CONSTRUCTAL THEORY AND FLOW ARCHITECTURES IN LIVING SYSTEMS

by

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We apply Constructal theory to the study of the flow structure of the human respiratory tree. We show that the flow architecture that would perform oxygenation of the blood and removal of carbon dioxide best, i. e. with lowest resistance, would be composed of a channel system with 23 bifurcation with a diffusive zone (alveolus) at the end. As this tree matches the human respiratory tree we conclude that nature has optimized it in time. Two constructal relationships also emerge: (1) the length λ*, defined by the ratio of the square of the airway diameter to its length, is constant for all individuals of the same species, and (2) the length* λ *is related to the volume of the space allocated to the respiratory process, to the length of the respiratory tree and to the area of the alveoli, and determines univocally the branching level of the respiratory tree.*

Keywords: *flow architectures, living systems, constructal theory*

Introduction

The Constructal law, first formulated by Adrian Bejan [1], states that if a system has freedom to morph it develops in time the flow architecture that provides easier access to the currents that flow trough it. In all classes of flow systems (animate, inanimate, engineered) the generation of flow architecture emerges as a universal phenomenon. According to Constructal theory the optimal structure is constructed by optimizing volume shape at every length scale, in a hierarchical sequence that begins with the smallest building block and proceeds towards larger building blocks (which are called "constructs"). A basic outcome of Constructal theory is that system shape and internal flow architecture do not develop by chance, but result from the permanent struggle for better performance and therefore must evolve in time [2].

Bejan used Constructal theory to successfully explain some allometric laws of living structures namely the rhythm of respiration in animals in relation with body size [1], the relation between metabolic rate and total body volume [1, 3], and the heartbeat frequency in relation to metabolic rate [1, 4]. Reis *et al*. [5] focused on the structure of the pulmonary airflow tree, which starts at the trachea and bifurcates 23 times before reaching the alveolar sacs. Until now, the reason for the existence of just 23 bifurcations in the respiratory tree has remained unexplained in the literature. Has this special flow architecture been developed by chance or does it represent the optimum structure for the lung's purpose, which is the oxygenation of the blood?

Purpose and trade-off between competing trends

Even though every living fluid tree works with a specific purpose, all share some general features. For example, every living fluid tree works either for the delivery of substances to a volume or a surface where a process occurs, *e. g.* a chemical reaction, or for the removal of other substances including the products of chemical reactions. At the smallest scale diffusion dominates while channeling of the flow and development of flow architectures occur at higher scales. Figure 1 illustrates how a first channel of width D_0 and conductivity K_0, collects the fluid that permeates the rectangular area and delivers it to a wider channel (D_1, K_1). Channels organize hierarchically in a flow architecture in which channels of lower conductivity are tributaries of a channel of higher conductivity.

As illustrated in fig. 1, increase in the number of the elementary areas through which fluid diffuses implies increasing number of channels, and therefore increasing resistance to fluid flow. On the other hand, if the number of channels is reduced, the resistance to fluid flow is also reduced accordingly but in this case, the elemental rectangle has to have a larger area, therefore increasing the resistance to fluid diffusion. The optimal flow architecture is the one that results from the trade-off between these two competing trends.

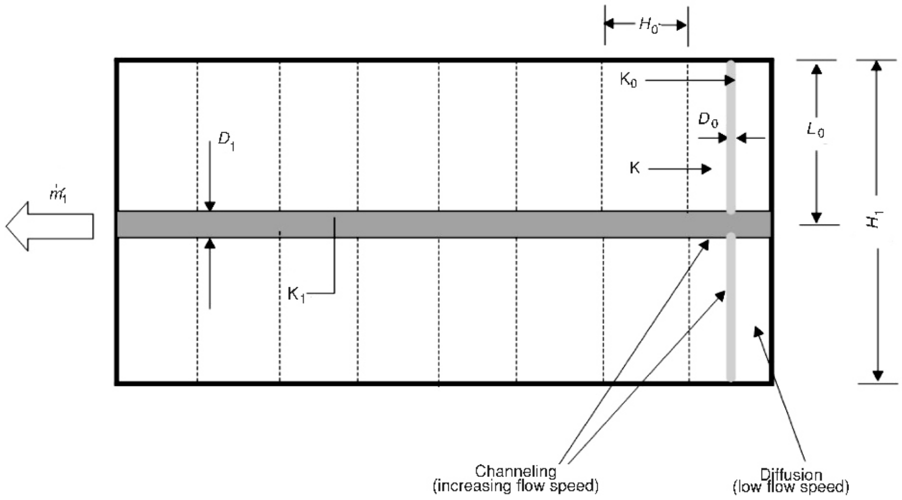

Figure 1. Area-to-point flow. The fluid seeps through the pores of the elemental volume of conductivity K and surface area (H_0 L_0) before reaching the first channel of width D_0 and conductivity K_0 that delivers it to the next higher conductivity (K_1) channel of width D_1. The flow rate at the end of the K_1 channel is $\dot{m}_1$. The width of this construct is $H_1 = 2$ $L_0 + D_1$ while its length is given by n_1 H_0 where n_1 is the number of the elemental volumes. Complex flow architecture may be constructed by repeating this process at higher scales

In line with Bejan's Constructal law, we believe that every living system has developed in time the flow architecture that provides easier access to the currents that flow trough it. As a leading example of a living flow structure, we focus on the human respiratory tree. By using Constructal theory, Reis *et al*. [5] addressed the study of the respiratory tree as a flow system with duct flow resistances (trachea and bronchial system) and diffusive resistances. In their study, to evaluate and compare the flow resistances, was considered that oxygen and carbon dioxide flow within the respiratory tree (bronchial tree plus alveolar sacs) as driven by the chemical potential. This is a rather convenient potential because pressure differences that drive the isothermal airflow in the bronchial system may be expressed in terms of chemical potential differences trough the generalized Gibbs-Duhem equation, $\Delta\mu \quad \rho^{\ 1}\Delta P \quad \Delta\varepsilon$, where ε stands for kinetic energy per unit mass (see [5] for details). The bronchial tree was assumed to be composed of cylindrical channels with Hagen-Poiseuille flow. Even though the bronchial channels are not perfectly round, Bejan has shown that nearly round channels perform almost as well as the exactly round channels [2].

For laminar flow, Constructal theory indicates that the minimum flow resistance at a bifurcation is achieved if the ratio between consecutive duct diameters is:

$$\frac{D_n}{D_{n \quad 1}} \quad 2^{\quad 1/3} \tag{1}$$

while the ratio of the respective lengths, is;

$$\frac{L_n}{L_{n \quad 1}} \quad 2^{\quad 1/3} \tag{2}$$

These relations theoretically predicted by Constructal theory, were discovered empirically long ago and correspond to the so-called Murray's laws.

After having considered every detail of the flow between the entrance of the trachea and the alveolar surface where oxygen meets the blood and carbon dioxide is removed from the blood, including the resistances due to bifurcations, Reis *et al.* [5] showed that the global resistance [Jskg^{-2}] to oxygen transportation within the respiratory tree is of the form:

$$R_{ox} \quad \frac{256 v L_0}{\pi D_0^4 [(\phi_{\mathrm{ox}})_0 \quad \phi_{\mathrm{ox}}]\rho}(N \quad 1) \quad \frac{0.13(R_{\mathrm{g}})_{\mathrm{ox}} T 2^{\quad 2N/3}}{\pi L_0 D_{\mathrm{ox}} \phi_{\mathrm{ox}} \rho} \tag{3}$$

In eq. (3), N matches the number of bifurcations of the bronchial tree, v is the kinematic viscosity of the air, L_0 and D_0 represent trachea length and diameter, respectively, T is temperature, ϕ_{ox} and $(\phi_{ox})_0$ represent the relative concentration of oxygen in the alveoli and in the outside air, respectively, D_{ox} is the diffusivity of the oxygen in the air, R_g is the air constant, and ρ stands for the density of the air.

The first term in the right hand side of eq. (3) represents the global channel flow resistance (bronchiolar tree) while the second term matches to the global diffusive resistance to oxygen transport in the alveoli. Equation (3) with the corresponding variables holds also for the resistance to carbon dioxide flow in the respiratory tree. These resistances can be evaluated by assuming a body temperature of 36 ºC and taking all pertinent values at this temperature. The average value of oxygen relative concentration within the respiratory tree, ϕ_{ox}, may be evaluated from the alveolar air equation in the form: $[(\phi_{ox})_0 - \phi_{ox})]Q - S = 0$, where $(\phi_{ox})_0 \sim 1/2(\phi_{air} + \phi_{ox})$ and ϕ_{air} are the oxygen relative concentration at the entrance of the trachea, and in the external air, respectively, Q is the tidal airflow, and S is the rate of oxygen consumption. With $(\phi_{ox})_{air} = 0.2095$, $Q \sim 6\ 10^{-3}$ m^3/min., and $S \sim 0.3\ 10^{-3}$ m^3/min. we obtain $\phi_{ox} \sim 0.1095$. The value of the average relative concentration of carbon dioxide in the respiratory tree, $\phi_{cd} = 0.04$. In this case we used $S = 0.24\ 10^{-3}$ m^3/min. since the respiratory coefficient is close to 0.8 and $(\phi_{cd})_{air} \sim 0.315\ 10^{-3}$. Anatomic treatises indicate that L_0 is typically 15 cm, while the trachea diameter, D_0, is approximately 1.5 cm. The global resistances to oxygen and carbon dioxide transportation in the respiratory tree are plotted against number of bifurcations in fig. 2.

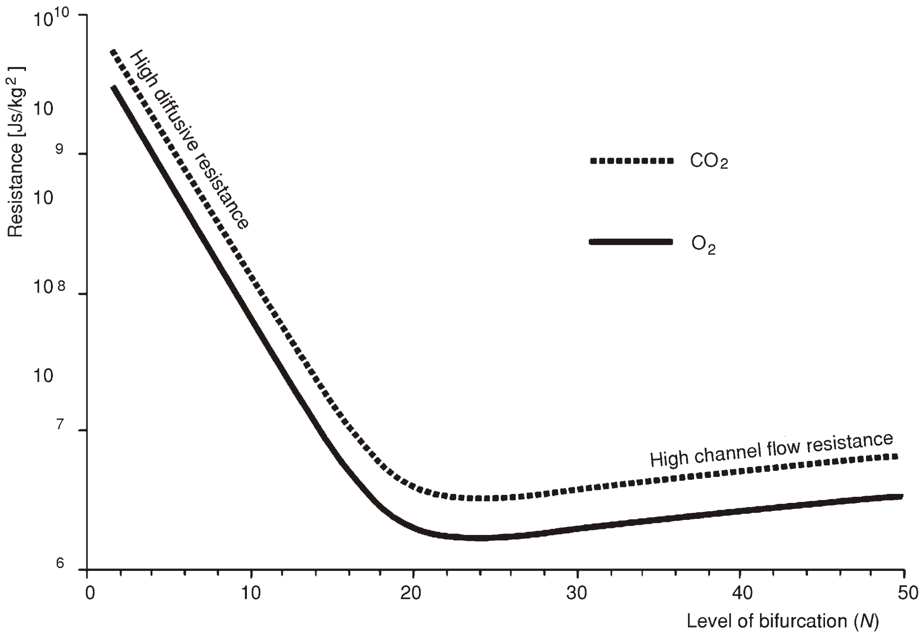

Figure 2. Total resistances to oxygen and carbon dioxide transport between the entrance of the trachea and the alveolar surface plotted as function of the level of bifurcation (*N*). The minimum resistance both to oxygen access and carbon dioxide removal matches *N* = 23

The minimum of each resistance occurs close to $N = 23$. This same result can be obtained by finding the number of bifurcations N_{opt} analytically that matches to the minimum of the global resistance – see eq. (3). This minimum is given by:

$$N_{opt} = 2.164 \ln \left[\frac{2.35 \times 10^{-4} D_0^4 (R_g)_{ox} T}{L_0^2 \nu D_{ox}} \left(\frac{(\phi_{ox})_0}{\phi_{ox}} - 1 \right) \right] \tag{4}$$

With the same values used for the respective curves in fig. 2 we found N_{opt} = = 23.4, and N_{opt} = 23.2 for the oxygen and the carbon dioxide transport, respectively. As the number of bifurcations must be an integer we conclude that it must be 23.

A first result is that the human respiratory tree that bifurcates 23 times between the trachea and the alveoli is optimized both for oxygen access and for carbon dioxide removal. The trade-off between resistance to flow in the bronchial tree and the resistance oxygen and carbon dioxide diffusion in the alveoli is achieved by the human respiratory tree, which has been optimized by nature in time. A second outcome is that the actual flow architecture of the respiratory tree can be anticipated theoretically based on Constructal theory.

Constructal relationships of the respiratory tree

We see from eq. (4) that the number of bifurcations that matches minimal global resistance to oxygen access an carbon dioxide removal is function of several environmental variables such as normal body temperature, oxygen and carbon dioxide diffusivities and concentrations in the air, air kinematic viscosity and only one morphological parameter, which is the length $\lambda = D_0^2/L_0$. As every individual lives with the same the average environmental parameters, we conclude that if the respiratory tree with 23 bifurcations is a characteristic of humankind therefore the number λ is also a characteristic of humankind. In other words, the ratio of the square of trachea diameter to its length is the same for every human being. This theoretical result is anticipated by Constructal theory and now awaits confirmation by the anatomists.

Another constructal relationship involving the length λ, the area allocated for the respiratory process (the total area of the alveoli) A, the volume of the lungs V, and the length of the respiratory tree (between the entrance of the trachea and the surface of the alveolus) L, was also derived by Reis *et al*. [5] and is the following:

$$\lambda = \frac{D_0^2}{L_0} = 8.63 \frac{AL}{V} \left[\frac{\nu D_{ox} \phi_{ox}}{(R_g)_{ox} T[(\phi_{ox})_0 - \phi_{ox}]} \right]^{1/2} \tag{5}$$

From eq. (5) we conclude that the non-dimensional number AL/V, determines the characteristic length $\lambda = D_0^2/L_0$ which in turn determines the number of bifurcations of the respiratory tree by eq. (4). This constructal relationship may be summarized as follows: "The alveolar area required for gas exchange *A*, the volume allocated to the respiratory system *V*, and the length of the respiratory tree *L*, which are constraints posed to the respiratory process determine univocally the structure of the lungs, namely the bifurcation level of the bronchial tree." Here we may observe the realm of Constructal theory, which is minimization of global resistances to fluid access under geometric constraints imposed to the process.

Conclusions

The flow architecture of the human respiratory tree can be anticipated based on the Constructal law. If, as engineers we had to design the flow architecture that would provide minimal resistance to oxygen access from the external air to a surface where a chemical reaction takes place and to carbon dioxide removal from this same surface, we would find out a flow tree with 23 bifurcations with alveoli at the end. The human respiratory tree is such a tree, therefore indicating that nature has optimized in time the human respiratory flow architecture.

Additional constructal relationships of the respiratory tree were anticipated theoretically: (1) the length λ (defined as the ratio of the square of the first air way diameter to its length) is constant for every individual of the same species and related to the characteristics of the space allocated for the respiratory process; (2) the length λ is univocally determined by a non-dimensional number, *AL/V*, which involves the characteristics of the space allocated to the respiratory system, namely the total alveolar area, *A*, the total volume *V*, and the total length of the airways, *L*.

Finally, we remark that the Constructal theory that was successfully employed in engineering (see [1]), promises to serve as fundamental tool to the study of living flow structures.

Nomenclature

A – total alveolar area, [m^2]
D – diffusion coefficient, [m^2s^{-1})
D_0 – diameter of the trachea; channel diameter in fig. 1, [m]
L – length from the entrance of the trachea to alveolus, [m]
L_0 – length of the trachea; channel length in fig. 1, [m]
N – total number of bifurcations, [–]
P – pressure, [N/m^2]
Q – tidal airflow, [m^3]
R – total resistance, [J $kg^{-2}s^{-1}$]
R_g – specific gas constant, [$Jkg^{-1}K^{-1}$]
S – rate of oxygen consumption, [kgs^{-1}]
T – temperature, [K]
V – volume, [m^3]

Greec letters

ε – mechanical energy per unit mass, [Jkg^{-1}]
λ – characteristic length (eq. 5), [m]
μ – chemical potential, [Jkg^{-1}]
ν – kinematic viscosity, [m^2s]
ρ – density, [kgm^{-3}]
ϕ – relative gas concentration, [–]

Subscripts

air – relative to air
cd – relative to carbon dioxide
n – order of bifurcation (0 for trachea)
opt – relative to the optimal value
ox – relative to oxygen

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