated using:

$$FAD = \frac{m_{od} R T_a}{p_a} - \frac{m_{os} R T_a}{p_a} \quad (49)$$

Volumetric efficiency (η_v) of the compressor is calculated using:

$$\eta_v = \frac{m_{od} R T_a}{p_a V_s} \quad (50)$$

Experimental details

The compressor with reed valve used in braking system of heavy passenger vehicles and trucks is tested using sophisticated test rig. The detailed experimental setup is shown in fig. 6. The compressor is run by an electric motor. The pressure inside the cylinder is captured by

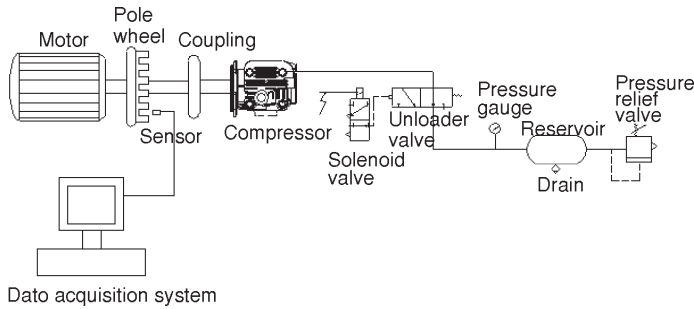


Figure 6. Experimental set up

piezo-electric pressure transducer and the data is stored using data acquisition system. The compressor speed is controlled by a speed pot in the control panel. The compressor is cooled by a fan. The compressor is connected with 50 liter reservoir and the pressure is maintained by using the governor valve. The air flow is measured by pump up time method. In this method the time taken for incremental pressure rise in the reservoir is measured. From the reservoir volume, pressure rise and the air flow rate is measured. The shaft power is measured using the energy meter.

Compressor details:

Bore diameter (D) – 66.67 mm, crank radius (r) – 23 mm, connecting rod length (l_c) – 70 mm, suction reed lift (h_s) – 2.2 mm, delivery reed lift (h_d) – 1.8 mm, mass of reciprocating parts (m_{rec}) – 0.245 kg, clearance volume (V_c) – 4.5 to 6.7 cm³, atmospheric pressure (p_a) – 1 bar, discharge pressure (p_d) – 5 to 9 bar (abs.), compressor speed (N) – 600 to 3000 rpm, diameter of suction

port (d_{os}) – 11 mm, diameter of delivery port (d_{od}) – 11 mm, distance (x_s) – 59 mm, distance (x_d) – 26.5 mm, effective length of suction reed (l_s) – 71 mm, effective length of delivery reed (l_d) – 45.5 mm, mass of suction valve (m_s) – 7 g, mass of delivery valve (m_d) – 2 g, number of suction ports (Z_s) – 4, and number of delivery ports (Z_d) – 2.

Results and discussions

The predicted results from the mathematical model and the corresponding experimental results are shown in tab. 1.

Table 1. Performance of compressor at different discharge pressures ($N = 3000$ rpm)

Results	6 bar Pred.	6 bar Exp.	7 bar Pred.	7 bar Exp.	8 bar Pred.	8 bar Exp.	9 bar Pred.	9 bar Exp.
Peak pressure [bar]	8.09	8.05	9.20	9.08	10.23	10.37	11.81	11.42
Free air delivered [litre per minute]	305.7	303	294	300	282.7	276	264.7	264
Volumetric efficiency [%]	63.5	63.1	61.0	62.5	58.7	57.5	54.9	55.0
Shaft power [W]	2284	2250	2206	2230	2300	2394	2534	2542

The cylinder air pressure variation with cylinder volume and crank angle is shown in figs. 7 and 8.

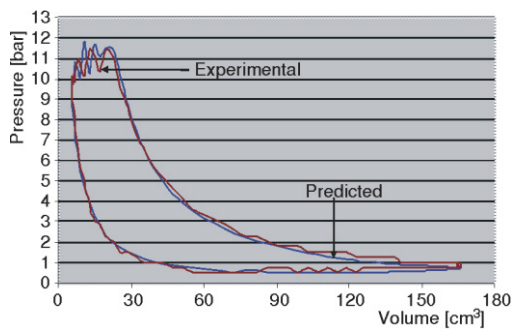


Figure 7. Pressure – volume diagram
(speed = 3000 rpm)
(color image see on our web site)

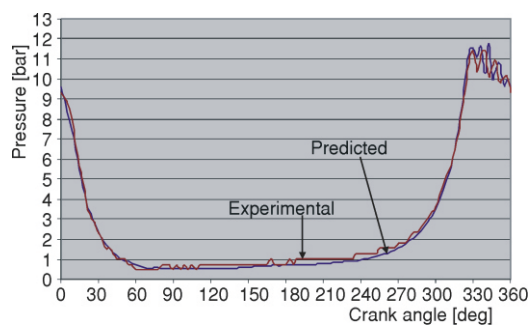


Figure 8. Pressure – crank angle diagram
(speed = 3000 rpm)
(color image see on our web site)

The increase in cylinder pressure for particular reservoir pressure during discharge process is due to the delay in full opening of the delivery valve. The delay and reduced volume of air through the discharge port compared to the volume displaced by the piston cause the compression of air along with discharge at the earlier stages of discharge process. Similar phenomenon exists in the suction side also. The predicted values of valve lift and torque at different crank angles are shown in figs. 9 and 10. The discharge and suction valves experience different pressure differences during the discharge and suction processes. This makes the valve fluttering and fluctuation in the pressure during the process. The loss in pressure during suction and increase in pressure during discharge directly affects the actual volume of air handled by the compressor.

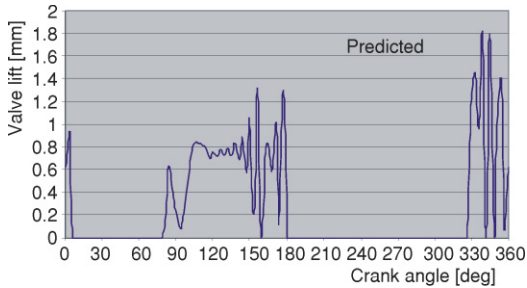


Figure 9. Valve lift – crank angle diagram (speed = 3000 rpm)

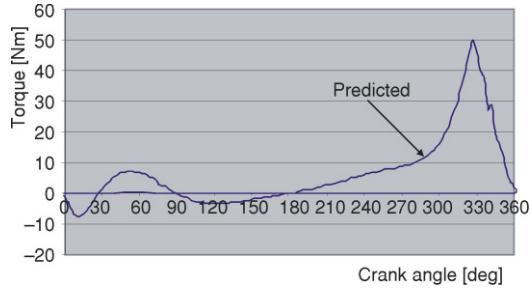


Figure 10. Torque – crank angle diagram (speed = 3000 rpm)

Due to excess peak pressure during discharge process, the indicated power of the compressor is always greater than the ideal indicated power for a particular FAD and thus increasing the shaft power. The volumetric efficiency is mainly dependent on the suction pressure. The effect of reduced suction pressure is to reduce the volumetric efficiency significantly. The model has been tested with different discharge pressures and compressor speeds. The simulated results are in good agreement with the experimental results which ensure the accuracy of the model. Figures 11, 12, and 13 show the comparison of predicted and experimental value of FAD, volumetric ef-

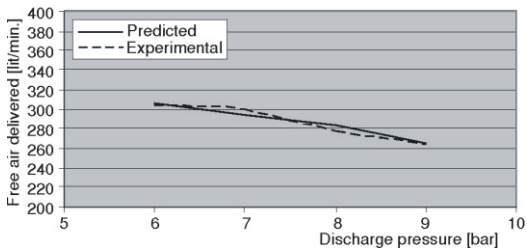


Figure 11. Free air delivered – discharge pressure diagram (speed = 3000 rpm)

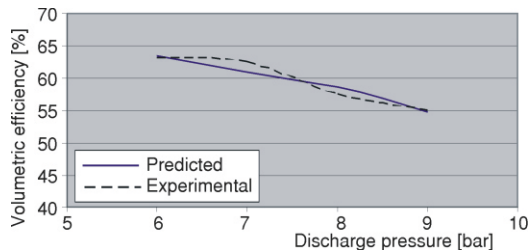


Figure 12. Volumetric efficiency – discharge pressure diagram (speed = 3000 rpm)

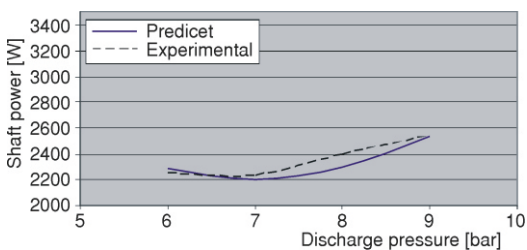


Figure 13. Shaft power – discharge pressure diagram (speed = 3000 rpm)

iciency, and shaft power at various discharge pressures.

Conclusions

The developed model predicts fluctuation of pressure during suction and discharge processes. It predicts valve fluttering during suction and discharge at all delivery pressures. The simulated results from the model are very much comparable with the experimental results.

It is possible to get volumetric efficiency, free air delivered, indicated power, cylinder air pressure, cylinder air temperature, resultant torque, and mass of air sucked-in or discharged out per cycle, by varying any operating parameters like, speed, discharge pressure, etc., and physical parameters like, clearance volume, crank radius, connecting rod length and cylinder diameter. The developed simulation model can be used for theoretical analysis of single stage, single

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