COMBINED OF MAGNETIC FIELD AND THERMOPHORESIS PARTICLE DEPOSITION IN FREE CONVECTION BOUNDARY LAYER FROM A VERTICAL FLAT PLATE EMBEDDED IN A POROUS MEDIUM

by

Ahmed Yousof BAKIER and Mohamed Ahmed MANSOUR

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Deals with heat and mass transfer by steady laminar boundary layer flow of Newtonian, viscous fluid over a vertical flat plate embedded in a fluid-saturated porous medium in the presence of thermophoretic and magnetic field. The resulting similarity equation are solved by finite difference marching technique. The nature of variation of particle concentration profile and magnetic field with respect to buoyancy force, Fw, and Prandtl number is found to be similar. Comparisons with previous published work are performed and the results are found to be in excellent agreement.

Key words: *mass transfer, magnetic filed, thermophoresis, free convection, boundary layer*

Introduction

Thermophoresis is a phenomenon by which submicron sized particles suspended in a non-isothermal gas acquire a velocity relative to the gas in the direction of decreasing temperature. The velocity acquired by the particles is called thermophoretic velocity and the force experienced by the suspended particles due to the temperature gradient is known as thermophoretic force. Thermophoretic deposition of radioactive particles is considered to be one of the important factors causing accidents in nuclear reactors. Thermophoresis causes small particles to deposit on cold surfaces. Repulsion of particles from hot objects will also take place and a particle-free layer is observed around hot bodies. Thermophoresis is considered to be important for particles of 10 μm in radius and temperature gradient of the order of 5 K/mm. A common example of this phenomenon is blackening of the glass globe of a kerosene lantern. The temperature gradient developed between the flame and the glass globe drives the carbon particles produced in the combustion process towards the globe, where they deposit. There are several other practical situations where we come across this phenomenon, like gas "clean up", corrosion of heat exchangers with attendant reduction of heat transfer coefficient, fouling of gas turbine equipment, coagulation of condensing/evaporating aerosols, in determining particle trajectories in the exhaust gas from combustion devices, in the transpiration cooling of gas turbine blades, *etc*. The initial study of thermophoretic transport involved simple one-dimensional flows for the measurement of the thermophoretic velocity and was undertaken by Goldsmith and May [1]. Talbot *et al*. [2] solved numerically for the velocity and temperature fields in the laminar boundary layer adjacent to a heated plate. Using several available theoretical expressions for the thermophoretic force, they calculated the trajectory of a particle entering the boundary layer. Measurements of the thickness of the particle-free layer next to the heated plate were compared with the calculated trajectories and it was found that the theory of Brock [3], modified slightly to fit the data for very small particles, gave the best overall agreement with the measurements. The first analysis of thermophoretic deposition in a geometry of engineering interest appears to be that of Hales *et al*. [4]. They solved the laminar boundary layer equations for simultaneous aerosol and steam transport to an isothermal vertical surface situated adjacent to a large body of an otherwise quiescent air-steam-aerosol mixture. Thermophoresis in laminar flow over a horizontal flat plate has been studied theoretically by Goren [5] where the analysis covered both cold and hot plate conditions. Selim *et al*. [6] consider the effect of surface mass transfer on mixed convection flow past a heated vertical flat permeable surface in the presence of thermophoresis. Previous work on this topic includes papers by Epstein *et al*. [7], who carried out a thermophoretic analysis of small particles in a free convection boundary layer adjacent to a cold vertical surface, and Mills *et al*. [8] and Tsai [9], who reported correlations for the deposition rate in the presence of thermophoresis and wall suction in laminar flow over a flat plate. Jia *et al*. [10] also investigated numerically the interaction between radiation and thermophoresis in forced convection laminar boundary layer flow and natural convective laminar flow over a cold vertical flat plate in the presence of thermophoresis was solved numerically by Jayaraj [11] and Jayaraj *et al*. [12] for constant and variable properties, respectively. Finally, Chiou [13] analyzed the effect of thermophoresis on submicron particle deposition from a forced laminar boundary layer flow on to an isothermal moving plate through similarity solutions and this analysis was extended by Chiou and Cleaver [14] convection from a vertical isothermal cylinder.

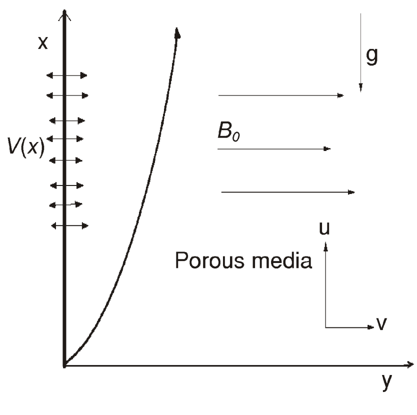

Figure 1. Sketch of physical model

Despite the practical importance of magnetic field with thermophoresis there is, to our best knowledge, almost no work devoted to this topic in porous media. Consideration is, therefore, given here to the similarity solutions of the boundary layer free convection magnetic field with thermophoretic deposition of aerosol particles on a vertical isothermal flat plate embedded in a fluid saturated porous medium. The Darcy and energy equations yield the velocity and temperature distributions in the boundary layer, which are then used in the coupled concentration equation to calculate the rates of particle deposition. The flow configuration and the coordinate system are as shown in fig. 1.

Analysis

Let us consider a steady, two-dimensional vertical natural convection MHD flows and boundary layer over a vertical flat plate of constant temperature T_w and concentration C_w, which is embedded in a fluid-saturated porous medium of ambient temperature T_∞ and concentration C_∞ where $T_w > T_\infty$ and $C_w > C_\infty$, respectively. Allowing for both Browian motion of particles and thermophoretic transport the governing boundary layer equations are, see [13, 15]:

$$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0 \tag{1}$$

$$u \quad \frac{gK}{\nu}[\beta_T(T \quad T_\infty) \quad \beta_c(C \quad C_\infty)] \quad \sigma B_0^2 u \tag{2}$$

$$u\frac{\partial T}{\partial x} \quad v\frac{\partial T}{\partial y} \quad \alpha_m \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} \quad v\frac{\partial C}{\partial y} \quad \frac{\partial}{\partial y}(V_T C) \quad D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions for the present problem are as follows:

$$\begin{aligned} &\text{at y} \quad 0: \quad v \quad V(x), \quad T \quad T_w, \quad C \quad C_w \\ &\text{as } y \quad \infty: \quad u \quad 0, \quad T \quad T_\infty, \quad C \quad C_\infty \end{aligned} \tag{5}$$

where u and v are the fluid velocity components along the x- and y-axes (which are parallel and normal to the plate, respectively), g is the gravitational force due to acceleration, β is the volumetric coefficient of thermal expansion, and T is the temperature of the fluid in the boundary layer. C is the species concentration in the boundary layer. ν, α_m, and D being the kinematics coefficients of viscosity, thermal diffusivity, and the Brownian diffusion coefficients, respectively. β_T and β_c are the thermal expansion coefficients of temperature and concentration, respectively. In eq. (5), $V(x)$ represents the permeability of the porous surface where its sign indicates suction (<0) or injection (>0). Here we confine our attention to the suction of fluid through the porous surface and for these. The effect of thermophoresis is usually prescribed by means of an average velocity which a particle will acquire when exposed to a temperature gradient. In boundary layer flow, the temperature gradient in the y-direction is very much larger than in the x-direction, and therefore only the thermophoretic velocity in y-direction is considered. As a consequence, the thermophoretic velocity V_T, which appears in eq. (4), may be expressed in the following form:

$$V_T \quad \frac{\kappa\nu}{T}\frac{\partial T}{\partial \mathrm{y}} \tag{6}$$

where T is some reference temperature, the value of $\kappa\nu$ represents the thermophoretic diffusivity, and κ is the thermophoretic coefficient, which ranges in value from 0.2 to 1.2

as observed by Batchelor and Shen [16] and is defined from the theory of Talbot *et al*. [2] by:

$$\kappa \quad \frac{2C_{\mathrm{s}} \quad \frac{\lambda_{\mathrm{g}}}{\lambda_{\mathrm{p}}} \quad C_{\mathrm{t}}\,\mathrm{Kn} \quad 1 \quad \mathrm{Kn}(C_1 \quad C_2 \mathrm{e}^{\frac{C_3}{\mathrm{Kn}}})}{(1+3C_{\mathrm{m}}\,\mathrm{Kn}) \;\; 1 \quad \frac{2\lambda_{\mathrm{g}}}{\lambda_{\mathrm{p}}} \quad 2C_{\mathrm{t}}\,\mathrm{Kn}} \tag{7}$$

where $C = 1 + \mathrm{Kn}[C_1 + C_2 \exp(-C_3/\mathrm{Kn})]$, Kn is the Knudsen number, $C_1 = 1.2$, $C_2 = 0.41$, and $C_3 = 0.88$, C_{m} and C_{s} are constants [16], λ_{g} and λ_{p} are the thermal conductivities of gas and diffused particles, respectively. As previously introduced by Mills *et al*. [8] and Tsai [9].

We now introduce the following non-dimensional variables:

$$X \quad \frac{x}{l}, \quad Y \quad \sqrt{\mathrm{Ra}}\,\frac{y}{l}, \quad U \quad \frac{u}{U_{\mathrm{c}}}, \quad V \quad \sqrt{\mathrm{Ra}}\,\frac{v}{U_{\mathrm{c}}},$$
$$V_T \quad \sqrt{\mathrm{Ra}}\,\frac{v_{\mathrm{t}}}{U_{\mathrm{c}}}, \quad \theta \quad \frac{T \quad T_\infty}{T_{\mathrm{w}} \quad T_\infty}, \quad \phi \quad \frac{C \quad C_\infty}{C_{\mathrm{w}} \quad C_\infty} \tag{8}$$

where $U_{\mathrm{c}} = \mathrm{g}\beta_{\mathrm{T}}K(T_{\mathrm{w}} - T_\infty)/\nu$ is the characteristic velocity, $\mathrm{Ra} = \mathrm{g}K\beta_{\mathrm{T}}(T_{\mathrm{w}} - T_\infty)l/\alpha_{\mathrm{m}}\nu$ is the Rayleigh number, and l is a characteristic length of the plate. Thus eqs. (1)-(4) and (6) take the following from:

$$\frac{\partial U}{\partial X} \quad \frac{\partial V}{\partial Y} \quad 0 \tag{9}$$

$$U = \theta + B\phi - MU \tag{10}$$

$$U\frac{\partial \theta}{\partial X} \quad V\frac{\partial \theta}{\partial Y} \quad \frac{\partial^2 \theta}{\partial Y^2} \tag{11}$$

$$U\frac{\partial \phi}{\partial X} \quad V\frac{\partial \phi}{\partial Y} \quad \frac{\partial}{\partial Y}(V_{\mathrm{T}}\phi) \quad \frac{1}{\mathrm{Le}}\frac{\partial^2 \phi}{\partial Y^2} \tag{12}$$

$$V_{\mathrm{T}} \quad -\frac{\kappa\,\mathrm{Pr}}{C_{\mathrm{T}} \quad \theta}\frac{\partial \theta}{\partial Y} \tag{13}$$

where Pr and Sc are the Prandtl and Schmidt numbers for a porous medium, C_{T} is the thermophoresis parameter, and B is the buoyancy parameter, which are defined as:

$$\mathrm{Pr} \quad \frac{\nu}{\alpha_{\mathrm{m}}}, \quad \mathrm{Le} \quad \frac{\alpha_{\mathrm{m}}}{D}, \quad C_{\mathrm{T}} \quad \frac{T_{\mathrm{w}} - T_\infty}{T_\infty}, \quad B \quad \frac{\beta_{\mathrm{c}}(C_{\mathrm{w}} - C_\infty)}{\beta_{\mathrm{T}}(T_{\mathrm{w}} - T_\infty)} \tag{14}$$

The boundary conditions (5) become:

$$\begin{array}{l} \text{at } Y \quad 0\text{:} \quad V \quad V(X), \quad \theta \quad 1, \quad \phi \quad 1 \\ \text{as } Y \quad \infty\text{:} \quad U \quad 0, \quad \theta \quad 1, \quad \phi \quad 1 \end{array} \tag{15}$$

We now look for a similarity solution of eqs. (9)-(12) of the form:

$$\begin{array}{c} \eta \quad \dfrac{\mathrm{y}}{\sqrt{\xi}}, \quad \psi \quad \sqrt{\xi}\mathrm{f}(\eta), \\ \theta \quad \theta(\eta), \quad \phi \quad \phi(\eta), \quad \xi \quad X \end{array} \tag{16}$$

We now define a stream function $\psi(x, y)$ which satisfies the continuity eq. (1) with:

$$u \quad \frac{\partial \psi}{\partial y}, \quad v \quad \frac{\partial \psi}{\partial x}$$

The transformed governing equations for bonundary layer flows become:

$$\begin{array}{c} (1 \quad M)f \quad \theta \quad B\phi \\ \theta \quad \dfrac{1}{2}f\theta \ = 0 \\ \dfrac{1}{\mathrm{Sc}}\phi \quad \dfrac{1}{2}f\theta \quad \kappa\dfrac{\mathrm{Pr}}{C_{\mathrm{T}} \quad \theta} \ \theta\,\phi \quad \phi\theta \ - \dfrac{\theta^{\,2}\phi}{\theta + C_{\mathrm{T}}} \quad 0 \end{array} \tag{17}$$

Subject to the boundary conditions becomes:

$$\begin{array}{l} \text{at } \eta \quad 0\text{:} \quad f \quad Fw, \quad \theta \quad 1, \quad \phi \quad 1, \\ \text{as } \eta \quad \infty\text{:} \quad f \quad 0, \quad \theta \quad 0, \quad \phi \quad 0 \end{array} \tag{18}$$

where a prime denotes ordinary differentiation with respect to η.

Of interest in this problem are the non-dimensional concentration profiles, $\phi(\eta)$ and the wall thermophoretic depostion velocity V_{m} which is given by:

$$V_{\mathrm{m}} \quad \frac{\kappa\,\mathrm{Pr}}{1 \quad C_{\mathrm{T}}}\theta\ (0) \tag{19}$$

We notice that for $\kappa = 0$ (absence of thermophoresis), eqs. (15)-(17) reduce to those of Cheng and Minkowycz [17] when $B = 0$ and to those of Bejan and Khair [18] when B 0, respectively.

The Sherwood number (Sh) is important physical parameter for this problem. This can be defined as:

$$\mathrm{Sh} \quad \frac{J_{\mathrm{w}} v}{CDv_{\mathrm{w}}}; \quad J_{\mathrm{w}} \quad D\left.\frac{\partial C}{\partial \mathrm{y}}\right|_{\mathrm{y} \ 0} \tag{20}$$

Results and discussion

For this present problem numerical computations have been carried out by employing the finite difference method known as the Shooting method. It is clearly seen that the results are given values of the parameters κ, Fw, Pr, Sc, B, C_T, V_m, and M. However, to check the present numerical results, we calculate the values of the reduced heat transfer, $-\theta'(0)$, and mass transfer, $-\phi'(0)$, from the plate for $\kappa = 0$, $B = 0$ and 1, and Sc = 1. Thus, for

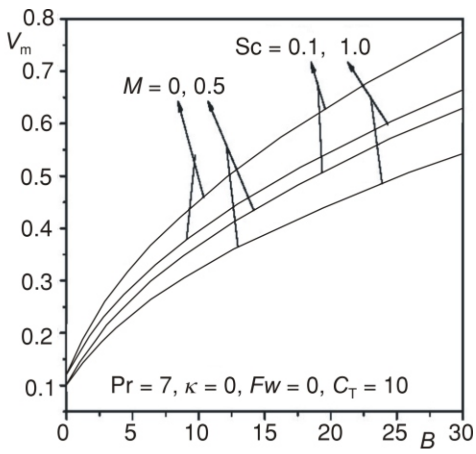

Figure 2. Effects of *B*, *M*, and Sc on thermophoretic deposition velocity

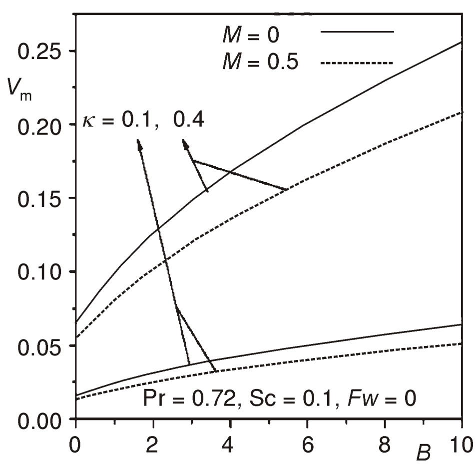

Figure 3. Effects of *B*, *M*, and κ on thermophoretic deposition velocity

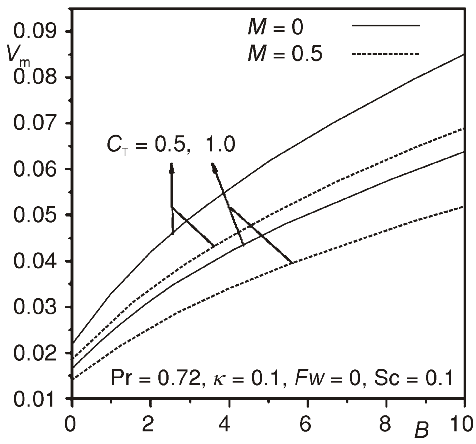

Figure 4. Effects of *B*, *M*, and C_T on thermophoretic deposition velocity

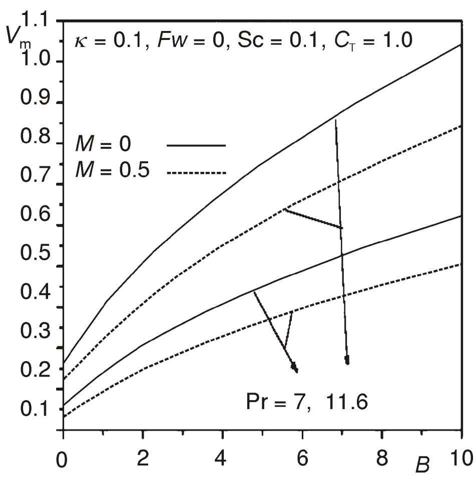

Figure 5. Effects of *B*, *M*, and Pr on thermophoretic deposition velocity

$B = 0$ we obtained $-\theta'(0) = 0.4438$ while the value found by Cheng and Minkowycz [17] is $-\theta'(0) = 0.444$. Also, for $\kappa = 0$, $B = \mathrm{Sc} = 1$, we get $-\theta'(0) = -\phi'(0) = 0.6276$ while Bejan and Khair [18] obtained $-\theta'(0) = -\phi'(0) = 0.628$. There is excellent agreement between the respective results.

Once the values of the functions $f(\eta)$, $\theta(\eta)$, $\phi(\eta)$, and their derivatives at $\eta = 0$ are known, the quantitie V_m can now be calculated, respectively, from the following expressions. Typical concentration profiles $\phi(\eta)$ and the wall thermophoretic velocity V_m are shown in figs. 2-11 for some values of the governing parameters κ, *Fw*, Pr, Sc, *B*, C_T, V_m, and *M*. These figures show how the magnetic filed and the concentration boundary layer and the wall deposition velocity react to changes with the parameters κ, *Fw*, Pr, Sc, *B*, C_T, V_m, and *M*.

The thermophoretic depostion velocity V_m and the buoyance parameter for different values of the magnetic field are depicted with Sc in fig. 2, κ (thermophoresis parameter) in fig. 3, C_T paramter in fig. 4, and Pr in fig. 5.

It is observed that the velocity increases with the decrease of *M* parameter. On the other hand, the thermophoretic depostion velocity V_m increases with the decrease of Sc and C_T, but increases with κ parameter and Pr parameter.

Figure 6 shows the corresponding effect of varying *Fw* on the skin-friction coefficient V_m. This figure confirms that as *Fw* increases, the wall thermophoretic deposition velocity increase.

Figures 7-11 represents the dimensionless concentration for different values of *Fw* parameter, buoyancy parameters, *M* parameters, Sc parameters, and C_T parameters, respectively. It is clear that the concentration of the fluid increases with the increase of *M* and conversily, while *Fw*, *B*, Sc, and C_T parameters decrease.

It can be noticed that the wall thermophoretic deposition velocity becomes sensitive to the variation of the parameters of particular benefit in processes.

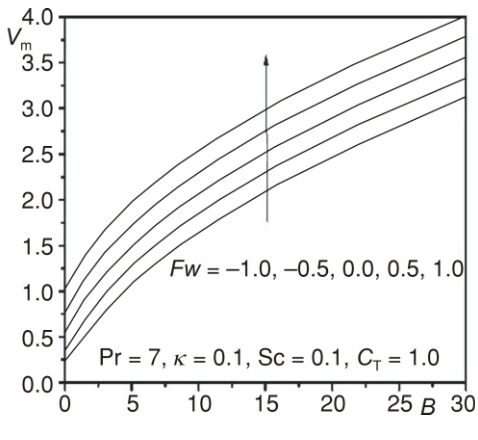

Figure 6. Effects of *B*, *Fw* on thermophoretic deposition velocity

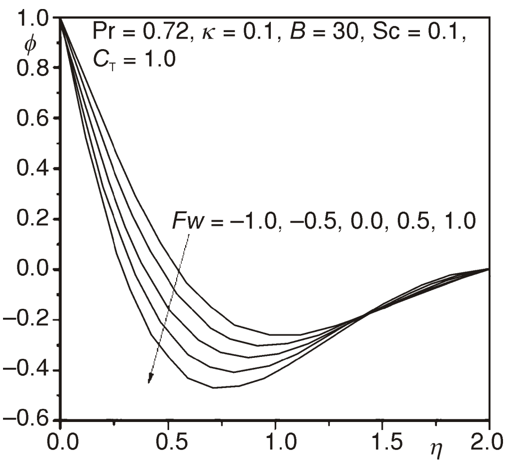

Figure 7. Effects of *Fw* on concentration profiles

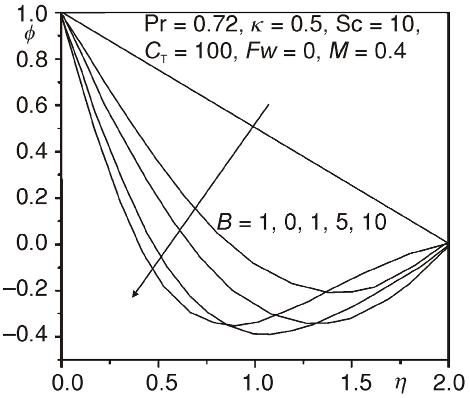

Figure 8. Effects of *B* on concentration profiles

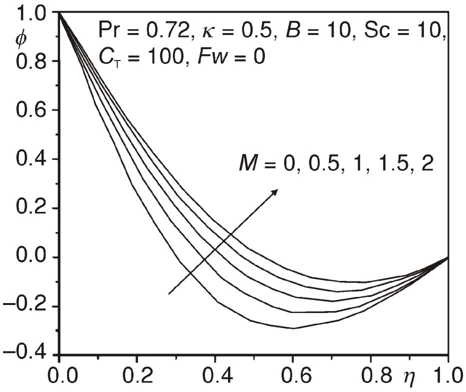

Figure 9. Effects of *M* on concentration profiles

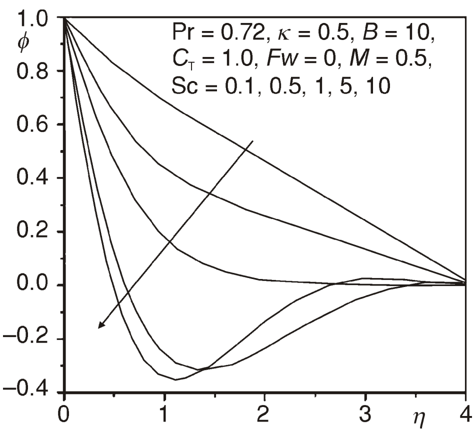

Figure 10. Effects of Sc on concentration profiles

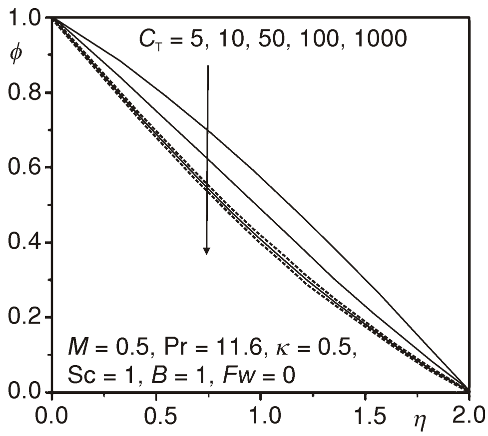

Figure 11. Effects of C_T on concentration profiles

Nomenclature

B	– buoyancy parameter $(=\beta_c(C_w - C_\infty)/\beta_T(T_w - T_\infty))$, [–]
B_0	– magnetic field, [T]
C	– species concentration in the boundary layer, [kgm^{-2}]
C_m, C_s, C_t, C_1, C_2, C_3	– constants in eq. (7), [–]
C_∞	– species concentration of the ambient fluid, [–]
D	– chemical molecular diffusivity, [–]
Fw	– dimensionless nonuniform surface mass flux, [–]

f – dimensionless stream function, [–]
g – acceleration due to gravity, [ms^{-2}]
J_w – rate of transfer of species concentration defined by eq. (20), [–]
K – permeability of the porous medium in eq. (2), [–]
Kn – Knudsen number, [–]
Le – Lewis number (= α_m/D), [–]
M – magentic field parameter (= σB_0^2), [–]
Pr – Prandtl number (= ν/α_m), [–]
Ra – Rayleigh number (= $gK\beta_T(T_w - T_\infty)l/\alpha_m\nu$), [–]
Sc – Schmidt number (ν/D), [–]
Sh – Sherwood number (= $J_w\nu/CD\nu_w$), [–]
T – temperature of the fluid in the boundary layer, [K]
T_∞ – temperature of the ambient fluid, [K]
T_w – temperature at the surface, [K]
u, v – x- and y-components of the velocity field, [ms^{-1}]
$V(x)$ – transpiration velocity, [–]
V_m – wall thermophoretic velocity, [–]
V_T – thermophoretic velocity, [–]
x, y – axis in direction along and normal to the plate, [m]

Greek symbols

α – thermal diffusivity, [m^{2}s^{-1}]
β – volumetric expansion coefficient of temperature, [K^{-1}]
η – non-dimensional pseudo-similarity variable, [–]
θ – dimensionless temperature function (= $(T - T_\infty)/(T_w - T_\infty)$), [–]
κ – the thermophoretic coefficient (eq. 7), [–]
λ_g, λ_p – thermal conductivity of gas and diffused particles, respectively, [–]
μ – fluid viscosity, [Pa s]
ν – kinematic coefficient of viscosity, [m^{2}s^{-1}]
σ – electrycal conductivity, [Wm^{-2}K^{-4}]
ϕ – dimensionless species concentration function (= $(C - C_\infty)/(C_w - C_\infty)$), [–]
ψ – stream function, [m^{2}s^{-1}]

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Author's adresses:

A. Y. Bakier, M. A. Mansour
Department of Mathematics
Faculty of Science, Assiut University
Assiut, Egypt

Corresponding author A. Y. Bakier
E-mail: aybakier@yahoo.com

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