ON PECULIAR PROPERTY OF THE VELOCITY FLUCTUATIONS IN WALL-BOUNDED FLOWS

by

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Statistical analysis of the velocity fluctuations is performed for the near-wall region of wall-bounded flows. By demanding that the small-scale part of the fluctuations satisfies constraints imposed by local axisymmetry, it was found that the small scales must be entirely suppressed in the near-wall region. This major conclusion is well supported by all available data from direct numerical simulations.

Key words: *turbulence, transition, boundary layer, turbulent channel flows, turbulent drag reduction*

Introduction

The question of how wall-bounded flows can be rationally controlled with *reasonable* cost is of fundamental and practical importance. This outstanding question has engineering relevance since it is directly related to the viscous drag and heat transfer processes. Substantial work has been done along these lines, but only partial success has been achieved. The purpose of this paper is to shed some light on the subject by considering the velocity fluctuations in the near-wall region which possesses specific invariance in the dissipation range. Constraints that lead to effective damping of the velocity fluctuations close to the wall have been examined.

Before we proceed with formal treatment of the fundamental issue related to wall turbulence, it is useful to analyse its evolution by looking into the anisotropy of the Reynolds stresses utilizing the databases of direct numerical simulations at low Reynolds numbers. The level of anisotropy of turbulence can be quantified, following analysis of Lumley and Newman [1], by introducing the anisotropy tensor:

$$a_{ij} = \frac{\overline{u_i u_j}}{q^2} - \frac{1}{3}\delta_{ij} \tag{1}$$

and its scalar invariants:

$$\mathrm{II}_a = a_{ij}a_{ji} \tag{2}$$

$$\mathrm{III}_a = a_{ij}a_{jk}a_{ki} \tag{3}$$

A plot of II_a versus III_a for axisymmetric turbulence, namely:

$$II_a = \frac{3}{2}\left(\frac{4}{3}|III_a|\right)^{2/3}$$

and two-component turbulence, namely:

$$II_a = \frac{2}{9} + 2III_a \tag{5}$$

defines the anisotropy invariant map according to Lumley [2]. This plot, shown in fig. 1 bounds all physically realizable turbulence. Two curves shown in this figure represent axisymmetric turbulence. The right curve corresponds to turbulence strained by axisymmetric expansion and the left curve corresponds to straining by axisymmetric contraction. Along the straight line resides two-component turbulence. Such turbulence exists in the region of viscous sublayer of wall-bounded turbulent flows. The limiting states of the turbulence are located at the corner points on the right- and left-hand sides of the anisotropy invariant map and correspond to one-component turbulence and isotropic two-component turbulence, respectively.

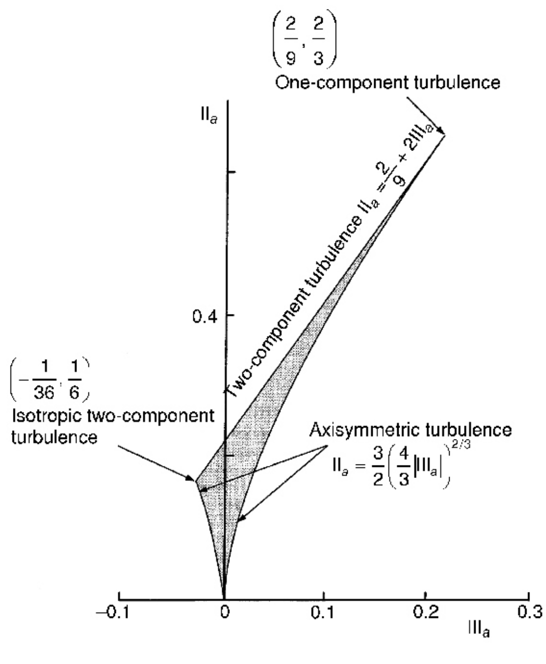


Figure 1. Anisotropy invariant map and the limiting values of invariants for the different states of the turbulence

Note that relations (4) and (5) for the invariants of a_{ij} also hold for the invariants of the anisotropy e_{ij} in the stress dissipation $\varepsilon_{ij} = \nu\overline{\partial u_i/\partial x_k \partial u_j/\partial x_k}$:

$$e_{ij} = \frac{\varepsilon_{ij}}{\varepsilon} - \frac{1}{3}\delta_{ij} \tag{6}$$

$$\mathrm{II}_e = e_{ij}e_{ji} \tag{7}$$

$$\mathrm{III}_e = e_{ij}e_{jk}e_{ki} \tag{8}$$

Therefore, in axisymmetric turbulence the invariants II_e and III_e are related by:

$$\mathrm{II}_e = \frac{3}{2}\left(\frac{3}{4}|\mathrm{III}_e|\right)^{2/3} \tag{9}$$

while in two-component turbulence,

$$\mathrm{II}_e = \frac{2}{9} + 2\mathrm{III}_e \tag{10}$$

The invariants of e_{ij} coincide with those corresponding to a_{ij}:

$$\mathrm{II}_e = \mathrm{II}_a \tag{11}$$

$$\mathrm{III}_e = \mathrm{III}_a \tag{12}$$

in the limiting states of the turbulence owing to [3]:

$$e_{ij} = a_{ij} \tag{13}$$

Looking at the two vertices of the anisotropy invariant map shown in fig. 1 and accounting for (1)-(13) we conclude that in the limiting states turbulence must satisfy the two-component limit and axisymmetry at large and small scales*.

The above relations for the invariants (11) and (12) also hold (very nearly) across the region of viscous sublayer of wall-bounded flows if the inhomogeneous parts of the dissipation correlations $1/4(\nu\partial^2 \overline{u_i u_j}/\partial x_k \partial x_k)$ are removed from ε_{ij} using the two-point correlation technique [8 (pp. 69-72), 9].

Anisotropy invariant mapping of wall turbulence

In an advanced approach, the dynamics of turbulence is studied across the functional space (shown in fig. 1) formed by the two scalar invariants, II_a and III_a, of the anisotropy tensor. In contrast to the real space where observations usually take place, turbu-

* Using the two-point correlation technique it is possible to show and demonstrate, utilizing the experimental [4] or numerical databases [5], that local isotropy, local axisymmetry, and local homogeneity for the small-scale part of turbulence must hold by definition in statistically isotropic, axisymmetric, and homogeneous fields, respectively in order to satisfy the constraint of coincidence for the two-point correlations [6- 8].

lence can appear in the invariant space only within a bounded domain which is quite narrow. This suggests that the behavior of turbulence within the domain cannot differ too much from the behavior at the surrounding boundaries [1]. Following this concept, the analysis to be presented produces not only insights relevant for understanding the dynamics of wall turbulence but also provides the link between the mechanisms responsible for transition and the continuous production of turbulence close to the solid boundaries. Studies of the dynamics of coherent structures close to the wall by Kline *et al.* [10], Kim, Kline and Reynolds [11], and Falco [12] showed remarkable analogies to the sequence of invents leading to the transition which led Laufer [13, 14] to the conclusions that these processes are very closely interconnected and therefore should be treated theoretically using the same mathematical tools.

The influence of the Reynolds number on the anisotropy of turbulence in a plane channel flow is shown in fig. 2. There is noticeable trend in these data that can be clearly

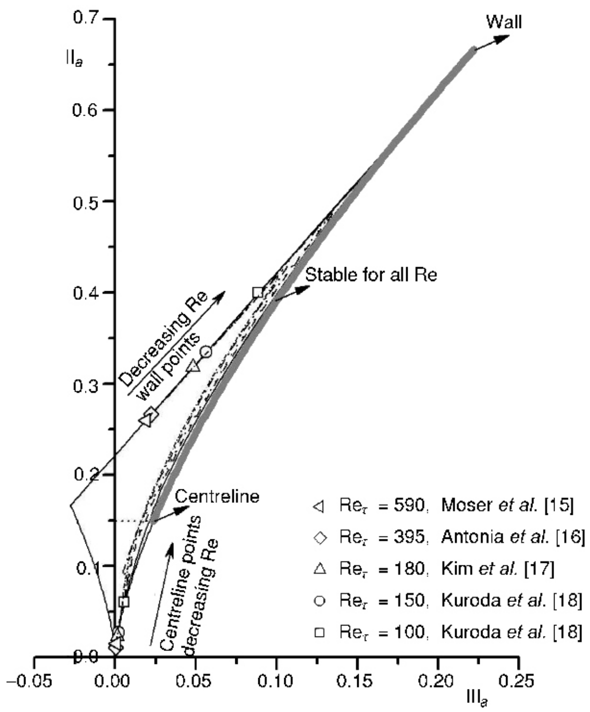


Figure 2. Anisotropy invariant mapping of turbulence in a channel flow. Data, which correspond to low Reynolds number, show the trend as Re $\quad Re_{crit}$ towards the theoretical solution valid for small, neutrally stable, statistically stationary axisymmetric disturbances [19]. The shading indicates the area occupied by the stable disturbances: for such disturbances it is expected that the laminar regime in a flat plate boundary layer will persist up to very high Reynolds numbers

distinguished. As the Reynolds number decreases towards the critical value valid for transition, from the laminar to the turbulent state, the anisotropy increases. Away from the near-wall region, x_2 8, the trend in the data indicates a tendency towards the right boundary of the anisotropy invariant map which corresponds to the axisymmetric state with the streamwise intensity larger than the intensities in the other two directions. Data which correspond to the region of viscous sublayer, 0 x_2 5, and lie along the two-component limit, tend towards the one-component state of turbulence. Here, x_2 represents normalized distance from the wall x_2 $x_2 u_\tau/\nu$ where u_τ is the wall shear velocity and ν is the kinematic viscosity of the working fluid.

The extrapolated trajectory of the stable disturbances, indicated in fig. 2 by the shading, which corresponds to Re Re_{crit}, coincides with the result of theoretical analysis of the transition process in a laminar boundary layer exposed to small (and therefore linear), neutrally stable (which are losing energy through the viscous dissipation process at the same rate approximately at which they gain energy from the mean flow), statistically stationary axisymmetric disturbances whose statistical properties are invariant under rotation about the flow direction [19]. This particular invariance is logical and consistent with the statistical dynamics of the disturbances far away from the near-wall region which shows u_1 u_3 u_2 that as x_2 ∞ [8]. The above-mentioned theoretical consideration of the transition process, based on the transport equations for the statistical properties of the disturbances, shows that the laminar regime in the boundary layer will persist up to very high Reynolds numbers if the anisotropy of the free stream is sufficiently high. This was tested experimentally, in a large wind tunnel facility of the Lehrsthul für Strömungsmechanik in Erlangen, by maintaining the laminar regime in a flat plate boundary layer up to $\mathrm{Re}_x = xU_\infty/\nu$ $4 \cdot 10^6$ where x is the distance from the leading edge of the plate and U_∞ is the free stream velocity [19].

Below we provide not only the proof but also quantitative support for the above statements which indicate the chief possibility for rational control of flow very close to the solid boundary: the effective way to suppress fluctuations in the near-wall region is to force them to be predominantly one-component. Wisdom, embedded into the anisotropy invariant map, suggests that this is equivalent to the requirement that fluctuations near the wall satisfy the two-component limit, axisymmetry at large and small scales and invariance under rotation about the flow direction.

The limiting state of near-wall turbulence

Let the mean flow be in the x_1 direction and consider the near-wall region lying in the plane parallel to the wall $x_2 = 0$. A Taylor series expansion of the instantaneous velocity fluctuations about the wall which satisfies the continuity equation $u_k / x_k = 0$ reads [20]:

$$\begin{array}{llll} u_1 & a_1 x_2 & a_2 x_2^2 & \ldots \\ u_2 & & b_2 x_2^2 & \ldots \quad \text{as} \quad x_2 \quad 0 \\ u_3 & c_1 x_2 & c_2 x_2^2 & \ldots \end{array} \tag{14}$$

where, the coefficients a_i, b_i, and c_i are functions of time and space coordinates x_1 and x_3.

If the small-scale part of the fluctuations is *locally* invariant to rotation about the x_1 coordinate, it is fairly easy to prove that the following relations hold for statistics of the velocity derivatives of the n^{th} order [21]:

$$\overline{\frac{\partial^n u_1}{\partial x_2^n}}^{\,2} \qquad \overline{\frac{\partial^n u_1}{\partial x_3^n}}^{\,2} \tag{15}$$

$$\overline{\frac{\partial^n u_2}{\partial x_2^n}}^{\,2} \qquad \overline{\frac{\partial^n u_3}{\partial x_3^n}}^{\,2} \tag{16}$$

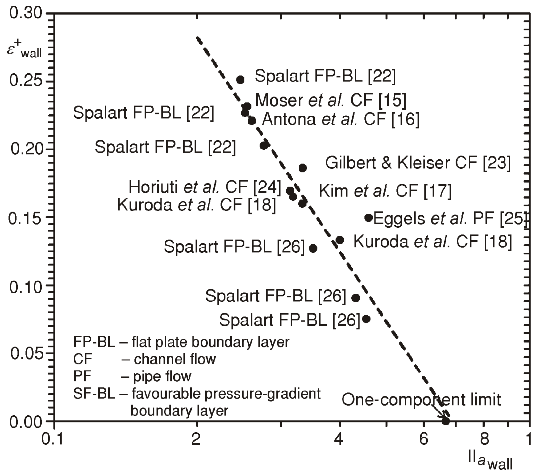
$$\overline{\frac{\partial^n u_2}{\partial x_3^n}}^{\,2} \qquad \overline{\frac{\partial^n u_3}{\partial x_2^n}}^{\,2} \tag{17}$$

Inserting the series expansion (14) for the velocity fluctuations into expressions (15)-(17), it is straightforward to show, by comparing terms corresponding to the same power of x_2 , that all coefficients a_i, b_i, and c_i, must vanish in order to satisfy constraints imposed by the local axisymmetry. We may conclude, therefore, that inhomogeneous but locally axisymmetric turbulence, that is, turbulence whose statistical properties in *the dissipation range* are invariant under rotation about the axis aligned with the mean flow direction, must vanish in close proximity of the solid boundaries.

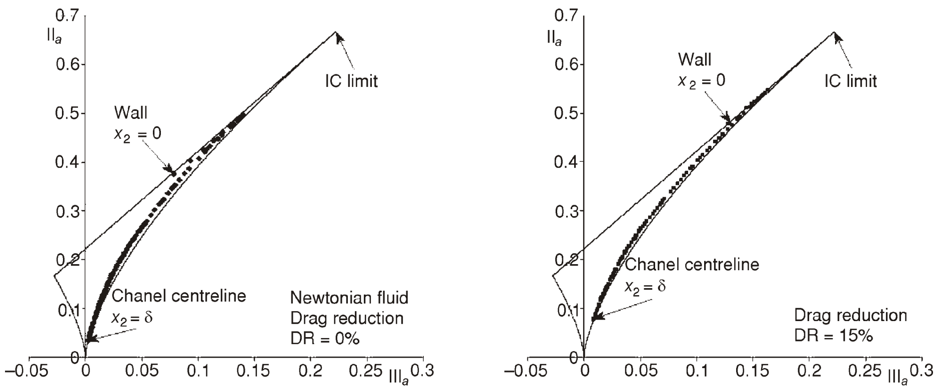
Logic, following the basics of invariant theory introduced by Lumley [2], suggests that we may expect fluctuations in the near-wall region to satisfy constraints imposed by the local axisymmetry when these tend asymptotically towards the one-component limit. For this very special case, two-component motions close to the wall must additionally satisfy axisymmetry at large and small scales, which are closely interrelated [3], and therefore also constraints imposed by local axisymmetry. Approaching the one-component limit, all of the coefficients of the Taylor series expansion for the instantaneous fluctuations about the wall vanish and therefore also the dissipation rate at the wall, causing significant suppression of small scale turbulence in the near-wall region. Figure 3 shows that this expectation is confirmed by all available data from direct numerical simulations of wall-bounded flows. This is demonstrated further in fig. 4, which shows trajectories of the joint variation of the invariants of a_{ij} across the anisotropy-invariant map from the numerical database of drag-reducing, fully developed, turbulent channel flow from Dimitropulos, Sureshkumar and Beris [27]. This figure confirms that, with increasing drag reduction (DR), which is accompanied by suppression of wall turbulence [28], the point that corresponds to the position at the wall $x_2 = 0$ moves upwards in the direction of the one-component limit.

Omitting the technical details of how to produce the desired componentality of the fluctuations close to the wall [29], which are not important to be explained here, the examples in figs. 3 and 4 strongly support the major conclusion which emerges from the

Figure 3. Turbulent dissipation rate at the wall, $\varepsilon \quad \nu(\overline{a_1^2} \quad \overline{c_1^2})$ normalized with the wall shear velocity and the kinematic viscosity of the flow medium versus the anisotropy of turbulence II_a at the wall. A best-fit line through the numerical data extrapolates fairly well the expected trend as the one-component limit (II_a = 2/3) is approached



present study: the effective way to suppress fluctuations in the near-wall region is to force them to be predominantly one-component.



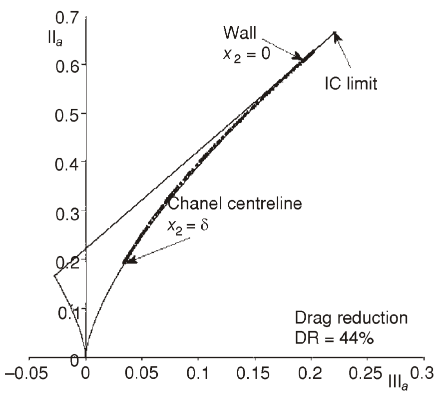


Figure 4. Anisotropy-invariant mapping of turublence in a fully developed channel flow with drag reduction from direct numerical simulations of Dimitropulos, Sureshkumar and Beris [27]. The trend in the data at the wall strongly supports the major conclusion which emerges from the presented analysis

Implications of the above-discussed property of the velocity fluctuations in the near-wall region for drag reduction, control of the laminar to turbulence transition process at huge Reynolds numbers and a dynamic description of the mechanism involved in self-maintenance of turbulence in wall-bounded flows are subjects of current research efforts [30, 31].

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References

[1] Lumley, J. L., Newman, G., The Return to Isotropy of Homogeneous Turbulence, *J. Fluid Mech., 82* (1977), pp. 161-178

[2] Lumley, J. L., Computational Modeling of Turbulent Flows, *Adv. Appl. Mech., 18* (1978), pp. 123-176

[3] Jovanović, J., Otić, I., Bradshaw, P., On the Anisotropy of Axisymmetric Strained Turbulence in the Dissipation Range, *J. Fluids Eng., 125* (2003), pp. 401-413

[4] Schenck, T., Jovanović, J., Measurements of the Instantaneous Velocity Gradients in Plane and Axisymmetric Wake Flows, *J. Fluids Eng., 124* (2002), pp.143-153

[5] Antonia, R. A., Kim, J., Browne, L. W., Some Characteristics of Small-Scale Turbulence in a Turbulent Duct Flow, *J. Fluid Mech., 233* (1991), pp. 369-388

[6] Chou, P. Y., On the Velocity Correlation and the Solution of the Equation of Turbulent Fluctuation, *Quart. Appl. Math., 3* (1945), pp. 38-54

[7] Kolovandin, B. A.,Vatutin, I. A., On Statistical Theory of Non-Uniform Turbulence, Presented at International Seminar on Heat and Mass Transfer in Flows with Separated Regions and Measurement Techniques, Herceg-Novi, Yugoslavia, September 1-13, 1969

[8] Jovanović, J., The Statistical Dynamics of Turbulence, Springer-Verlag, Berlin, 2004

[9] Jovanović, J., Ye, Q.-Y., Durst, F., Statistical Interpretation of the Turbulent Dissipation Rate in Wall-Bounded Flows, *J. Fluid Mech., 293* (1995), pp. 321-347

[10] Kline, S. J., Reynolds, W. C., Schraub, F. A., Runstadlet, W. P., The Structure of Turbulent Boundary Layres, *J. Fluid Mech., 30* (1967), pp. 741-773

[11] Kim, H. T., Kline, S .J., Reynolds, W. C., The Production of Turbulence Near Smooth Wall in a Turbulent Boundary Layer, *J. Fluid Mech., 50* (1971), pp. 133-160

[12] Falco, R. E., The Role of Outer Flow Coherent Motions in the Production of Turbulence Near a Wall, in: Coherent Structure of Turbulent Boundary Layer (Eds. C. R. Smith, D. E. Abbott), AFOSR/Lehigh University, 1978, pp. 448-461

[13] Laufer, J., New Trends in Experimental Turbulence Research, *Ann. Rev. Fluid Mech., 7* (1975), pp. 307-326

[14] Laufer, J., Flow Instability and Turbulence, *Proceedings*, Symposium on Heat and Mass Transfer and the Structure of Turbulence, Dubrovnik, Yugoslavia, October 6-10, 1980,

Structure of Turbulence in Heat and Mass Transfer (Ed. Z. Zarić), Hemisphere Publ. Comp., pp. 3-5

[15] Moser, R. D., Kim, J., Mansour, N. N., Direct Numerical Simulation of Turbulent Channel Flow up to $Re_\tau = 590$, *Phys. Fluids, 11* (1999), pp. 943-945

[16] Antonia, R. A., Teitel, M., Kim, J., Browne, L. W. B., Low-Reynolds-Number Effects in a Fully Developed Turbulent Channel Flow, *J. Fluid Mech., 236* (1992), pp. 579-605

[17] Kim, J., Moin, P., Moser, R., Turbulence Statistics in a Fully Developed Channel Flow at Low Reynolds Numbers, *J. Fluid Mech., 177* (1987), pp. 133-166

[18] Kuroda, A., Kasagi, N., Hirata, M., Direct Numerical Simulation of the Turbulent Plane Couette-Poiseulle Flows: Effect of Mean Shear on the Near Wall Turbulence Structures, *Proceedings,* Ninth Symposium on Turbulent Shear Flows, Kyoto, Japan, 1993, 8.4.1–8.4.6, http://www.thtlab.t.u-tokyo.ac.jp/

[19] Jovanović, J., Hillerbrand, R., Pashtrapanska, M., The Statistical Analysis of the Origin of Turbulence Using Numerical Databases, *KONWIHR Quartl, 31* (2001), pp. 6-8

[20] Monin, A. S., Yaglom, A. M., The Statistical Fluid Mechanics, Vol. 1, The MIT Press, Cambridge, MA, USA, 1987

[21] George, W. K., Hussein, H. J., Locally Axisymmetric Turbulence, *J. Fluid Mech., 233* (1991), pp. 1-23

[22] Spalart, P. R., Direct Simulation of a Turbulent Boundary Layer up to R = 1410, *J. Fluid Mech.,* 187 (1988), pp. 61-98

[23] Gilbert, N., Kleiser, L., Turbulence Model Testing with the Aid of Direct Numerical Simulation Results, *Proceedings,* Eighth Symposium on Turbulent Shear Flows, Munich, Germany, 1991, pp. 26.1.1-26.1.6

[24] Horiuti, K., Establishment of the Direct Numerical Simulation Data Base of Turbulent Transport Phenomena, Ministry of Education, Science and Culture Japan, Co-Operative Research No. 012302043, 1992, http://www.thtlab.t.u-tokyo.ac.jp/

[25] Eggels, J. G. M., Unger, F., Weiss, M. H., Westerweel, J., Adrian, R. J., Friedrich, R., Nieuwstadt, F. T. M., Fully Developed Turbulent Pipe Flow: A Comparison between Direct Numerical Simulation and Experiment, *J. Fluid Mech., 268* (1994), pp. 175-209

[26] Spalart, P. R., Numerical Study of Sink-Flow Boundary Layers, *J. Fluid Mech., 172* (1986), pp. 307-328

[27] Dimitropulos, C. R., Sureshkumar, R., Beris, A. N., Direct Numerical Simulation of Viscoelastic Turbulent Channel Exhibiting Drag Reduction: Effect of the Variation of Rheological Parameters, *J. Non-Newtonian Fluid Mech., 79* (1998), pp. 443-468

[28] Warholic, M. D., Massah, H., Hanratty, T. J., Influence of Drag-Reducing Polymers on Turbulence: Effects of Reynolds Number, Concentration and Mixing, *Exp. Fluids, 27* (1999), pp. 461-472

[29] Wang, Z.Y., Jovanović, J., Drag Characteristics of Extra-Thin-Fin Riblets in an Air Conduit, *J. Fluids Eng., 155* (1993), pp. 222-226

[30] Jovanović, J., Pashtrapanska, M., On the Criterion for the Determination Transition Onset and Breakdown to Turbulence in Wall-Bounded Flows, *J. Fluids Eng., 128* (2004), pp. 1-8

[31] Lammers, P., Jovanović, J., Durst, F., Numerical Experiment on Wall Turbulence at Very Low Reynolds Number, *Phys. Fluids*, 2004, submitted

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