THERMAL INSTABILITY OF WALTERS B' VISCOELASTIC FLUID PERMEATED WITH SUSPENDED PARTICLES IN HYDROMAGNETICS IN POROUS MEDIUM

by

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The effect of suspended particles on the thermal instability of Walters B' viscoelastic fluid in hydromagnetics in porous medium is considered. For stationary convection, Walters B' viscoelastic fluid behaves like a Newtonian fluid. The medium permeability and suspended particles hasten the onset of convection whereas the magnetic field postpones the onset of convection, for the case of stationary convection. The magnetic field and viscoelasticity introduce oscillatory modes in the system which was non-existent in their absence.

Key words: *Walters B' viscoelastic fluid, thermal instability, suspended particles, magnetic field, porous medium*

Introduction

The theoretical and experimental results of the onset of thermal instability (Be'nard convection) in a fluid layer under varying assumptions of hydrodynamics has been treated in detail by Chandrasekhar [1]. Chandra [2] observed that in an air layer, convection occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion "Columnar instability". However, a Be'nard-type cellular convection was observed for layers deeper than 10 mm. Chandra [2] added an aerosol to mark the flow pattern. Thus there is a decades-old contradiction between the theory and the experiment. Scanlon and Segel [3] have considered the effect of suspended particles on the onset of Be'nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles. The effect of suspended particles was thus found to destabilize the layer. Palaniswamy and Purushotham [4] have considered the stability of shear flow of stratified fluids with fine dust and have found the effect of fine dust to increase the region of instability.

In all the above studies, the medium has been considered to be non-porous and the fluid to be Newtonian. Lapwood [5] has studied the stability of convective flow in a porous medium using Rayleigh's procedure. Wooding [6] has considered the Rayleigh instability of a thermal boundary layer in flow through porous medium. The gross effect when the fluid slowly percolates through the pores of the rock is represented by the well known Darcy's law. The problem of thermal instability in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The development of geothermal power resources has increased general interest, in the properties of convection in porous media. The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region.

The importance of non-Newtonian fluids in modern technology and industries is ever increasing and the investigations on such fluids are desirable. One such class of non-Newtonian fluids is Walters B' fluid. Chakraborty and Sengupta [7] have studied the flow of unsteady viscoelastic (Walters B' liquid) conducting fluid through two porous concentric non-conducting infinite circular cylinders rotating with different angular velocities in the presence of uniform axial magnetic field. Sharma and Kumar [8] have studied the stability of two superposed Walters B' viscoelastic liquids. In another study, Sharma and Kumar [9] have studied the Rayleigh-Taylor instability of two superposed conducting Walters B' elastico-viscous fluids in hydromagnetics. Kumar [10] has studied the stability of two superposed viscoelastic (Walters B') fluid-particle mixtures in porous medium. It is this class of elastico-viscous fluids we are interested in, particularly to study the effect of suspended (or dust) particles on the Walters B' viscoelastic fluid heated from below in porous medium in the presence of a uniform horizontal magnetic field.

Formulation of the problem and perturbation equations

Here we consider an infinite horizontal layer of on electrically conducting Walters B' elastico-viscous fluid permeated with suspended particles and bounded by the planes $z = 0$ and $z = d$ in a porous medium. This layer is heated from below so that, the temperatures and densities at the bottom surface $z = 0$ are T_0 and ρ_0 and at the upper surface $z = d$ are T_d and ρ_d respectively and that a uniform temperature gradient $\beta(\ |dT/dz|)$ is maintained. A uniform horizontal magnetic field $\vec{H}(H,0,0)$ and gravity field $\vec{g}(0,0,\ g)$ pervades the system.

The equations of motion and continuity for Walters B' viscoelastic fluid in the presence of suspended particles and magnetic field in porous medium are:

$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}\ \ \frac{1}{\varepsilon}(\vec{q}\ \ \ )\vec{q}\ \ \ \ \frac{1}{\rho_0}\ \ p\ \ \mathrm{g}\ 1\ \ \frac{\delta\rho}{\rho_0}\ \vec{\lambda}\ \ \frac{1}{k_1}\ \ \upsilon\ \ \upsilon'\frac{\partial}{\partial t}\ \ \vec{q}$$
$$\frac{KN}{\rho_0\varepsilon}(\vec{q}_d\ \ \ \vec{q})\ \ \frac{\mu_e}{4\pi\rho_0}(\ \ \ \vec{H})\ \ \vec{H} \tag{1}$$

$$\vec{q} \quad 0 \tag{2}$$

where $p, \rho, T, \vec{q}(u, \upsilon, w), \vec{q}_d(\bar{x}, t), N(\bar{x}, t), \upsilon$, and υ' denote fluid pressure, density, temperature, fluid velocity, suspended particles velocity, suspended particles number density, kinematic viscosity, and kinematic viscoelasticity, respectively. Symbol ε is the medium porosity, k_1 is the medium permeability, g is the acceleration due to gravity, $\bar{x} \quad (x, y, z)$, $\vec{\lambda} \quad (0, 0, 1)$, and $K \quad 6\pi\mu\eta'$, η' being the particle radius, is the Stokes' drag coefficient. Assuming a uniform particle size, a spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term in the equation of motion (1), proportional to the velocity difference between the particles and the fluid.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Interparticle reactions are ignored because the distances between the particles are assumed to be quite large compared with their diameter. The effects due to pressure, gravity, Darcy's force and magnetic field on the particles are small and so are ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are:

$$mN \quad \frac{\partial \vec{q}_d}{\partial t} \quad \frac{1}{\varepsilon}(\vec{q}_d \quad)\vec{q}_d \quad KN(\vec{q} \quad \vec{q}_d) \tag{3}$$

$$\varepsilon \frac{\partial N}{\partial t} \quad (N\vec{q}_d) \quad 0 \tag{4}$$

If C_v, C_{pt}, T, and q' denote the heat capacity of fluid at constant volume, heat capacity of the particles, temperature and *effective thermal conductivity* of the pure fluid, respectively. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives:

$$[\rho_0 C_v \varepsilon \quad \rho_s C_s (1 \quad \varepsilon)] \frac{\partial T}{\partial t} \quad \rho_0 C_v (\vec{q} \quad)T \quad mNC_{pt} \quad \varepsilon \frac{\partial}{\partial t} \quad \vec{q}_d \quad T \quad q' \quad {}^2 T \tag{5}$$

where ρ_s and C_s are the density and the heat capacity of the solid (porous matrix) material, respectively.

The Maxwell's equations yield:

$$\varepsilon \frac{\partial \vec{H}}{\partial t} \quad (\vec{H} \quad)\vec{q} \quad \varepsilon\eta \quad {}^2 \vec{H} \tag{6}$$

$$\vec{H} \quad 0 \tag{7}$$

where η stands for the electrical resistivity.

The equation of state for the fluid is:

$$\rho = \rho_0[1-\alpha(T-T_0)] \tag{8}$$

where α is the coefficient of thermal expansion and the subscript zero refers to values at the reference level $z = 0$. The kinematic viscosity υ, kinematic viscoelasticity υ', magnetic permeability μ_e, electrical resistivity η, and coefficient of thermal expansion α are all assumed to be constants.

The basic motionless solution is:

$$\vec{q} \quad (0,0,0), \quad \vec{q}_d \quad (0,0,0), \quad T \quad T_0 \quad \beta z, \quad \rho \quad \rho_0(1 \quad \alpha\beta z), \quad N \quad N_0 \quad \text{const.} \tag{9}$$

Assume small perturbations around the basic solution and let δp, $\delta\rho, \theta,$ and $\vec{h}(h_x, h_y, h_z)$ denote respectively the perturbations in fluid pressure p, density ρ, temperature T, and magnetic field $\vec{H}$. In basic (motion less) solution holds: fluid velosity $\vec{q}$ (0, 0, 0), suspended particle velocity $\vec{q}_d$ (0, 0, 0), suspended particle number N_0, and magnetic field $\vec{H}(H,0,0)$ as it is shown in conditions (9). The change in density δp caused mainly by the perturbation θ in temperature, is given by:

$$\delta\rho = -\alpha\rho_0\theta \tag{10}$$

Then the linearized perturbed equations of Walters B viscoelastic fluid become:

$$\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} \quad \frac{1}{\rho_0} \quad \delta\rho \quad \mathrm{g}\alpha\theta\vec{\lambda} \quad \frac{1}{k_1} \quad \upsilon \quad \upsilon'\frac{\partial}{\partial t} \quad \vec{q} \quad \frac{KN_0}{\rho_0\varepsilon}(\vec{q}_d \quad \vec{q}) \quad \frac{\mu_e}{4\pi\rho_0}(\quad \vec{h}) \quad \vec{H} \tag{11}$$

$$\vec{q} \quad 0 \tag{12}$$

$$mN_0\frac{\partial \vec{q}_d}{\partial t} \quad KN_0(\vec{q} \quad \vec{q}_d) \tag{13}$$

$$(E \quad h\varepsilon)\frac{\partial \theta}{\partial t} \quad \beta(w \quad hs) \quad \kappa \quad {}^2\theta \tag{14}$$

$$\varepsilon\frac{\partial \vec{h}}{\partial t} \quad (\vec{H} \quad)\vec{q} \quad \varepsilon\eta \quad {}^2\vec{h} \tag{15}$$

$$\vec{h} \quad 0 \tag{16}$$

where $E \quad \varepsilon \quad (1 \quad \varepsilon)\rho_s C_s/\rho_0 C_v, \quad h \quad mN_0 C_{pt}/\rho_0 C_v,$ and $\kappa \quad q'/\rho_0 C_v$.

Eliminating $\vec{q}_d$ in eq. (11) with the help of eq. (13), writing the scalar components of resulting equation and eliminating u, v, h_x, h_y, and δp between them, by using eq. (12) and eq. (16), we obtain:

$$n'(\ ^2 w)\ \frac{\varepsilon}{k_1}\ \upsilon\ \ \upsilon'\frac{\partial}{\partial t}\ \ ^2 w\ \ \varepsilon g\alpha\ \frac{\partial^2\theta}{\partial x^2}\ \ \frac{\partial^2\theta}{\partial y^2}$$
$$\frac{\mu_e \varepsilon H}{4\pi\rho_0}\frac{\partial}{\partial x}(\ ^2 h_z)\ \ 0 \tag{17}$$

$$\frac{m}{K}\frac{\partial}{\partial t}\ \ 1\ \ \overline{E\ \ h\varepsilon}\frac{\partial}{\partial t}\ \ \kappa\ \ ^2\ \ \theta\ \ \beta\ \frac{m}{K}\frac{\partial}{\partial t}\ \ 1\ \ h\ \ w \tag{18}$$

$$\varepsilon\ \frac{\partial}{\partial t}\ \ \eta\ \ ^2\ \ h_z\ \ \ H\frac{\partial w}{\partial x} \tag{19}$$

where

$$n'\ \ \frac{\partial}{\partial t}\ \ 1\ \ \frac{mN_0 K|\rho_0}{m\frac{\partial}{\partial t}\ \ K}$$

Dispersion relation

Here we analyze the disturbances into normal modes and assume that the perturbation quantities are of the form:

$$[w,\theta,h_z]\ \ [W(z),\Theta(z),X(z)]\exp(ik_x x\ \ \ ik_y y\ \ \ nt) \tag{20}$$

where k_x, and k_y are wave numbers along the x-and y-directions respectively. $k\ \ \sqrt{k_x^2\ \ k_y^2}$ is the resultant wave number and n is, in general, a complex constant. Using expression (20), eqs. (17-19) in non-dimensional form become:

$$\frac{\sigma'}{\varepsilon}\ \ \frac{1}{P_l}(1\ \ F\sigma)\ (D^2\ \ \ a^2)W\ \ \ \frac{g\alpha d^2 a^2\Theta}{\upsilon}\ \ \ \frac{ik_x\mu_e H d^2}{4\pi\rho_0\upsilon}(D^2\ \ \ a^2)X\ \ \ 0 \tag{21}$$

$$\frac{\tau\upsilon\sigma}{d^2}\ \ 1\,[D^2\ \ \ a^2\ \ \ \overline{E\ \ h\varepsilon}\mathrm{Pr}_1\sigma]\Theta\ \ \ \frac{\beta d^2}{\kappa}\ \ H'\ \ \frac{\tau\upsilon\sigma}{d^2}\ \ W \tag{22}$$

$$[D^2\ \ \ a^2\ \ \ \mathrm{Pr}_2\,\sigma]X\ \ \ \ \frac{ik_x H d^2}{\varepsilon\eta}W \tag{23}$$

where we have expressed the coordinates x, y, and z in the new unit of length d, time t in the new unit of length d^2/κ and put $a = kd$, $\sigma = nd^2/\upsilon$, $\Pr_1 = \upsilon/\kappa$ is the Prandtl number, $\Pr_2 = \upsilon/\eta$ is the magnetic Prandtl number, $P_l = k_1/d_2$ is the dimensionless medium permeability, $F = \upsilon/d^2$ is the dimensionless kinematic viscoelasticity, $\sigma' = n'd^2/\upsilon$, $H' = h + 1$, $\tau = m\kappa/Kd^2$, and $D = d/dz$.

Eliminating T and X between eqs. (21-23), we obtain

$$1 \quad \frac{\upsilon\tau\sigma}{d^2}[D^2 \quad a^2 \quad \overline{E \quad h\varepsilon}\Pr_1 \sigma] \quad \frac{\sigma'}{\varepsilon} \quad \frac{1}{P_l}(1 \quad F\sigma)\,(D^2 \quad a^2 \quad \Pr_2 \sigma)$$
$$\frac{k_x^2 Q}{\varepsilon} \quad (D^2 \quad a^2)W \quad \mathrm{R}a^2 \quad H' \quad \frac{\upsilon\tau\sigma}{d^2}[D^2 \quad a^2 \quad \Pr_2 \sigma]W \tag{24}$$

where $\mathrm{R} = g\alpha\beta d^4/\upsilon\kappa$ is the Rayleigh number and $\mathrm{Q} = \mu_e H^2 d^2/4\pi\rho_0\upsilon\eta$ is the Chandrasekhar number.

Here we consider the case in which both the boundaries are free, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The case of two free boundaries is little artificial but allows us to have analytical solution. The boundary conditions, appropriate to the problem, are [1]:

$$W = 0, \quad D^2W = 0, \quad \Theta = 0, \quad X = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1 \tag{25}$$

Using the above boundary conditions (25), it can be shown with the help of eqs. (21-23) that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of W characterizing the lowest mode is:

$$W = W_0 \sin \pi z, \tag{26}$$

where W_0 is a constant.

Substituting the proper solution (26) in eq. (24), we obtain the dispersion relation

$$\mathrm{R}_1 \quad \frac{(1 \quad x)(1 \quad x \quad \overline{E \quad h\varepsilon} i\sigma_1 \Pr_1) \; 1 \quad \frac{i\upsilon\tau\pi^2\sigma_1}{d^2} \quad \frac{i\sigma'_1}{\varepsilon} \quad \frac{1}{P}(1 \quad i\pi^2 F\sigma_1)(1 \quad x \quad i\sigma_1 \Pr_2) \quad \frac{\mathrm{Q}_1 x \cos^2\theta}{\varepsilon}}{x \;\; H' \quad \frac{i\upsilon\tau\pi^2\sigma_1}{d^2} \quad (1 \quad x \quad i\sigma_1 \Pr_2)} \tag{27}$$

where $x = a^2/\pi^2$, $i\sigma_1 = \sigma/\pi^2$, $P = \pi^2 P_l$, $\mathrm{R}_1 = \mathrm{R}/\pi^4$, $i\sigma'_1 = \sigma'/\pi^2$, $\mathrm{Q}_1 = \mathrm{Q}/\pi^2$, and $k_x = k\cos\theta$.

The stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (27) reduces to:

$$\mathrm{R}_1 \qquad \frac{(1\quad x)\ \dfrac{1\quad x}{P}\quad \dfrac{\mathrm{Q}_1 x\cos^2\theta}{\varepsilon}}{xH} \tag{28}$$

We thus find that for stationary convection the viscoelastic parameter F vanishes with σ and Walters B' viscoelastic fluid behaves like an ordinary Newtonian fluid.

To study the effects of magnetic field, suspended particles and medium permeability, we examine the natures of $d\mathrm{R}_1/d\mathrm{Q}_1$, $d\mathrm{R}_1/dH'$, and $d\mathrm{R}_1/dP$. Eq. (28) yields:

$$\frac{d\mathrm{R}_1}{d\mathrm{Q}_1} \qquad \frac{(1\quad x)\cos^2\theta}{H'\varepsilon} \tag{29}$$

$$\frac{d\mathrm{R}_1}{dH} \qquad \frac{(1\quad x)\ \dfrac{1\quad x}{P}\quad \dfrac{\mathrm{Q}_1 x\cos^2\theta}{\varepsilon}}{xH'^2} \tag{30}$$

$$\frac{d\mathrm{R}_1}{dP} \qquad \frac{(1\quad x)^2}{xH' P^2} \tag{31}$$

It is clear from eqs. (29-31) that for stationary convection the magnetic field postpone the onset of convection whereas the suspended particles and medium permeability hasten the onset of convection in Walters B' viscoelastic fluid permeated with suspended particles, heated from below in porous medium in presence of a uniform horizontal magnetic field.

Graphs have been plotted between R_1 and x for various values of Q_1, P, and H'. It is evident from figs. (1-3) that the magnetic field postpones the onset of convection while medium permeability and suspended particles hasten the onset of convection.

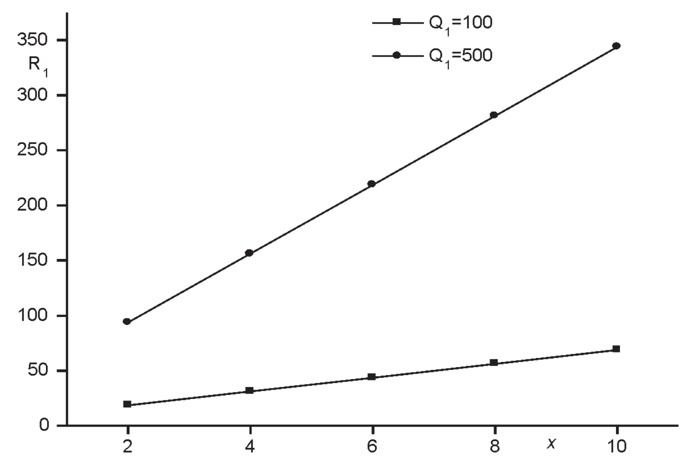

Figure 1. Variation of R_1 with x for a fixed $\varepsilon = 0.4$, $\theta = 45°$, $P = 10$, $H' = 20$ for different values of $\mathrm{Q}_1(=100, 500)$

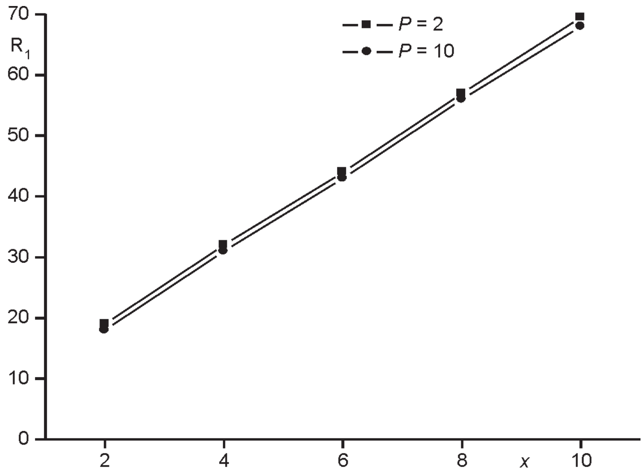

Figure 2. Variation of R_1 with x for a fixed ε = 0.4, θ = 45°, Q_1 = 100, H' = 20 for different values of P (=2, 10)

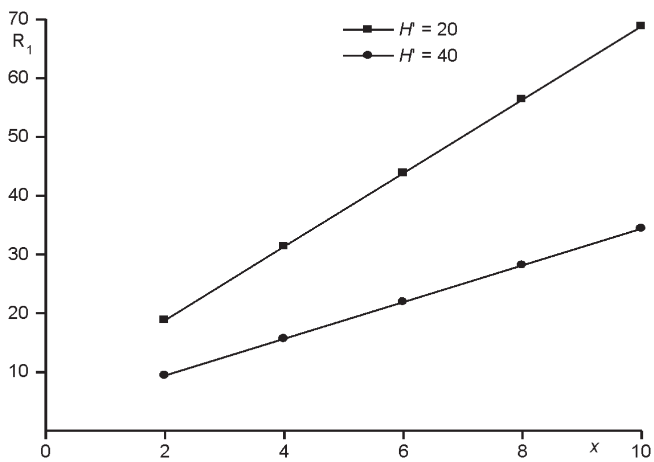

Figure 3. Variation of R_1 with x for a fixed ε = 0.4, θ = 45°, Q_1 = 100, P = 5 for different values of H' (= 20, 40)

Stability of the system and oscillatory modes

Multiplying eq. (21) by W^*, the complex conjugate of W, integrating over the range of z and using eqs. (22) and (23) together with the boundary conditions (25), we obtain:

$$\frac{\sigma'}{\varepsilon} \quad \frac{1}{P_l}(1 \quad F\sigma) \; I_1 \quad \frac{g\alpha\kappa a^2}{\upsilon\beta} \quad \frac{d^2 \quad \upsilon\tau\sigma^*}{H'd^2 \quad \upsilon\tau\sigma^*} \; [I_2 \quad \overline{E \quad h\varepsilon}\, \mathrm{Pr}_1 \, \sigma^* I_3]$$

$$\frac{\mu_e \eta\varepsilon}{4\pi\rho_0\upsilon}[I_4 \quad \mathrm{Pr}_2 \, \sigma^* I_5] = 0 \tag{32}$$

where

$$I_1 \quad \int_0^1 (|DW|^2 \quad a^2|W|^2)dz, \qquad I_2 \quad \int_0^1 (|D\Theta|^2 \quad a^2|\Theta|^2)dz$$
$$I_3 \quad \int_0^1 |\Theta|^2 dz \qquad\qquad I_4 \quad \int_0^1 (|D^2X|^2 \quad 2a^2|DX|^2 \quad a^4|X|^2)dz \tag{33}$$
$$I_5 \quad \int_0^1 (|DX|^2 \quad a^2|X|^2)dz$$

and σ^* is the complex conjugate of σ. The integrals $I_1, I_2, \ldots, I_5$ are all positive definite.

Putting $\sigma = i\sigma_i$, $f = mN_0/\rho_0$ and equating the imaginary parts of eq. (32), we obtain:

$$i\sigma_i \quad \frac{1}{\varepsilon} \quad 1 \quad \frac{f}{1 \quad \mathrm{Pr}_1^2 \tau^2\sigma_i^2} \quad \frac{F}{P_l} \quad I_1 \quad \frac{g\alpha\kappa a^2}{\upsilon\nu\beta(H'^2 d^4 \quad \upsilon^2\tau^2\sigma_i^2)}[d^2\upsilon\tau h I_2$$
$$\mathrm{Pr}_1 \overline{E \quad h\varepsilon}(H'd^4 \quad \upsilon^2\tau^2\sigma_i^2)I_3] \quad \frac{\mu_e\eta\varepsilon\,\mathrm{Pr}_2}{4\pi\rho_0\upsilon} I_5 \quad 0 \tag{34}$$

Equation (34) yields that $\sigma_i = 0$ or $\sigma_i \quad 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of magnetic field and viscoelasticity, eq. (34) reduces to:

$$i\sigma_i \quad \frac{1}{\varepsilon} \quad 1 \quad \frac{f}{1 \quad \mathrm{Pr}_1^2 \tau^2\sigma_i^2} \quad I_1 \quad \frac{g\alpha\kappa a^2}{\upsilon\beta(H'^2 d^4 \quad \upsilon^2\tau^2\sigma_i^2)}[d^2\upsilon\tau h I_2$$
$$\mathrm{Pr}_1 \overline{E \quad h\varepsilon}(H'd^4 \quad \upsilon^2\tau^2\sigma_i^2)I_3] \quad 0 \tag{35}$$

and the quantity inside the brackets is positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid. The magnetic field and viscoelasticity introduce oscillatory modes (as σ_i may not be zero) in the system which was non-existent in their absence.

Conclusions

A layer of Newtonian fluid heated from below, under varying assumptions of hydrodynamics and hydromagnetics, has been studied by Chandrasekhar [1]. With the growing importance of non-Newtonian fluids in chemical engineering, modern technology and industry, the investigations on such fluids are desirable. The Walters B' fluid is one such important non-Newtonian (viscoelastic) fluid. Keeping in mind the importance of non-Newtonian fluids, the present paper considered the effect of suspended particles on the Walters B' viscoelastic fluid heated from below in porous medium in the presence of a uniform horizontal magnetic field.

For stationary convection, Walters B' viscoelastic fluid behaves likes a Newtonian fluid. Here, the magnetic field is found to postpone the onset of convection whereas the medium permeability and suspended particles hasten the onset of convection. The magnetic field and viscoelasticity introduce oscillatory modes in the system which was non-existent in their absence.

Nomenclature

a – dimensionless wave number, [–]
C_{pt} – heat capacity of particles, [J/kgK]
C_s – heat capacity of the solid (porous matrix) material, [J/kgK]
C_v – heat capacity of fluid at constant volume, [J/kgK]
d – depth of layer, [m]
F – dimensionless kinematic viscoelasticity, [–]
g – acceleration due to gravity [m/s₂]
$\vec{g}$ – gravity field, [m/s^2]
$\vec{H}$ – magnetic field, [G]
K – Stokes' drag coefficient, [kg/s]
k – wave-number, [1/m]
k_x, k_y – horizontal wave-numbers, [1/m]
k_1 – medium permeability, [m^2]
m – mass of single particle, [g]
N – suspended particle number density, [1/m^3]
n – growth rate, [1/s]
P_l – dimensionless medium permeability, [–]
Pr_1 – Prandtl number (= ν/κ), [–]
Pr_2 – magnetic Prandtl number (= ν/η), [–]
p – fluid pressure, [Pa]
Q – Chandrasekhar number, [–]
q' – efective thermal conductivity, [W/m·K]
$\vec{q}$ – filter velocity, [m/s]
$\vec{q}_d$ – suspended particle velocity, [m/s]
R – Rayleigh number, [–]
T – temperature, [K]
t – time, [s]

Greek letters

α – coefficient of thermal expansion, [1/K]
β – uniform temperature gradient, [K/m]
ε – medium porosity, [–]
η – electrical resistivity, [m^2/s]
η' – particle radius, [m]
θ – perturbation in temperature, [K]
κ – thermal diffusivity, [m^2/s]
μ – dynamic viscosity, [kg/m·s]
μ_e – magnetic permeability, [-]
υ – kinematic viscosity, [m^2/s]
υ' – kinematic viscoelasticity, [m^2/s]
ρ – density, [kg/m^3]
ρ_s – density of the solid (porous matrix) material, [kg/m^3]

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