RADIATIVE HEAT TRANSFER WITH HYDROMAGNETIC FLOW AND VISCOUS DISSIPATION OVER A STRETCHING SURFACE IN THE PRESENCE OF VARIABLE HEAT FLUX

by

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The boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field is studied. The equations of motion and heat transfer are reduced to non-linear ordinary differential equations and the exact solutions are obtained using properties of confluent hypergeometric function. It is assumed that the prescribed heat flux at the stretching porous wall varies as the square of the distance from origin. The effects of the various parameters entering into the problem on the velocity field and temperature distribution are discussed.

Key words: *heat transfer, radiation, magnetic field, viscous dissipation*

Introduction

The analysis of the flow through porous medium has become the basis of several scientific and engineering applications. Flow and heat transfer phenomena over a moving flat surface are important in many technological processes, such as the aerodynamic extrusion of plastic sheet, rolling, purification of molten metals from non-metallic inclusion by applying magnetic field, and extrusion in manufacturing processes. In continuous casting, that is the process consists of pouring molten metal into a short vertical metal die or mould (at a controlled rate), which is open at both ends, cooling the melt rapidly and withdrawing the solidified product in a continuous length from the bottom of the mould at a rate consistent with that of pouring, the casting solidified before leaving the mould. The mould is cooled by circulating water around it. The process is used for producing blooms, billets and slabs for rolling structural shaped, it is mainly employed for copper, brass, bronze, and aluminum and also increasingly with cast iron (C. I.) and steel.

However, in real situation one has to encounter the boundary layer flow over a stretching sheet. For example, in a melt-spinning process, the extradite is stretched in to a filament or sheet while it is drawn from the die. Finally, this sheet or filament solidifies while it passes through controlled cooling system.

Sakiadis [1] first investigated the boundary layer flow of a viscous fluid due to the motion of a plate in its own plane and Erickson *et al.* [2] and Gupta *et al*. [3] extended this problem to the case for which suction or blowing existed at the moving surface. Crane [4] and Mc Cormack *et al*. [5] studied the boundary layer flow of a Newtonian fluid caused by stretching of an elastic flat sheet which moves in its own plane with the velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. The uniqueness of the exact analytical solutions followed by two different approaches [4, 5] was proved simultaneously by McLeod *et al.* [6] and Troy *et al.* [7]. Both the basic flow and the heat transfer problems for linear stretching of the sheet have since been extended in various ways. One may, for example, refer to Vleggaar [8], Soundalgekar *et al.* [9], Carragher *et al.* [10], Dutta [11], and Chen *et al.* [12]. Afzal *et al.* [13], Kuiken [14], and Banks [15] considered the power law stretching of the plate ($u \alpha x^{m}$). Banks and Zaturska [16] considered the eigenvalue problem for boundary layer over the stretching plate. The hydromagnetic flow and heat transfer case for linearly stretching plate has been studied by Chiam [17] and Abo-Eldahab *et al.* [18].Unsteady boundary layer flow due to stretching sheet has been considered by Shafie, *et al.* [19], Kechil *et al.* [20], and Liao [21].

These all writers have completely neglected the radiative heat and dissipation due to viscous. Whenever the temperature of surrounding fluid is high, the radiation effects play an important role and this situation does exist in space technology. In such cases one has to take into account the effects of radiation and free convection. In steady flows, such studies presented by Cess [22], Arpaci [23], Cheng *et al.* [24], Hossain *et al.* [25, 26], and Hossain *et al.* [27]. For an impulsively started infinite vertical isothermal plate, Ganeshan *et al.* [28] studied the effects of radiation and free convection, by using Rosseland approximation, and Brewster [29]. Recently, Rashad [30] has studied, numerically, the radiative effects on heat transfer from a stretching surface in a porous medium neglecting the viscous dissipation. In this paper the study of radiation and viscous dissipation effects over a stretching surface subjected to variable heat flux in presence of transverse magnetic field is presented and it has been found that these parameters do affect the plate temperature (recovery temperature).

Governing equations and analysis

Consider a steady two-dimensional incompressible viscous laminar flow of an electrically conducting fluid in the presence of a magnetic field B_0 and radiation transfer over a moving sheet. The x-axis is chosen along the sheet and the y-axis perpendicular to it, the applied magnetic field B_0 is along y-axis. The sheet issues from a thin slit at the origin (0, 0). It is assumed that the speed of a point on the plate is proportional to its distance from the slit, and the boundary layer approximations still applicable. It is also assumed that the prescribed heat flux at the stretching wall varies as the square of the distance from the origin. The steady-state boundary layer equations of mass, momentum, and energy governing the flow are:

$$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} \quad v\frac{\partial u}{\partial y} \quad \vartheta\frac{\partial^2 u}{\partial y^2} \quad \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} \quad v\frac{\partial T}{\partial y} \quad \frac{k}{\rho c_{p}}\frac{\partial^2 T}{\partial y^2} \quad \frac{1}{\rho c_{p}}\frac{\partial q_r}{\partial y} \quad \frac{\vartheta}{c_{p}} \quad \frac{\partial u}{\partial y}^{\;2} \tag{3}$$

where u and v are the fluid velocities in the x- and y- directions, respectively, T is the temperature, ϑ – the kinematics viscosity, ρ – the density, k – the thermal conductivity, c_{p} – the specific heat at constant pressure, and q_{r} – the radiative heat flux in the y-direction.

The appropriate boundary conditions for the problem are:

$$u \quad cx, \quad v \quad v_0, \quad \frac{\partial T}{\partial y} \quad Ax^2, \text{ at } \quad y \quad 0$$
$$u \quad 0, \quad T \quad T_\infty, \quad y \quad \infty \tag{4}$$

where A and c are given positive constants.

We assume the Rosseland approximation [29] for radiative heat flux, which leads to:

$$q_r \quad \frac{4\sigma}{3\kappa^*}\frac{\partial T^4}{\partial y}$$

where σ is the Stefan-Boltzmann constant and κ^* is the mean absorption coefficient.

If the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series for T^4 about T_∞, after neglecting higher order terms, is given by:

$$T^4 \quad 4T_\infty^3 T \quad 3T_\infty^4 \tag{5}$$

The solution of eqs. (1) and (2), satisfying the boundary conditions (4) is:

$$u \quad cxf \ (\eta),$$
$$v \quad \sqrt{\vartheta c} f(\eta),$$
$$\eta \quad \sqrt{\frac{c}{\vartheta}} y \tag{6}$$

where prime denotes differentiations df/dη, and

$$f(\eta) \quad a \quad b\mathrm{e}^{\ \alpha\eta},$$
$$a \quad \frac{\alpha^2 \quad M}{\alpha}, \quad b \quad \frac{1}{\alpha}, \quad \alpha \quad \frac{\lambda \quad \sqrt{\lambda^2 \quad 4(1 \quad M)}}{2} \tag{7}$$

$M =\ \ B^2{}_0/\rho c$ (the magnetic parameter), and $\lambda \quad v_0/\sqrt{\vartheta c}$ (the injection parameter).

In order to solve eq. (3) we assume:

$$T \quad T_\infty \quad A\sqrt{\frac{\vartheta}{c}} x^2\ \theta(\eta) \tag{8}$$

By using eqs. (5), (6), and (8), eq. (3) takes the following form:

$$1 \quad \frac{4}{3N} \ \theta \quad \mathrm{Pr}\, f\theta \ -2\,\mathrm{Pr}\, f\ \theta \quad \mathrm{Pr}\,\mathrm{Ec}\,(f\ \ )^2 \quad 0 \tag{9}$$

subject to the boundary conditions

$$\theta \quad 1 \quad \text{at} \quad \eta \quad 0$$
$$\theta(\infty) = 0 \tag{10}$$

where Ec $\ c^{5/2}/Ac_\mathrm{p}\vartheta^{1/2}$ is the Eckert number, Pr = $\rho\vartheta c_\mathrm{p}/k$ is the Prandtl number, and $N =$ $= \kappa^* k/4\sigma T_\infty^3$ is the radiation parameter.

If we assume $1 + 4/3N = \beta$, then to obtain the solution of eq. (9), we introduce a new variable ξ as follows:

$$\xi \quad \frac{\mathrm{Pr}}{\alpha^2\beta}\mathrm{e}^{\ \alpha\eta} \tag{11}$$

Hence the eq. (9) reduces to:

$$\xi \frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} \quad 1 \quad \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad \xi \; \frac{\mathrm{d}\theta}{\mathrm{d}\xi} \quad 2\theta \quad Q\xi \tag{12}$$

where $Q \quad \mathrm{Ec}\alpha^4\beta/\mathrm{Pr}$.

The corresponding boundary conditions are:

$$\theta \;\; \xi \quad \frac{\mathrm{Pr}}{\alpha^2\beta} \quad \frac{\alpha\beta}{\mathrm{Pr}} \quad \text{and} \quad \theta(\xi \quad 0) \quad 0 \tag{13}$$

The solution of eq. (12) satisfying boundary conditions (13) in terms of the confluent hypergeometric function ${}_1F_1(a\,,c, x)$ [31] is given by:

$$\theta \quad \frac{Q\xi^2}{2(\gamma \quad 1)} \quad C_2\xi^{\mathrm{Pr}\;\;1\;\;\frac{M}{\alpha^2}} \; {}_1F_1 \;\; \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 2;\; \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 1; \xi \tag{14}$$

where $\gamma \quad 1 \quad \mathrm{Pr}[1 \quad (M/\alpha^2)]$ and assuming $(\alpha\beta/\mathrm{Pr}) \quad Q\,\mathrm{Pr}/\alpha^2\beta(\gamma \quad 1) \quad L_1$, the constant C_2 is given by the following equation:

$$L_1 \quad C_2 \quad \frac{\mathrm{Pr}}{\alpha^2\beta}^{\mathrm{Pr}\;\;1\;\;\frac{M}{\alpha^2}} \quad [\beta(M \quad \alpha^2)]_1 F_1 \;\; \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 2;\, \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 1; \quad \frac{\mathrm{Pr}}{\alpha^2\beta}$$

$$C_2 \quad \frac{\mathrm{Pr}}{\alpha^2\beta}^{\mathrm{Pr}\;\;1\;\;\frac{M}{\alpha^2}} \quad \frac{\mathrm{Pr}(\alpha^2 \quad M) \quad 2\alpha^2}{\mathrm{Pr}(\alpha^2 \quad M) \quad 2\alpha^2}\, {}_1F_1 \;\; \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 1;\, \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 2; \quad \frac{\mathrm{Pr}}{\alpha^2\beta}$$

Now, θ in terms of variable η may be expressed as:

$$\theta(\eta) \quad L_2\mathrm{e}^{\;\;2\alpha\eta} \quad L_3\,(\mathrm{e}^{\;\;\alpha\eta})^{\mathrm{Pr}\;\;1\;\;\frac{M}{\alpha^2}} \; {}_1F_1 \;\; \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 2;\, \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 1; \quad \frac{\mathrm{Pr}}{\alpha^2\beta}\mathrm{e}^{\;\;\alpha\eta}$$

where

$$L_2 \quad \frac{\mathrm{Pr}}{\alpha^2\beta}^{2} \quad \frac{Q}{2(\gamma \quad 1)},\, L_3 \quad C_2 \quad \frac{\mathrm{Pr}}{\alpha^2\beta}^{\mathrm{Pr}\;\;1\;\;\frac{M}{\alpha^2}}$$

Recovery temperature

The recovery temperature at the stretching plate is given by:

$$\theta(0) \quad L_2 \quad L_{3\,1}F_1 \;\; \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 2;\, \mathrm{Pr} \;\; 1 \quad \frac{M}{\alpha^2} \quad 1; \quad \frac{\mathrm{Pr}}{\alpha^2\beta}$$

Discussions and results

The flow and heat transfer of a viscous incompressible fluid subjected to variable heat flux with viscous dissipation effect in the presence of a transverse magnetic field caused by a stretching wall is governed by the parameters, namely, the magnetic parameter M, the injection parameter λ, the Prandtl number Pr, the radiation parameter N, and the Eckert number Ec.

The dimensionless temperature distribution $\theta \;\; \eta)$ is plotted against η for different values of M and λ (figs. 1 and 2). It is seen that $\theta \;\; \eta)$ decreases as M increases and it increases with λ. It is also seen that $\theta \;\; \eta)$ is negative.

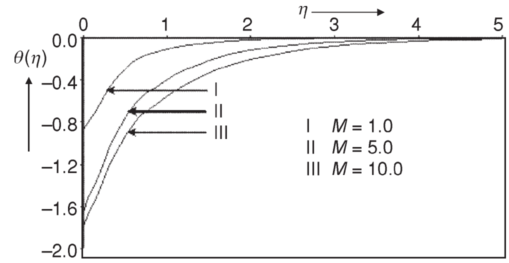

Figure 1. Dimensionless temperature against η for different values of M with Pr = 2.0, N = 1.67, λ = 0.2, and Ec = 0.4

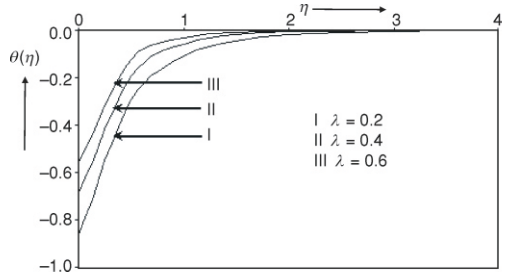

Figure 2. Dimensionless temperature against η for different values of λ with Pr = 2.0, N = 1.67, M = 1.0, and Ec = 0.4

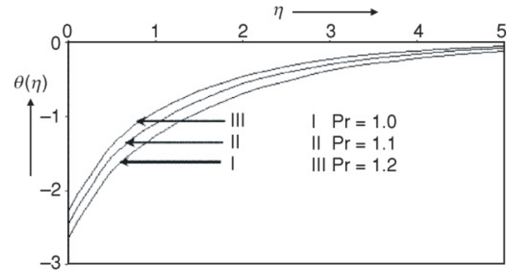

Figure 3. Dimensionless temperature against η for different values of Pr with M = 5.0, N = 1.67, λ = 0.2, and Ec = 0.6

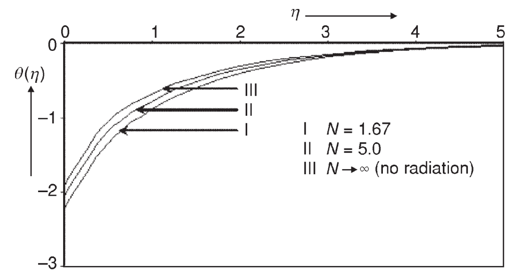

Figure 4. Dimensionless temperature against η for different values of N with M = 5.0, Pr = 1.0, λ = 0.4, and Ec = 0.6

In fig. 3, the dimensionless temperature distribution $\theta(\eta)$ is plotted against η for different values of Pr. The profiles of function $\theta(\eta)$ are all negative and the function increases as Pr increases.

The dimensionless temperature distribution $\theta(\eta)$ is plotted against η for different values of N in fig. 4. The case of no radiation is also shown in the figure (*i. e.* when N is infinitely large). It may be noted that temperature function $\theta(\eta)$ increases as radiation parameter N increases. Also, the function $\theta(\eta)$ is negative for all values of N.

In fig. 5, the dimensionless temperature distribution $\theta(\eta)$ is plotted against η for different values of Ec. It is noted that $\theta(\eta)$ increases with Ec, which shows the effect of viscous dissipation. It may be noted that Ec = 0.0 corresponds to the case of absence of viscous dissipation.

The recovery temperature $\theta(0)$ is plotted against Pr for different values of M and λ in figs. 6 and 7, respectively. It is seen that $\theta(0)$ decreases as M increases, but it increases as λ in-

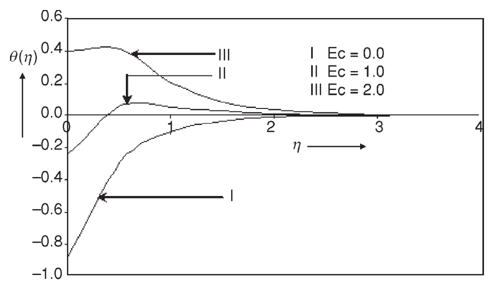

Figure 5. Dimensionless temperature against η for different values of Ec with M = 1.0, N = 5.0, λ = 0.4, and Pr = 2.0

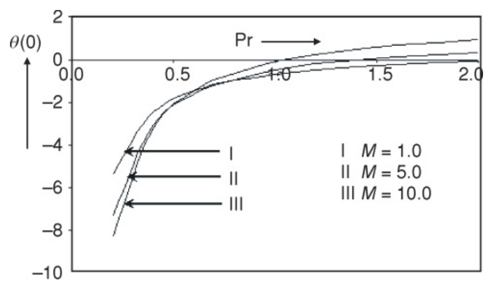

Figure 6. Recovery temperature against Pr for different values of M with λ = 0.2, N = 1.67, and Ec = 1.0

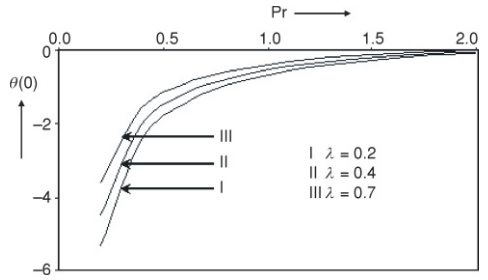

Figure 7. Recovery temperature against Pr for different values of λ with $M = 1.0$, $N = 1.67$, and Ec = 1.0

creases. The results for M are being changed after Pr 0.7. It is also seen that $\theta(0)$ is negative for both the cases.

The recovery temperature $\theta(0)$ is plotted against Pr for different values of N in fig. 8. It is being observed that $\theta(0)$ increases with N or $\theta(0)$ decreases as radiation increases.

The recovery temperature $\theta(0)$ is plotted against Pr for different values of Ec in fig. 9. It is seen that $\theta(0)$ increases with Ec.

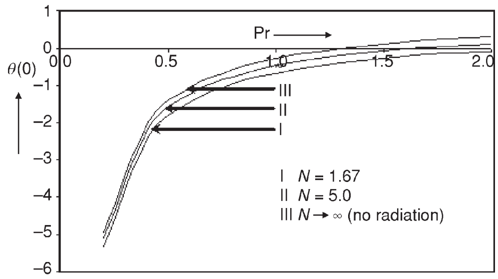

Figure 8. Recovery temperature against Pr for different values of N with $M = 1.0$, $\lambda = 0.2$, and Ec = 1.0

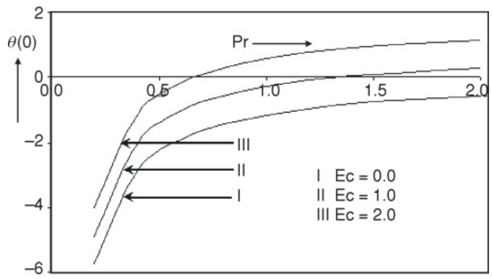

Figure 9. Recovery temperature against Pr for different values of λ with $M = 5.0$, $N = 1.67$, and $\lambda = 0.5$

Conclusions

The presence of radiation decreases the temperature in the boundary layer.

The effect of viscous dissipation is to increase the temperature in the boundary layer.

The recovery temperature $\theta(0)$ is decreased in presence of radiation whereas it is increased in presence of viscous dissipation.

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