EFFECTS OF OHMIC HEATING AND VISCOUS DISSIPATION ON STEADY MHD FLOW NEAR A STAGNATION POINT ON AN ISOTHERMAL STRETCHING SHEET

by

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Aim of the paper is to investigate effects of ohmic heating and viscous dissipation on steady flow of a viscous incompressible electrically conducting fluid in the presence of uniform transverse magnetic field and variable free stream near a stagnation point on a stretching non-conducting isothermal sheet. The governing equations of continuity, momentum, and energy are transformed into ordinary differential equations and solved numerically using Runge-Kutta fourth order with shooting technique. The velocity and temperature distributions are discussed numerically and presented through graphs. Skin-friction coefficient and the Nusselt number at the sheet are derived, discussed numerically, and their numerical values for various values of physical parameters are compared with earlier results and presented through tables.

Key words: *steady, MHD, stagnation point, stretching sheet, viscous dissipation, ohmic heating, skin-friction coefficient, Nusselt number*

Introduction

Flow and heat transfer of an incompressible viscous fluid over stretching sheet find applications in manufacturing processes such as the cooling of the metallic plate, nuclear reactor, extrusion of polymers, *etc*. Flow in the neighbourhood of a stagnation point in a plane was initiated by Hiemenz [1]. Crane [2] presented the flow over a stretching sheet and obtained similarity solution in closed analytical form. Fluid flow and heat transfer characteristics on stretching sheet with variable temperature condition have been investigated by Gurbka *et al*. [3]. Watanabe [4, 5] discussed stability of boundary layer and effect of suction/injection in MHD flow under pressure gradient. Noor [6] studied the characteristics of heat transfer on stretching sheet. Chiam [7] discussed the heat transfer in fluid flow on stretching sheet at stagnation point in presence of internal dissipation, heat source/sink and Ohmic heating. Chamka *et al*. [8] considered Hiemenz flow in the presence of magnetic field through porous media. Sharma *et al.* [9] investigated steady MHD flow through horizontal channel: lower being a stretching sheet and upper being a permeable plate bounded by porous medium. Mahapatra *et al*. [10] investigated the magnetohydrodynamic stagnation-point flow towards isothermal stretching sheet and reported that velocity decreases/increases with the increase in magnetic field intensity when free stream velocity is smaller/greater than the stretching velocity. Mahapatra *et al.* [11] studied heat transfer in stagnation-point flow on stretching sheet with viscous dissipation effect. Attia [12] analysed the hydromagnetic stagnation point flow on porous stretching sheet with suction and injection. Pop *et al*. [13] discussed the flow over a stretching sheet near a stagnation point taking radiation effect.

Aim of the present paper is to investigate effects of Ohmic heating, viscous dissipation, and variable free stream on flow of a viscous incompressible electrically conducting fluid and heat transfer near a stagnation point on an isothermal non-conducting stretching sheet.

Formulation of the problem

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid in the vicinity of a stagnation point on a stretching sheet in the presence of transverse magnetic field of constant intensity B_0. The stretching sheet has constant temperature T_w, linear velocity $u_w(x)$ and free stream velocity is $U(x)$. It is assumed that external field is zero, the electric field owing to polarization of charges and Hall effect are neglected. Stretching sheet is placed in the plane $y = 0$ and x-axis is taken along the sheet as shown in fig. 1. The fluid occupies the half plane ($y > 0$).

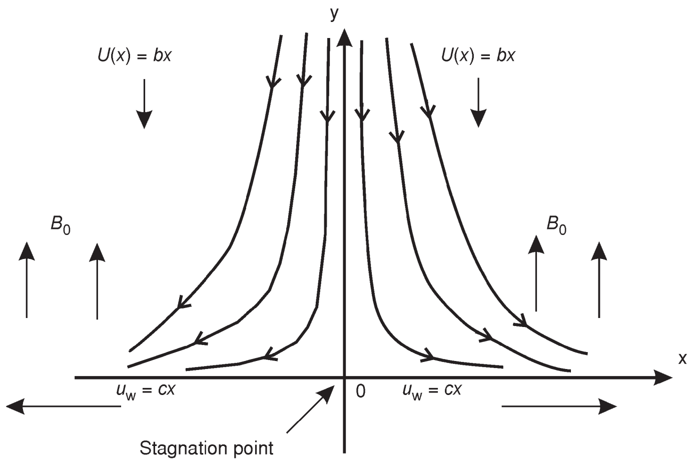

Figure 1. Physical model

The governing equations of continuity, momentum, and energy (Pai [14], Bansal [15], Schlichting *et al.*[16], *etc.*) under the influence of uniform transverse magnetic field (Jeffery [17], Bansal [18]) with Ohmic dissipation are:

$$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} \quad v\frac{\partial u}{\partial y} \quad U\frac{\mathrm{d}U}{\mathrm{d}x} \quad \nu\frac{\partial^2 u}{\partial y^2} \quad \frac{\sigma B_0^2}{\rho}[u \quad U(x)] \tag{2}$$

$$\rho C_\mathrm{p} \quad u\frac{\partial T}{\partial x} \quad v\frac{\partial T}{\partial y} \quad \kappa\frac{\partial^2 T}{\partial y^2} \quad \sigma B_0^2[u \quad U(x)]^2 \quad \mu \quad \frac{\partial u}{\partial y}^{2} \tag{3}$$

The boundary conditions are:

$$\begin{aligned} y \quad 0&: \quad u \quad u_w(x) \quad cx, \quad v \quad 0, \quad T \quad T_w \\ y \quad \infty&: \quad u \quad U(x) \quad bx, \quad T \quad T_\infty \end{aligned} \tag{4}$$

Method of solution

Introducing the stream function ψ (x, y) as defined by:

$$u \quad \frac{\partial \psi}{\partial y} \quad \text{and} \quad v \quad \frac{\partial \psi}{\partial x} \tag{5}$$

and the similarity variable

$$\eta = y\sqrt{\frac{c}{\nu}} \quad \text{and} \quad \psi(x, y) = \sqrt{c\nu}\, x f(\eta) \tag{6}$$

into the eqs. (2) and (3), we get:

$$f''' + ff'' - f'^2 - \mathrm{M}^2(f' - \lambda) + \lambda^2 = 0 \tag{7}$$

$$\theta'' + \mathrm{Pr}\,\theta' f + O_h \mathrm{Pr}\,\mathrm{Ec}\mathrm{M}^2(f' - \lambda)^2 + \mathrm{Pr}\,\mathrm{Ec}\, f''^2 = 0 \tag{8}$$

where $O_\mathrm{h} = 0$ or 1 is the Ohmic heating parameter. It is observed that eq. (1) is identically satisfied.

The corresponding boundary conditions are reduced to:

$$f(0) = 0, \; f'(0) = 1, \; \theta(0) = 1, \; f'(\infty) = \lambda, \; \theta(\infty) = 0 \tag{9}$$

The governing eqs. (7) and (8) with the boundary conditions (9) are solved using Runge-Kutta fourth order technique (Jain *et al*. [19], Krishnamurthy *et al.* [20], Jain [21], *etc.*) along with shooting technique (Conte *et al.* [22]). First of all, higher order non-linear differential eqs. (7) and (8) are converted into simultaneous linear differential equations of order first and they are further transformed into initial value problem applying the shooting technique. Once the problem is reduced to initial value problem, then it is solved using Runge-Kutta fourth order technique.

Skin-friction

Skin-friction coefficient at the stretching sheet is given by:

$$C_\mathrm{f} = \frac{\tau_w}{\rho c\sqrt{c\nu}} = x f''(0) \tag{10}$$

where $\tau_\mathrm{w} = \mu(\partial u/\partial y + \partial v/\partial x)_{y=0}$ is the shear stress at the stretching sheet.

Nusselt number

The rate of heat transfer in terms of the Nusselt number at the stretching sheet is given by:

$$\mathrm{Nu} = \sqrt{\frac{\nu}{c}}\frac{q_\mathrm{w}}{\kappa(T_\mathrm{w} - T_\infty)} = -\theta'(0) \tag{11}$$

where $q_\mathrm{w} = -\kappa(\partial T/\partial y)_{y=0}$

Particular cases

(1) In the absence of magnetic field *i. e.* M = 0, the results of the present paper are reduced to those obtained by Pop *et al.* [13] and Mahapatra *et al.* [11].

(2) In the absence of magnetic field and viscous dissipation *i. e.* M = 0 and Ec = 0, the results of the present paper are reduced to those obtained by Mahapatra *et al*. [11].

(3) At $\lambda = 1$, there is no formation of boundary layer as the surface velocity is equal to fluid velocity (Pai [14]).

Results and discussion

Equations (7) and (8) are solved using Runge-Kutta fourth order technique for different values of M, λ, Ec, and O_h when Pr = 1.0 taking step size 0.005.

Table 1. Values of f (0) for different values of λ are compared with the results obtained by Pop *et al.* [13] and Mahapatra *et al.* [11]

λ	f (0)		
	Pop *et al.* [13]	Mahapatra *et al.* [11]	Present paper results
0.1	–0.9694	–0.9694	–0.969386
0.2	–0.9181	–0.9181	–0.9181069
0.5	–0.6673	–0.6673	–0.667263
2.0	2.0174	2.0175	2.01749079
3.0	4.7290	4.7293	4.72922695

Table 2. Values of f (0) for different values of λ and M

λ	f (0)		
	M = 0.0	M = 0.5	M = 1.0
0.1	–0.969386	–1.0678983	–1.321111
0.5	–0.667263	–0.7118914	–0.8321261
2.0	2.017502	2.0777247	2.249103

Table 3. Values of $-\theta'(0)$ for different values of λ are compared with the results obtained by Mahapatra *et al.* [11] when Pr = 1.0

λ	θ (0)	
	Mahapatra *et al.* [11]	Present paper results
0.1	0.603	0.602157
0.5	0.692	0.692445
2.0	0.974	0.978726

Table 4. Values of $-\theta'(0)$ for different values of λ, M, and Ec when Pr = 1.0 and O_h = 1.0

λ= 0.1	θ (0)		
M	Ec = 0.0	Ec = 1.0	Ec = 2.0
0.0	0.602157	0.228906	–0.144344
1.0	0.550806	–0.217832	–0.986471
2.0	0.474701	–1.037764	–2.550229
λ = 2.0	θ (0)		
0.0	0.978726	0.060048	–0.858629
1.0	0.985968	–0.205953	–1.397875
2.0	1.001650	–0.858963	–2.719576

It is observed from tab. 1 that the numerical values of f (0) of the present paper when M = 0 are in good agreement with those obtained by Pop *et al.* [13] and Mahapatra *et al.* [11].

It is observed from tab. 2 that shear stress at the sheet decreases due to increase in the magnetic field intensity when $\lambda < 1$, while it increases with the increase in the magnetic field intensity when $\lambda > 1$. It also increases due to increase in λ when the magnetic field intensity is fixed.

It is seen from tab. 3 that the numerical results of $-\theta'(0)$ of the present paper when M = = 0, O_h = 0, and Pr = 1.0 are in full agreement with the results obtained by Mahapatra *et al.* [11].

It is seen from tab. 4 that the Nusselt number decreases with the increase in the magnetic field intensity when λ = 0.1. For λ = 2.0, due to increase in the magnetic field intensity, the Nusselt number slightly increases in the absence of viscous dissipation; while it decreases when Ec = 1.0 and 2.0. Hence reversal effect in heat transfer rate is observed in presence of Ohmic heating.

Table 5 reveals that the Ohmic heating (when O_h = 1.0) decreases the Nusselt number when only viscous dissipation is considered and effect is more pronounced as magnetic field intensity increases.

Figure 2 shows that the boundary layer thickness decreases considerably as λ increases in the absence of magnetic field intensity, which is shown by dotted vertical lines at the points where f' reaches at boundary condition. Hence for $\lambda > 1$, the boundary layer is thin. Figure 3 depicts that for λ = 0.1 with the increase in the magnetic field intensity, the fluid velocity decreases which is fully agree with physical phenomena. However, reverse phenomena is observed for λ = 2.0, which is due to the fact that inverted boundary layer is formed for $\lambda > 1$ *i. e.* when free stream velocity is greater than stretching sheet parameter (Mahapatra *et al.* [10, 11]).

It is observed from fig. 4 that at λ = 0.1 with the increase in magnetic field intensity,

Table 5. Values of $-\theta'(0)$ for different values of λ, M and O_h when Pr = 1.0 and Ec = 2.0

θ (0)				
M	$\lambda = 0.1$		$\lambda = 2.0$	
	$O_h = 0.0$	$O_h = 1.0$	$O_h = 0.0$	$O_h = 1.0$
1.0	–0.507073	–0.986471	–1.054078	–1.397875
2.0	–1.235249	–2.550229	–1.568088	–2.719576

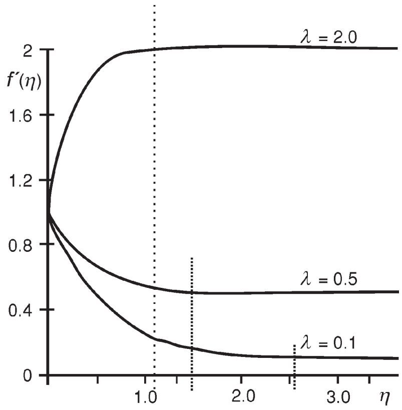

Figure 2. Velocity distribution *vs.* η when M = 0.0

fluid temperature increases and is even higher than the sheet temperature. In the absence of viscous dissipation and $\lambda = 2.0$, the results in tab. 4 show that

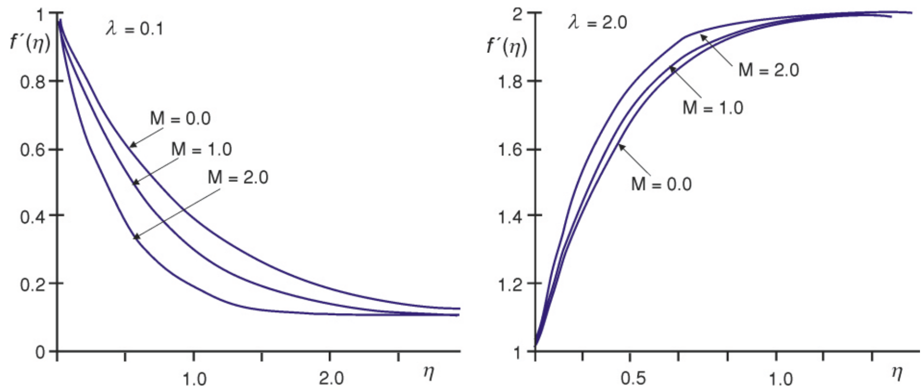

Figure 3. Velocity distribution *vs.* η for different values of λ

the fluid temperature decreases with the increase in the magnetic field intensity because inverted thermal boundary layer formation but when Ec 0.0 with the increase in magnetic field intensity, fluid temperature increases near the sheet and is approximately same as the distance from

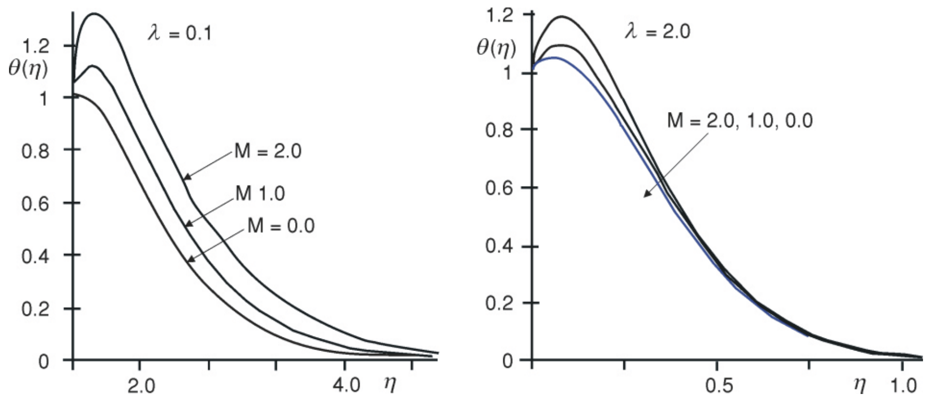

Figure 4. Temperature distribution *vs.* η for different values of λ when Ec = 2.0 and O_h = 1.0

sheet increases. Hence a reversal characteristic in fluid temperature is observed in the presence of Ohmic heating. The thermal boundary layer thickness increases with the increase in magnetic field intensity for $\lambda = 0.1$ and change is negligible for $\lambda = 2.0$.

It is seen from fig. 5 that with the increase in the viscous dissipation parameter, fluid temperature increases and effect is more pronounced with the increase in magnetic field intensity, because of the effect of Ohmic heating when $\lambda = 0.1$. The change in the thermal boundary layer thickness is negligible. Figure 6 shows the effects of Ec and M on fluid temperature when $\lambda = 2.0$. The temperature profiles are same as in the case of $\lambda = 0.1$ except that steep increase in fluid temperature near the sheet is observed when $\lambda = 0.1$ in comparison to the case when $\lambda = 2.0$.

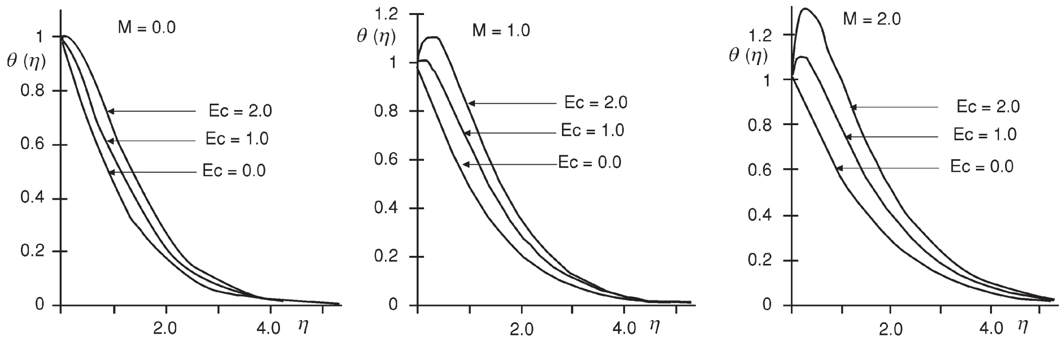

Figure 5. Temperature distribution *vs.* η for different values of M and Ec when λ= 0.1

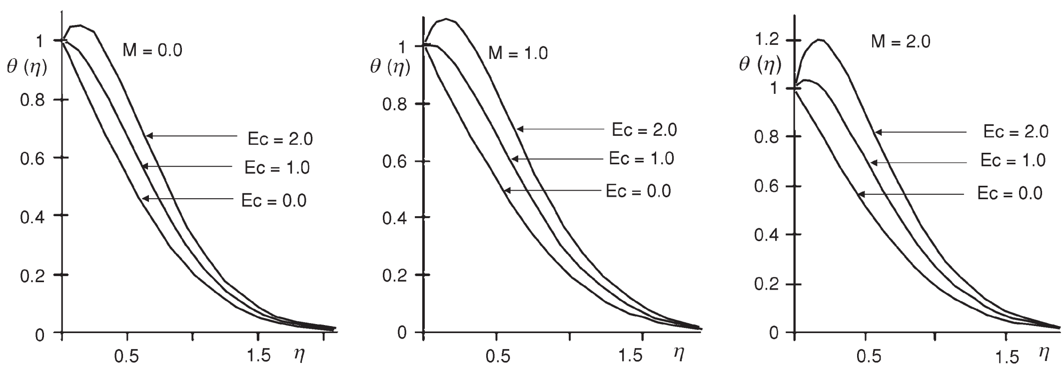

Figure 6. Temperature distribution *vs.* η for different values of M and Ec when λ= 2.0

Figure 7 represents that the fluid temperature increases in the presence of viscous dissipation and Ohmic heating. Increase of fluid temperature near the sheet is observed in the presence of Ohmic heating, when $\lambda = 0.1$ in comparison to the case when $\lambda = 2.0$.

Conclusions

Fluid velocity increases and boundary layer thickness decreases with the increase in λ *i. e.* when the free stream parameter is dominating. Fluid velocity decreases due to increase in magnetic field intensity when λ= 0.1, while it increases due increase in the magnetic field intensity when $\lambda = 2.0$.

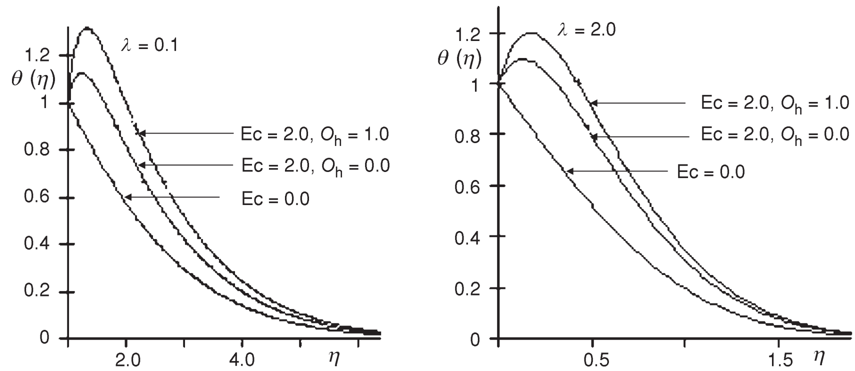

Figure 7. Temperature distribution *vs.* η for different values of Ec and λ when M = 2.0

The skin-friction coefficient at the sheet decreases due to increase in the magnetic field intensity when λ = 0.1, while reverse behaviour is observed when λ = 2.0.

Fluid temperature increases due to increase in magnetic field intensity when λ = 0.1. Fluid temperature decreases with the increase in the magnetic field intensity in the absence of viscous dissipation, but with the increase in magnetic field intensity fluid temperature increases near the sheet and is approximately same as the distance from sheet increases in the presence of viscous dissipation when λ = 2.0. Steep increase in fluid temperature near the sheet is observed when λ = 0.1 in comparison to the case when λ = 2.0.

The thermal boundary layer thickness increases with the increase in magnetic field intensity when λ = 0.1 and negligible change is observed when λ = 2.0. Fluid temperature increases due to increase in the Eckert number and effect is more pronounced at higher Hartmann number, irrespective values of λ. The effect of increase in the Eckert number on thermal boundary layer thickness is negligible.

Ohmic heating parameter cannot be neglected for large values of the Eckert number and Hartmann number.

Nomenclature

B_o – magnetic field intensity, [T]
b – free stream velocity parameter, [s^{-1}]
C_f – skin-friction coefficient, [–]
C_p – specific heat at constant pressure, [$Jkg^{-1}K^{-1}$]
c – stretching sheet parameter, [s^{-1}]
Ec – Eckert number [$= c^2x^2/C_p(T_w - T_\infty)$], [–]
f – dimensionless stream function, [–]
M – Hartmann number [$= (\sigma B_0^2/\rho c)^{1/2}$], [–]
Nu – Nusselt number, [–]
O_h – Ohmic heating parameter, [–]
Pr – Prandtl number ($= \mu C_p/\kappa$), [–]
q_w – rate of heat transfer, [Wm^{-2}]
T – fluid temperature, [K]
T_w – temperature of stretching sheet, [K]
T_∞ – free stream temperature, [K]
$U(x)$ – free stream velocity, [ms^{-1}]
u, v – velocity components along x- and y-axes, respectively, [ms^{-1}]
u_w – velocity of stretching sheet, [ms^{-1}]
x, y – Cartesian coordinates along x- and y-axes, respectively, [m]

Greek letters

θ – dimensionless temperature [$(T - T_\infty)/(T_w - T_\infty)$], [–]
κ – thermal conductivity, [$Wm^{-1}K^{-1}$]
λ – ratio of free stream velocity parameter and stretching sheet parameter, [–]
μ – coefficient of viscosity, [$kgms^{-1}$]
ν – kinematic viscosity ($= \mu/\rho$), [m^2s^{-1}]
η – similarity variable [$= (c/\nu)^{1/2}y$]
ρ – density of fluid, [kgm^{-3}]
σ – electrical conductivity , [$Wm^{-2}K^{-4}$]
τ_w – shear stress, [Pa]

Superscript

$'$ – differentiation with respect to η

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