MATHEMATICAL MODEL OF UNSTEADY GAS TO SOLID PARTICLES HEAT TRANSFER IN FLUIDIZED BED

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Original scientific paper UDC: 519.677:536.24-912 BIBLID: 0354-9836, *13* (2009), 1, 55-68 DOI: 10.2298/TSCI0901055S

The mathematical model of unsteady one-dimensional gas to particles heat transfer for non-isothermal fluidized bed with periodic heating of solid particles has been described. The method of numerical solution of governing differential equations, the algorithm and the computer program, have been presented. By using mathematical model and computer program, the temperature profiles for interstitial gas, gas in bubbles, and solid particles along the height of fluidized bed in function of time, have been determined. The results obtained on the basis of prediction method are compared to the experimental results of the authors; the satisfactory agreement has been found for interstitial gas temperature and solid particle temperature. On the basis of this comparison, the mathematical model has been verified.

Key words: fluidized bed, heat transfer, temperature field, gas-to-particles heat transfer

Introduction

The phenomena of gas-to-particle heat transfer accompanied with mass transfer and chemical reaction are present in many technological processes (drying of particulated materials, combustion of coal, thermal treatment of metal products ...) taking place in fluidized bed. For successful design and construction of reactors performing such processes with fluidized bed it is necessary to have reliable information on gas to particle heat transfer coefficient, variations of gas and particle temperatures along fluidized bed height, as well as on height of active zone for heat transfer.

Experimental investigations are usually long and expensive, which demand complex and expensive measuring techniques, therefore arises the need for prediction and understanding of such processes with minimum experimentation.

That is the reason why the development of mathematical model for those processes in form of differential equations is necessary condition for understanding of experimental results as well as for prediction of processes in reactors.

A great number of theoretical and experimental researches on the phenomenon of fluid-to particle heat transfer in fluidized bed have been performed in recent years, but the problem is not yet solved in a satisfactory way. In literature related to this problem one can find very often quite opposite conclusions.

Determination of fluid-to-particle heat transfer coefficients for both cases when particles are cooled or warmed under stationary or un-stationary conditions was carried out many times. In the case of wet particles the heat transfer is accompanied with the mass transfer.

Kettenring et al. [1] have performed the investigation of fluidized bed heat and mass transfer with Al₂O₃ and silica gel particles under constant particle surface temperature that is secured by evaporation of moisture from particle surfaces. The air temperatures were measured along the bed height by thermocouples unprotected of particle contacts, while the particle temperature was assumed to be equal with air temperature at bed outlet. Walton et al. [2] have determined air-to-coal particle heat transfer in a test section with inner diameter of 102 mm, under steady thermal conditions. The air temperature was measured by movable aspiration thermocouple, while the particle temperature was measured by unprotected thermocouple. Shachova [3] has performed investigation of gas to particle heat transfer under unsteady thermal conditions. In paper of Wamsley et al. [4] the method of unsteady thermal regime was applied. In a tube of inner diameter 59 mm the solid particles various materials were fluidized by using air or carbon dioxide. The particle sizes were within the interval $d_p = 0.135-0.913$ mm. Heertjes *et al.* [5] were determining particle to air heat transfer under steady thermal conditions in fluidized bed with drying of silica gel particles within the interval of constant drying velocity. The air temperature in fluidized bed was measured by movable aspiration thermocouple, which hot end was protected from contact with particles by using a grid. The solid particle temperature was treated to be constant within the bed and it is assumed to be equal with air temperature at bed outlet. A very interesting investigation of air to dry particle heat transfer under low fluidization numbers was carried out by Sharlovskaya [6, 7]. The experiments were performed under quasi-stationary conditions, it means that both the bed temperature and the air temperature were varied with respect to the same linear relation and so the temperature difference was remained to be constant. Richardson et al. [8] were determining gas to particle heat transfer coefficient in fluidized bed of small height ($H_0 = 10$ mm), and rectangular cross-section (100 50 mm) under quasi-stationary conditions – constant material flow rate, and constant temperatures of both inlet air and material. Glass and lead spheres of size within 0.114-0.55 mm were used as fluidizing particles. Fluidization was performed by air and carbon dioxide. Donnadieu [9] was investigating the air to particle heat transfer in fluidized bed under the conditions of unsteady heating and cooling of bed. The glass sphere bed ($d_p = 0.430$ mm) was fluidized in tube of diameter 200 mm. He assumed the air temperature dependence on bed height to be for every time moment of exponential form. During the experiment the temperature was measured by thermocouples with unprotected hot end, in the same time in a series of bed points it was accepted that this temperature is the air temperature. The solid particle temperature is assumed to be constant within the bed and equal to air temperature at bed outlet.

A large number of researchers in their investigations have found that the heat exchange between gas and particles was almost completely done within a thin layer above the distribution plate. This layer is so-called "active heat transfer zone". Above this layer the heat exchange could be neglected. For instance, in the paper of Zenz *et al.* [10] it is pointed out that heat exchange between gas and particles will terminate in the region up to 25 mm from distribution plate. The results are in nice correlation with the results of Kazakova [11] obtained for codling of sodium nitrate spheres by air in fluidized bed.

In the paper Ciesielczyk [12], the air to particle heat transfer has been investigated, as particles the spheres of silica gel, sand, sodium nitrate, and polytetrafluorethylen were used with the first drying period. The special attention was paid to the influence of the particle shape factor φ , on the air to particle heat transfer coefficient. It was found that shape factor has a significant influence on the values of heat transfer coefficient, as well as there is the increase of heat transfer coefficient with increase of φ .

Beside cited papers, there are more papers related to the investigation gas to particle heat transfer in fluidized bed. A detailed review of various investigations as well as criterial equations for air to particle heat transfer in fluidized bed is given in paper [13].

It is necessary to point out that there were many trials in the generalization of experimental results on air to particle heat transfer in fluidized bed. One of interesting suggestions for generalization of a part of published experimental results are the correlations of Franz [14] based on the results of many authors for the case when the gas temperature was measured by aspiration thermocouples, and particularly when it was measured by unprotected thermocouples.

However, after many trials the generalization was not successfull in sense of obtaining a general relation which could approximate the majority of published experimental results. The reason why is in large diffusivity of results published by many authors. For instance the published results for air to particle heat transfer coefficients obtained under similar conditions by different authors differ between each other for more than 2-3 orders of magnitude. The reasons for such a situation are numerous and they are analyzed in details in [15]. Such situation is the result of using different methods for measuring air and particle temperatures, different experimental conditions (stationary and unstationary operation regimes), different method for treatment of results (plug flow or ideal mixing flow model).

Real experiments certainly do not follow neither of the above regimes (plug flow model or ideal gas mixing), they usually follow the model which is the mixture of the above two. Kunii *et al.* [16] have shown that results based on the ideal mixing model have a large diffusion, whereas, the results based on ideal mixing model tend to unique expression. So they recommend the ideal mixing model as the most appropriate for determination of air to particle heat transfer coefficient. Experimental results of some authors they generalized by one unique equation [16, 17].

On the basis previous works review one can conclude that up to day investigations did not solve many problems within the field of air to particle heat transfer in fluidized bed, although many investigations have been done which have treated this problem.

Also one can conclude that a reliable model which could be successfully applied for prediction the air to particle heat transfer in fluidized bed still does not exist. Existing models are very complex and valid for particular cases for which they are designed.

The lack of reliable data enabling the determination of air to particle heat transfer coefficient with sufficient accuracy and also a booming need for reliable methods of designing, sizing of fluidized bed devices, and apparatuses provoke undertaking such kind of study with deeper theoretical and experimental investigation of processes in fluidized bed [15].

In this paper, the mathematical model for unsteady gas to solid particles heat transfer in fluidized bed is presented, the numerical solution method of differential equations is given, as well as the algorithm and the computer program. By using mathematical model and numerical procedure the temperature profiles for interstitial gas, gas in bubbles and solid particles along the height of fluidized bed in function of time have been determined. The results obtained for interstitial gas temperature and solid particle temperature on the basis of prediction method are compared to the experimental results given in [15].

Experimental investigation of the air to particle heat transfer in fluidized bed

The main goal of this investigation is to give a closer description of the phenomenon of air to particle heat transfer in fluidized bed, since the experimental results obtained by many authors differs significantly between each other and relying on empirical and semi-empirical expressions is unreliable.

The basic aim of the experimental investigation is the determination of the temperature profiles for interstitial gas, and solid particles along the height of fluidized bed under unsteady

heat transfer condition, as well as their dependence on different parameters. The special attention is paid to the determination of the height of active heat transfer zone in which practically the complete heat transfer is realized.

Experimental investigation of air to particle heat transfer in fluidized bed was carried out under unsteady thermal regime in apparatus of 120 mm in diameter and 900 mm in height with three different sand particles fractions of equivalent diameters: $d_p = 0.250, 0.500, and 0.850$ mm. For each of these fractions the basic characteristics were experimentally determined, as follows: d_p – equivalent particle diameter, ρ_p – sand density, ρ_n – volumetric density, ε_{mf} – porosity, and U_{mf} – minimal fluidization velocity. The minimal fluidization velocity is also calculated from expression by Stojiljković in [15].

The obtained results according to [15] are given in tab. 1.

| Fraction | Equivalent diameter | Density | Volumetric density | Porosity | Minimal fluidization velocity | |
|----------|----------------------------|-----------------------------------|-----------------------------------|---------------------------|---------------------------------|------------------------------|
| | | | | | Experimental | Calculated |
| [mm] | <i>d</i> _p [mm] | $ ho_{ m p}$ [kgm ⁻³] | $ ho_{ m n}$ [kgm ⁻³] | $\varepsilon_{ m mf}$ [–] | $U_{\rm mfexp} [{ m ms}^{-1}]$ | $U_{ m mfcal} [m ms^{-1}]$ |
| 0.2-0.3 | 0.250 | 2654 | 1383 | 0.48 | 0.11 | 0.111 |
| 0.4-0.6 | 0.500 | 2632 | 1446 | 0.45 | 0.32 | 0.318 |
| 0.8-1.0 | 0.850 | 2632 | 1492 | 0.43 | 0.55 | 0.551 |

Table 1. Sand characteristics

Specific heat transfer and thermal conductivity of sand, are taken to be $c_p = 712 \text{ J/kgK}$ and $\lambda = 0.930 \text{ W/mK}$ from appropriate literature.

In order to define the influence of particular parameters on the air to particle heat transfer coefficient, the experiments were conducted in such a way that for each sand fraction there different stagnant bed heights $H_0 = 25$, 50, and 100 mm and two fluidization velocities were varied. The first fluidization velocity was $\approx 3 U_{mf}$ for all fractions. The second fluidization velocity was $\approx 5 U_{mf}$ for particles of $d_p = 0.250$ and 500 mm, while for particles of $d_p = 0.850$ mm it is $\approx 4.5 U_{mf}$, due to limited capacity of fan. For the case when the smallest sand fraction $d_p = 0.250$ mm was fluidized, the inlet air temperature was varied within the interval of 65-72 °C and for biggest fraction, $d_p = 0.850$ mm within the interval of 72-77 °C. For middle sand fraction $d_p = 0.500$ mm, the experiments were carried out for two temperature regimes of inlet air: 56-64 °C, and 79-86 °C.

For measuring of air and particles temperatures in fluidized bed the four temperature probes were made, which description is given in [15]. The probe has two chromel-alumel thermocouples conducted through one small tube. The hot end of one thermocouple is protected by fine grid protecting contact of particles. The other thermocouple hot end is not protected, in such a way the protected thermocouple measures the air temperature while unprotected the particle temperature.

The probes are positioned at different heights through the holes (first at 3-4 mm above the distribution plate, second 8-9 mm, third 13-14 mm, and fourth at 19-20 mm). For measurements the inlet and outlet air temperature, the unprotected thermocouples were used. The first was placed just beneath the distributor and second at the height corresponding to the height of free board for a given experimental run.

In order to determine the air to particle heat transfer coefficient, five zones have been defined within the fluidized bed. Height from distribution plate up to corresponding thermocou-

ple, is defining the zone. Depending on the particular experimental run the heights of the zones are as follow is: first 3-4 mm, second 8-9 mm, third 13-14 mm, and fourth 19-20 mm. The fifth zone height is equal to the height of entire fluidized bed, $H_{\rm f}$.

Within the frame of experimental investigation of the air to particle heat transfer in fluidized bed the following parameters are determined: $H_{\rm f}$ -height of fluidized bed, δ - fraction of bubble phase volume, profiles of the interstitial phase and solid particle temperatures along the height of fluidized bed in function of time, height of heat transfer active zone, and air to particle heat transfer coefficient. The influence of parameters as: $d_{\rm p}$, $U_{\rm o}$, $t_{\rm g in}$, and $H_{\rm o}$ on the variations of previous quantities has been shown in [15].

On the basis of experimental data the following variations are shown in the diagram *t*-*H*: air and solid particles temperatures along fluidized bed height for different times measured from the start of experiment. For illustration purposes, in fig. 1 two such diagrams are show for one experimental run, for different time intervals. On each diagram the time from start of experimental run as well as basic quantities (d_p , H_o , U_o , K) are pointed out.



Figure 1. Variations of gas and solid particles temperatures in function of bed height

Also, the diagrams showing the air and particle temperatures vs. time (t- τ diagrams) are shown. In fig. 2 one of such diagram is shown for illustrative purpose. The quantities characterizing each experimental run are given also in such diagrams.

In $t-\tau$ diagrams measured air temperatures at bed inlet and outlet are shown, as well as particle temperatures measured by the second thermocouple. Temperatures measured by the third and fourth thermocouple are not shown in these diagrams, since at these bed heights they are very close between each other and almost equal to air temperature at bed outlet, as it can be seen from *t*-*H* diagrams.



Figure 2. Variations of gas and solid particles temperatures in function of time

Also, the values of air to particle heat transfer coefficients for particular zones and for the whole bed have been determined [15]. On the basis of experimental data obtained results, the criterial equation for air to particle heat transfer has been suggested.

Results of the author's own research have shown that almost entire air to particle heat transfer in fluidized bed has been realized almost entirely within the zone up to 20 mm above the distribution plate. Consequently this value is assumed to be the height oh heat transfer active zone.

Mathematical model

Presented mathematical model, has been developed for unsteady gas-to-solid particles heat transfer in fluidized bed with periodical heating of solid particles. A three phase bed model that defines bed phases as bubble phase, interstitial gas phase, and solid phase, has been used in the development of this model. Each phase has been considered as pseudo homogeneous media characterized by effective transport coefficients.

In this model one dimensional problem has been considered, *i. e.* it is assumed the variation of the analyzed quantities (such as gas temperature in bubbles, interstitial gas temperature, temperature of solid particles) only in longitudinal direction while its variations in transversal direction have been neglected.

The basic characteristic of the presented model is that it takes into consideration the influence of both the bubble-to-emulsion phase heat transfer and the solid particles mixing on the gas to solid particles heat transfer. The previous models have not included these effects.

In development of this model the following assumptions are used [15]:

- emulsion phase is in the state of minimal fluidization, and excess air above the needed quantity for minimal fluidization, flows through the bed in the form of bubbles,

- bubbles are spherical and consist of no solid particles,
- bubbles are uniform in size at a given height of bed, growing of bubbles during rising in bed is at the expense of coalescence,
- bubble phase is in plug flow with no axial dispersion, gas temperature in bubble is uniform throughout the bubble and depends only on the vertical position,
- interstitial gas can be considered to be in plug flow *i. e.* there is no backflow mixing,
- solid particles are uniform in size, shape and density and its thermo-physical properties are independent on temperature variation,
- gas-to-particles heat transfer occurs only on the particle surface, since the particles are small, the temperature gradient inside the particles can be neglected, and
- radiation heat transfer is not included into consideration.
- Presented mathematical model includes energy, continuity, and momentum equations for each of three phases and equation of state for gas phase [15].

Energy equation

Energy conservation equations for each of the phases are given below. For bubble phase there is:

$$\rho_{b}c_{b}V_{b}\frac{\partial T_{b}}{\partial \tau} - \rho_{b}c_{b}V_{b}U_{b}\frac{\partial T_{b}}{\partial x} - H_{be}V_{b}(T_{g} - T_{b})$$
(1)

The appropriate initial and boundary conditions are:

where $T_b^0(x)$ is the initial temperature profile of gas in bubble phase and T_{gu} – the gas temperature at bed inlet.

For interstitial gas there is:

$$\rho_{g}c_{g}V_{gef}\frac{\partial T_{g}}{\partial \tau} = \rho_{g}c_{g}V_{gef}U_{g}\frac{\partial T_{g}}{\partial x} = h_{pg}a_{p}V(T_{p}-T_{g}) = H_{be}V_{b}(T_{b}-T_{g})$$
(3)

The appropriate initial and boundary conditions are:

$$T_{g}(x,0) \quad T_{g}^{0}(x) \quad \text{for} \quad \tau \quad 0, \quad 0 \quad x \quad H_{f} \\ T_{g}(0,\tau) \quad T_{gu} \quad \text{for} \quad x \quad 0, \quad \tau \quad 0$$
(4)

where $T_g^0(x)$ is the initial temperature profile of gas in emulsion phase.

For solid phase there is:

$$\rho_{\rm p}c_{\rm p}V_{\rm p}\frac{\partial T_{\rm p}}{\partial \tau} = \rho_{\rm p}c_{\rm p}V_{\rm p}U_{\rm p}\frac{\partial T_{\rm p}}{\partial x} - h_{\rm pg}a_{\rm p}V(T_{\rm g}-T_{\rm p}) - k_{\rm w}x(T_{\rm o}-T_{\rm p}) - V_{\rm p}\frac{\partial}{\partial x} - \lambda_{\rm e}\frac{\partial T_{\rm p}}{\partial x}$$
(5)

The appropriate initial and boundary conditions are:

$$T_{\rm p}(x,0) \quad T_p^0 \quad \text{for} \quad \tau \quad 0, \quad 0 \quad x \quad H_{\rm f}$$

$$\frac{\partial T_{\rm p}}{\partial x} \quad 0 \quad \text{for} \quad x \quad 0 \quad \text{and} \quad x \quad H_{\rm f}$$
(6)

where $T_{p}^{0}(x)$ is the initial temperature of solid particles.

In eq. (5) the last term accounts for diffusion. Introduction of the diffusion term into energy equation for solid particles has been aimed to taking into consideration in this model (in contrast to previous ones) the influence of solid particles mixing on the gas-to-solid particles heat transfer, since these processes are coupled. The coefficient of effective thermal conductivity in fluidized bed, λ_e , has been used for estimation of solid particles mixing intensity.

Continuity equation

Volumes of some phases in fluidized bed are defined by the following relations. Volume of bubble phase:

$$V_{\rm b} = \delta V \tag{7}$$

Volume of interstitial gas (gas in emulsion phase):

$$V_{\rm gef} = (1 - \delta)\varepsilon_{\rm mf} V \tag{8}$$

Volume of solid particles:

$$V_{\rm p} = (1 - \delta)(1 - \varepsilon_{\rm mf})V \tag{9}$$

Momentum equation

Rising velocity of a crowd of bubbles in bed can be determined from equation [2]:

$$U_{\rm b} = U_{\rm o} = U_{\rm mf} = 0.711 \sqrt{{\rm g}d_{\rm b}} (10)$$

if the size of bubbles in bed is known.

If such information is not available, the bubble velocity can be determined from balance equation [15] describing the fact that the total mass flow rate of gas through the bed can be divided into mass flow rates of gas through emulsion phase and through bubble phase, fig. 3. From this relation one can obtain:

$$U_{\rm b} \quad U_{\rm mf} \quad \frac{U_{\rm o} \quad U_{\rm mf}}{\delta} \quad (11)$$

Figure 3. Schematic layout of bubbling fluidized bed

Velocity of the gas in emulsion phase (interstitial gas) is equal to the velocity achieved at minimum fluidization conditions:

$$U_{\rm g} = \frac{U_{\rm mf}}{\varepsilon_{\rm mf}} \tag{12}$$

For the solid particles velocity is assumed:

$$U_{\rm p} = 0 \tag{13}$$

since there is no preferent direction of particle motion.



Equation of state and thermal properties of gas

In order to complete the mathematical model, the equation of state and the relations of thermo-physical properties of gas phase have to be added to the above set of equations.

Numerical procedure of solution

Differential equations (1), (3), and (5) expressing the variation of temperature for some phases along the height of fluidized bed can be generalized by the following form of differential equation for general variable Φ :

$$\frac{\partial}{\partial \tau}(\rho \Phi) \quad \operatorname{div}(\rho \vec{U} \Phi) \quad \operatorname{div}(\Gamma \operatorname{grad} \Phi) \quad S \tag{14}$$

From this equation for $\Phi = 1$ and $\Gamma = 0$, one can obtain the continuity equation:

$$\frac{\partial \rho}{\partial \tau} \quad \text{div}(\rho \vec{U}) \quad 0 \tag{15}$$

Differential equation for general variable Φ , written in Cartesian coordinate system, for one dimensional problem, obtains the form:

$$\frac{\partial}{\partial \tau}(\rho \Phi) \quad \frac{\partial}{\partial x}(\rho U \Phi) \quad \frac{\partial}{\partial x} \Gamma \frac{\partial \Phi}{\partial x} \quad S \tag{16}$$

and the continuity equation becomes:

$$\frac{\partial \rho}{\partial \tau} \quad \frac{\partial}{\partial x} (\rho U) \quad 0 \tag{17}$$

The fact, that the foregoing equations expressing the variation of temperature for some phases along the height of fluidized bed can be reduced in the same form eq. (14), enables the simpler numerical solution.

In order to solve the foregoing equations in a numerical way, the discretization method based on control volume procedure has been used. This method is a modification of the finite difference method [18].

Fluidized bed domain by using orthogonal grid is divided into finite number of control volumes, fig. 4. In the discretization points located inside control volumes, the values of dependent variables from the considered differential equations have to be determined. The grid density influences accuracy and economy of the numerical procedure. It is obvious that denser grid gives more accurate field of dependent variable but in the same time demands the stronger computers. Therefore, the grid must be denser where the greater dependent variable gradients are expected.



From eq. (16) by using discretization method based on the method of control volumes, the discretization (algebraic) equation for control volume surrounding point P and for one-dimensional problem, is obtained in the form [18]:

$$a_{\rm P}\Phi_{\rm P} = a_{\rm E}\Phi_{\rm E} + a_{\rm W}\Phi_{\rm W} + b \tag{18}$$

where

The main discretization coefficients a_E and a_W in eq. (18) are dependent on convective and diffusion heat transfer between neighboring grid points (E, P, W).

Equation of type (18) is constructed for each control volume of the domain. For each variable Φ it is necessary to solve a set of algebraic equations which number is equal to the number of grid points [18].

In the prediction of the unsteady gas-to-solid particles heat transfer in fluidized bed, the computer program FLURT (for detailed description of algorithm see [15]) is used.

Results and discussions

The basic concept of this model assumes that gas-to-solid particles heat transfer in fluidized bed is composed of two individual parallel mechanisms: gas-to-solid particles heat transfer in emulsion bed phase (occurring in the same way as in packed bed) and heat transfer due to mixing of solid particles.

It is started from the basic statement of two-phase fluidization theory that the emulsion phase is always at minimum fluidization conditions and all excess gas above the needed quantity for minimal fluidization flows through the bed in the form of bubbles, fig.1. Since the voidage and height of bed at minimal fluidization conditions are approximately equal to the same parameters for packed bed, therefore the bed at minimal fluidization conditions can be treated as packed bed with gas flow velocity equal to minimal fluidization velocity. In that case gas-to-solid particles heat transfer in emulsion phase can be treated in the same way as heat transfer in packed bed. That is the reason why in eqs. (3) and (5) the gas-to-solid particles heat transfer coefficient for packed bed is introduced. This coefficient is determined from relations:

Nu_p
$$0.0114 \operatorname{Pr}^{1/3} \operatorname{Re}_{p}^{1.625}$$
, for Re_p 100 (21)

In the same time, in contrast to packed bed, there is in fluidized bed a heat transfer due to intensive solid particles mixing, caused by motion of bubbles through the bed. This effect is taken into account in diffusion term in eq. (5). For determination of the effective coefficient of thermal conductivity in longitudinal direction λ_{ea} the following relation is used:

$$\lambda_{\rm ea} = D_{\rm a} \rho_{\rm p} (1 - \varepsilon_{\rm mf}) c_{\rm p} \tag{22}$$

In relation (22) the coefficient of longitudinal diffusion of solid particles D_a is determined on the base of Todes and Citovic model [19] by the following expression:

$$D_{\rm a} = \frac{1}{60} \sqrt{{\rm g}L^3} \; \frac{U_{\rm o}}{U_{\rm mf}} \; 1$$
 (23)

where L represents the height of packed bed H_0 .

The bubble-emulsion phase heat transfer coefficient, H_{be} , is determined from the expression [20]:

$$\frac{1}{H_{be}} \quad \frac{1}{H_{bc}} \quad \frac{1}{H_{ce}} \tag{24}$$

where the bubble to cloud heat transfer, $H_{\rm he}$, is taken into account according to relation:

$$H_{\rm bc} = 45 \frac{U_{\rm mf} \rho_{\rm g} c_{\rm g}}{d_{\rm b}} = 5.85 \frac{\sqrt{\rho_{\rm g} c_{\rm g} \lambda_{\rm g}} \sqrt[4]{g}}{\sqrt[4]{d_{\rm b}^{\rm f}}}$$
(25)

while the cloud to emulsion heat transfer, H_{ce} , is neglected, since the uniform temperature field in the cloud is achieved very quickly.

Bubble diameter, $d_{\rm b}$, is defined according to Darton's expression [21]:

$$d_{\rm b} = \frac{0.54(U_{\rm o} - U_{\rm mf})^{0.4}(H - 4\sqrt{A_{\rm o}})^{0.8}}{g^{0.2}}$$
(26)

Values for other quantities existing in model equations: $d_{\rm p}$, $\rho_{\rm p}$, $\rho_{\rm n}$, $H_{\rm o}$, $H_{\rm f}$, δ , $\varepsilon_{\rm mf}$, $U_{\rm mf}$, and $U_{\rm o}$ are taken from the experimental data [15]. Some of them are given.

On the basis of presented mathematical model and by using already described computer program, the numerical experiments were carried out under the same conditions as in real experiments. From the numerical experiments the data as: gas temperature in bubble, interstitial gas temperature, and solid particles temperature along the fluidized bed height, have been obtained. The results of only one numerical experiment are shown in figs. 5-9. On each figure, in the interest of clarity, the variations of both the interstitial gas temperature and the solid particles temperature in function of time for a given bed height (H). On the same figures, for the purpose of comparison, values of the interstitial gas temperature and the solid particles temperature obtained from experimental measurements at the same bed height (H) are shown. On each figure, the quantities d_p , H_o , U_o , and K that characterize each experiment are shown, as well as bed height measured from the distribution plate (h) to which calculated and measured temperature values are related.



Since the measurement of gas temperature in bubbles was not possible due to experimental difficulties, the computed values of gas temperature in bubbles have not been compared to experimental ones.

By analyzing of presented diagrams one can conclude that computed values for both the interstitial gas temperature and the solid particles temperature are in good agreement with the measured values of same parameters. Especially good agreement between experiment and prediction one can see in the steady region of process. There is generally a better agreement be-



Figure 7. Comparison of experiment and prediction



Figure 9. Comparison of experiment and prediction



Figure 8. Comparison of experiment and prediction

tween predicted and measured values of interstitial gas temperature than for the values of solid particles temperature. The prediction method gives the lower values of solid particles temperature in comparison to the measured ones. This discrepancy is more significant at lower heights in bed, while at greater heights in bed (h) it can be neglected. These discrepancies are not so significant and they are the result of some assumptions during the mathematical modeling of process.

Conclusions

Comparison of the results obtained from the developed mathematical model to the authors

experimental results, shows a good agreement. On the basis of the foregoing considerations one can conclude:

- the assumed concept of the physical model is correct,
- the presented mathematical model and computer program FLURT can be used in prediction of heat transfer in fluidized bed and determination of temperature profiles of certain phases along the height of the fluidized beds in thermal devices with fluidized beds in practical applications,
- Todes and Citovich equation (23) for D_a, given in [19], gives satisfactory results for solid particles mixing in fluidized bed, and is recommended for determination of the coefficient of longitudinal diffusion of solid particles in fluidized bed, and
- the developed mathematical model is conceptualized in such a way that an easy extension is
 possible to the case when heat transfer is accompanied with mass transfer and chemical
 reaction, but it is necessary to apply the results of additional researches related to a given
 problem.

Nomenclature

| a _n | _ | surface area per unit volume of particles, |
|---------------------------|---|--|
| Р | | $[= 6(1 - \varepsilon_{\rm mf})(1 - \delta)/\phi d_{\rm n}], [m^2 m^{-3}]$ |
| $a_{\rm P}, a_{\rm W}, a$ | Е | - main discretization coefficients, [-] |
| $a_{\mathbf{p}}^{0}$ | _ | discretization coefficient in neighboring |
| 1 | | noint with respect to time [_] |
| C | _ | heat capacity $[Ika^{-1}K^{-1}]$ |
| ם ח | | coefficient of longitudinal diffusion of |
| D_{a} | _ | solid particles $[m^2s^{-1}]$ |
| d | | bubble diameter [m] |
| $d_{\rm b}$ | _ | mean diameter of the particles [m] |
| <i>u</i> _p | _ | appaleration of gravity [mg ⁻²] |
| g II | _ | acceleration of gravity, [IIIS] |
| П 1. | _ | listen an forma distribution whether formal |
| n II | _ | distance from distribution plate, [III] |
| H _{bc} | _ | bubble to cloud near transfer coefficient |
| | | per unit bubble volume, [wm K] |
| H _{be} | _ | bubble to emulsion heat transfer |
| | | coefficient per unit bubble volume, $-3x-1$ |
| | | |
| $H_{\rm ce}$ | _ | cloud to emulsion heat transfer coefficient |
| | | per unit bubble volume, [Wm ^{-K}] |
| $h_{\rm pg}$ | _ | gas to particle heat transfer coefficient, |
| | | $[Wm^{-2}K^{-1}]$ |
| K | - | fluidization number (= $U_{\rm o}/U_{\rm mf}$), [–] |
| K _{be} | - | bubble to emulsion mass transfer |
| | | coefficient per unit bubble volume, [s ⁻¹] |
| $k_{\rm w}$ | _ | overall heat transfer coefficient for |
| | | apparatus wall, [Wm ⁻² K ⁻¹] |
| L | - | characteristic length, [m] |
| 'n | _ | mass flow rate of gas, [kgs ⁻¹] |
| $S, S_{\rm P}, S_{\rm C}$ | - | source term |
| Т | _ | temperature, [K] |
| t | _ | temperature, [°C] |
| U | _ | velocity, [ms ⁻¹] |
| $U_{\rm o}$ | _ | superficial gas velocity, [ms ⁻¹] |
| V | _ | volume, [m ³] |
| x | _ | coordinate |

Greek letters

| a | - heat transfer coefficients between gas |
|-------------------|---|
| | and fluid bed particles, $[Wm^{-2}K^{-1}]$ |
| Γ | diffusion coefficient |
| 8 | fraction of bubble phase volume |
| 5 | voidage fraction |
| ર | – heat conductivity, [Wm ⁻¹ K ⁻¹] |
| l _{ea} | coefficient of effective thermal |
| | conductivity of fluidized bed in |
| | longitudinal direction, [Wm ⁻¹ K ⁻¹] |
| и | – dynamic viscosity, [kgm ⁻¹ s ⁻¹] |
| 0 | - density, [kgm ⁻³] |
| \mathcal{O}_{n} | volumetric density, [kgm⁻³] |
| τ | – time, [s] |
| \mathcal{D} | general variable |
| φ | shape factor, [-] |
| Subscrip | ots |
| b | – bubble |
| E, W, P | grids points |
| e | - effective |
| ef | emulsion phase |
| f | fluidized bed |
| g | – gas |
| in | – inlet |
| mf | minimal fluidization |
| D | packed bed |
| р | – particle |
| Dimens | ionless groups |
| Nu | - Nusselt number (= $\alpha_{\rm p} d_{\rm p} / \lambda_{\rm g}$), [-] |

| INUp | _ | $(-\alpha_p \alpha_p / \alpha_g)$ | , [_] |
|------|---|--|-------|
| Pr | _ | Prandtl number (= $\mu_a c_a / \lambda_a$). | [_] |

- Prandtl number (= $\mu_g c_g / \lambda_g$), [-] Rep - Reynolds number (= $d_p \rho_g U_o / \mu_g$), [-]

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Paper submitted: September 11, 2008 Paper revised: February 7, 2009 Paper accepted: February 10, 2009