A Message from the Guest Editor

A BRIEF APPRAISAL OF THE GOODMAN METHOD WITH SOME PERSONAL STANDPOINTS

Dear readers, this issue of the journal *Thermal Science* contains 11 papers employing *the Goodman heat-balance integral method (HBIM) in solution of variety of problems.* The idea to create this collection came to my mind in 2007 when I solved a specific problem pertinent to heat transfer problems of the fire boilover – see *Thermal Science,* vol. 11 (2007), issue 2. Even though the method is simple and almost 50 years old, the problem solved and the solution outcomes encouraged me to do a deep look what was published for 50 years. The result of this was astonishing amount of articles employing directly or to some extent the HBIM. In the journals of Elsevier, for example, there were published of 450 articles approximately employing this amazing method. Then, the idea to create a special issue dedicated to the 50th anniversary of the Goodman method was realized and approved by the Editor-in-chief. The collection of articles was not easy but finally, I am glad that the dream came true. In fact this special issue is a unique collection of articles devoted to HBIM ever created for 50 years since the seminal Goodman article in 1958.

Commonly the editorial messages stress the attention how great is the problem at issue. I will avoid this point since everybody working on approximate analytical solutions of diffusion equations knows the importance of HBIM and the problems solved by it. I will refer to some moments, which try to highlight the physical background of the method rather than the mathematical tricks. I will express some personal standpoints which might be accepted or rejected by the readers, but they mainly try to explain that the physics behind each mathematical model should be clear and well defined.

The Goodman method is simple as a mathematical idea. However, behind its formal simplicity there is a deep understanding of the physics of heat diffusion process. The conventional constitutive equations of Fourier and Fick relate irreversible diffusional fluxes of heat and mass, respectively, to gradients of temperature and concentrations. Combining these with conservation laws leads to parabolic equations of change. However, all standard equations with parabolic terms have a non-physical property: a disturbance at any point in the medium is felt instantly at every other point; that is, the velocity of propagation of disturbances is infinite. This paradox is clear, in the simple case of heat conduction in semi-infinite solid whose surface temperature may suddenly increase from $T = 0$ to a constant non-zero T_{surface}. The classical *exact* solution of Carslaw and Eager is expressed through error integral and provides $T = 0$ at the time $t = 0$, but for any arbitrary short time and arbitrate large distance x from the wall, the temperature $T(x, t)$ is non-vanishing, implying infinitely fast propagation of the disturbances. This non-physical behaviour has been pointed by many authors and the dilemma has been resolved by acceptance of the concept of flux relaxation leading to the hyperbolic Cattaneo equation. While *hyperbolic* rather than *parabolic* equations are used, the wall heat flux does not start instantaneously, but rater grows gradually with a rate which depends on the relaxation time constant. After some time the wall heat flux reaches a maximum and then decreases, similar to the Fourier case. However, the hyperbolic case of heating of a semi-infinite has a quite realistic feature: two regions exist in a solid; the first in which the heat transfer has already taken place (disturbed region) and the second where the disturbances is not yet present (undisturbed region). In contrast, the Fourier theory predicts the appearance of the disturbances everywhere, even for distance in the undisturbed region, which is of course a non-physical behaviour.

This preceding note on the properties of the *hyperbolic* and the *parabolic* equations and their physical adequacies was especially inserted in the Preface. The Goodman method has *three basic innovations*: (1) **Physical one**, *i. e*. definition of the *heat penetration depth*, coming from the hyperbolic model. This allows the non-physical parabolic model to be *repaired* by a simple tool, *the penetration depth*; (2) **Classical mathematical approach** to solve approximately differential equations by expression of the solutions as series; (3) **The averaging of the heat energy** over the disturbed region, which is a *physical principle*, but allows the Leibniz rule to be applied. The final result is well-known.

In the existing literature on HBIM, the authors usually mention that the Goodman method comes from the idea of the Karman-Polhausen integral method (KPIM) applied to solve the boundary layer problems. However, to my point of view this is not entirely true. The KPIM uses mainly the two last steps, while the heat penetration depth is a concept coming mainly from the true hyperbolic model (and the physics of the diffusion process, of course) rather that the parabolic Fourier theory. Hence, the HBIM might be considered as a successful repairing of the parabolic model by a purely physically based concept of the penetration depth.

The common complain against the HBIM refers to the arbitrary choice of the function used to approximate the temperature distribution in the disturbed region. This could be considered as a matter of arguments. We have two options: (1) to use the *parabolic Fourier theory* and the *error-function solutions*, both non-physical and no-exact, since *the error function solution is also an approximation*, and (2) to simplify the expression of the profile and apply an adequate physical restriction through the definition of the heat penetration depth. To my personal point of view, the Goodman method drew the realistic way to solve complex problems *via* the second approach. We have to remember that many complex problems pertinent to Stefan problems with practical importance in the field of mass transfer, solidification, and thermal protection of rockets and spacecrafts were solved by the inexact Goodman method.

These complain address mainly the mathematical inexactness of HBIM and do not refer the physics behind it. The common approach is to calibrate the HBIM solution to the exact ones expressed through the error integral. The common question is: why we have to calibrate the approximate solution, when the exact one already exists. The answer is straightforward; the HBIM solution is practical, while that assumed as exact is hard to handle in applications. To be exact and correct with respect to the scholars working on HBIM this personal standpoint could be expressed simply: the issue is not to get a solution of a certain problem, but *how it can be used after that.* The first part of this ideology refers to the mathematical approach, while the second one, the practical side of the solution implementation. The HBIM derived solutions are more practical rather than those considered as exact solutions. The calibrated approximated solution can be used many times without a significant loss of exactness, which is quite important when subroutines of large computer codes have to be created. However, the problem addressing the exactness of the solution still remains.

The present collection of articles shows different approaches to improve the Goodman method through:

refining mathematical tools addressing the approximating functions and the numerical methods, and

additional constraints based on first thermodynamic principles.

To my personal point of view this collection of articles is a worthy work done by the entire team of the authors. As Guest editor, another important objective of this special issue was to update the information about the current status of the Goodman method and to make it useful for novel researchers and experts. In this sense, I expect that the articles included here could provide a good initial point for those starting their research in the approximate solution of diffusion equations and, at the same time, they could be a good review of some advances in the Goodman method of heat-balance integral.

I wish to thank the authors for their willingness to contribute to this special issue of the journal *Thermal Science* dedicated to the 50^{th} anniversary of the Goodman method and the referees who reviewed the quality of the submitted contributions. Last but not least, I express my gratitude to Executive editor, Dr. Vukman Bakić who, in fact, did the entire work on the issue completion in time. Finally, a special word of thanks goes to the Editor-in-chief, Prof. Simeon Oka for the encouragement and support of my idea to create this special issue.

May 2009

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