UNSTEADY PLANE MHD BOUNDARY LAYER FLOW OF A FLUID OF VARIABLE ELECTRICAL CONDUCTIVITY

by

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This paper is devoted to the analysis of unsteady plane laminar magneto-hydrodynamic boundary layer flow of incompressible and variable electrical conductivity fluid. The present magnetic field is homogeneous and perpendicular to the body surface. Outer electric filed is neglected and magnetic Reynolds number is significantly lower then one i. e. considered problem is in inductionless approximation. Free stream velocity is an arbitrary differentiable function. Fluid electrical conductivity is decreasing function of velocity ratio. In order to solve the described problem multi-parametric (generalized similarity) method is used and so-called universal equations are obtained. Obtained universal equations are solved numerically in appropriate approximation and a part of obtained results is given in the form of figures and corresponding conclusions.

Key words: *magneto-hydrodynamic, laminar boundary layer, variable electrical conductivity, generalized similarity method, universal equation*

Introduction

The problem of boundary-layer separation and control has attracted considerable attention over several decades because of the fundamental flow physics and technological applications. Some of the essential ideas related to boundary-layer separation and the need to prevent the same from occurring have been addressed by Prandtl [1]. A number of methods may be employed to control the boundary layer separation that occurs due to the adverse pressure gradient: admit the body motion in stream-wise direction, increasing the boundary layer velocity, boundary layer suction, second gas injection, body cooling, introducing a transverse magnetic field, *etc.*

Interest in effect of outer magnetic field on heat-physical processes appears sixty years ago [2]. The study of magneto-hydrodynamic flow of an electrically conducting fluid

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past an arbitrary shape surface has attracted the interest of many researchers in view of its important applications in many engineering problems.

Recently the problem of magneto-hydrodynamic (MHD) flow over surfaces has become more important due to the possibility of applications in areas like nuclear fusion, chemical engineering, medicine, and high-speed, noiseless printing. Problem of MHD flow in the vicinity of plate has been studied intensively by a number of investigators [3-7]. Most of previous investigations were concerned with studies of the steady flow of fluid whose electrical conductivity is constant.

The subject of the present research is to give an analytic investigation to the problem of unsteady laminar MHD boundary layer flow of a viscous incompressible fluid. The external magnetic field is homogeneous and perpendicular to the body. The fluid which forms the boundary layer is incompressible and its electrical conductivity is variable and can be assumed in the following form:

$$\sigma = \sigma_0 \left(1 - \frac{u}{U}\right)^n, \quad n \in \mathbb{N} \cup \{0\} \tag{1}$$

where: u – stream-wise velocity in the boundary layer, U – free stream velocity, $\mathbb{N}$ – the set of natural numbers. It should be noted that the free stream velocity is an arbitrary differentiable function of the coordinate x and time t.

Mathematical model

The problem in question is considered in an induction less approximation, and its mathematical model is expressed by the following equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \begin{Bmatrix} NU\left(1 - \frac{u}{U}\right) & \text{for} \quad n = 0 \\ -NU\left(1 - \frac{u}{U}\right)^n & \text{for} \quad n \in \mathbb{N} \end{Bmatrix} \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

and the boundary and initial conditions:

$$u = 0, v = 0 \quad \text{for} \quad y = 0; \;\; u \to U(x,t) \quad \text{for} \quad y \to \infty \tag{4}$$

$$u = u_1(x,y) \quad \text{for} \quad t = t_0 \quad u = u_0(t,y) \quad \text{for} \quad x = x_0 \tag{5}$$

In the equations (2) and (3) and in the boundary and initial conditions (4) and (5) the parameter labeling used is common for the theory of MHD boundary layer: y – transversal coordinate, v – transversal velocity in the boundary layer, ν – kinematic viscosity of fluid, $N = \sigma B^2/\rho$, B – magnetic field induction, ρ – density of fluid, $u_1(x,y)$ – velocity distribution in the boundary layer at the moment $t = t_0$, $u_0(t,y)$ – velocity distribution in the cross section $x = x_0$ of the boundary layer.

Generalized similarity method

For the analysis of the described problem it is necessary to solve the system of equations (2)-(3) with the corresponding boundary and initial conditions (4)-(5), which can be done by using different numerical methods. The system of equations can be solved for specific values of the parameter n, magnetic induction, and for given function of the free stream velocity. The results thus obtained, and on the basis of which conclusions can be drawn, pertain only to that particular case of the boundary layer. By the same token, for every other particular case of the boundary layer, a complete calculation would have to be made anew.

For further studying of the problem in this paper, the ideas of generalized similarity method have been used [8-10] which is extended in papers [11, 12]. This method leads to the so-called universal equation of the described problem, and its universal solution enable drawing general conclusions on the development of the boundary layer, and can be also used for calculations concerning particular cases of the boundary layer. It should be noted that the obtained universal equation has to be solved only once, and that the obtained universal results can be conveniently stored and reused. This method proved to be valid for different problems of the boundary layer [13-17], which recommends it for further employment.

Following the ideas expressed in [8], we take into consideration the stream function $\Psi = \Psi(x,y,t)$ by the relations:

$$\frac{\partial \Psi}{\partial x} = -v \, , \quad \frac{\partial \Psi}{\partial y} = u \tag{6}$$

which transform the system of equations (2)-(3) into the equation:

$$\frac{\partial^2 \Psi}{\partial t \partial y} + \frac{\partial \Psi}{\partial y}\frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial \Psi}{\partial x}\frac{\partial^2 \Psi}{\partial y^2} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \nu \frac{\partial^3 \Psi}{\partial y^3} + \left\{ \begin{array}{ll} NU\left(1 - \dfrac{1}{U}\dfrac{\partial \Psi}{\partial y}\right) & \text{for } n = 0 \\ -N\dfrac{\partial \Psi}{\partial y}\left(1 - \dfrac{1}{U}\dfrac{\partial \Psi}{\partial y}\right)^n & \text{for } n \in \mathbb{N} \end{array} \right\} \tag{7}$$

and the boundary (4) and initial conditions (5) into conditions:

$$\Psi = 0, \ \frac{\partial \Psi}{\partial y} = 0 \ \text{ for } \ y = 0; \ \ \frac{\partial \Psi}{\partial y} \to U(x,t) \ \text{ for } \ y \to \infty \tag{8}$$

$$\frac{\partial \Psi}{\partial y} = u_1(x,y) \ \text{ for } \ t = t_0; \ \ \frac{\partial \Psi}{\partial y} = u_0(t,y) \ \text{ for } \ x = x_0 \tag{9}$$

By introducing new variables:

$$t=t,\ x=x,\ \eta=\frac{Dy}{h(x,t)},\ \varphi(x,\eta,t)=\frac{D\Psi(x,y,t)}{U(x,t)h(x,t)} \tag{10}$$

where: $h(x,t)$ – a certain characteristic linear size of the transversal coordinate in the boundary layer, D – standardization constant, the equation (7) is transformed into the equation:

$$D^2\frac{\partial^3\varphi}{\partial\eta^3}+z\frac{\partial U}{\partial x}\left[1-\left(\frac{\partial\varphi}{\partial\eta}\right)^2+\varphi\frac{\partial^2\varphi}{\partial\eta^2}\right]+\frac{z}{U}\frac{\partial U}{\partial t}\left(1-\frac{\partial\varphi}{\partial\eta}\right)+\frac{1}{2}U\frac{\partial z}{\partial x}\varphi\frac{\partial^2\varphi}{\partial\eta^2}+\frac{1}{2}\eta\frac{\partial z}{\partial t}\frac{\partial^2\varphi}{\partial\eta^2}-$$

$$-z\frac{\partial^2\varphi}{\partial\eta\partial t}+UzX(x;\eta)+\left\{\begin{array}{ll} Nz\left(1-\dfrac{\partial\varphi}{\partial\eta}\right) & \text{for } n=0 \\ -Nz\dfrac{\partial\varphi}{\partial\eta}\left(1-\dfrac{\partial\varphi}{\partial\eta}\right)^n & \text{for } n\in¥ \end{array}\right\}=0 \tag{11}$$

where the following marks have been used:

$$z=\frac{h^2}{\nu}\ ;\ \ X(x_1;x_2)=\frac{\partial\varphi}{\partial x_1}\frac{\partial^2\varphi}{\partial\eta\partial x_2}-\frac{\partial\varphi}{\partial x_2}\frac{\partial^2\varphi}{\partial\eta\partial x_1} \tag{12}$$

The corresponding boundary conditions are:

$$\varphi=0\ ,\ \frac{\partial\varphi}{\partial\eta}=0\ \text{ for }\ \eta=0;\ \ \frac{\partial\varphi}{\partial\eta}\to 1\ \text{ for }\ \eta\to\infty \tag{13}$$

Now we introduce sets of parameters;

– dynamical:

$$f_{k,\,n}=U^{k-1}\frac{\partial^{k+n}U}{\partial x^k\partial t^n}z^{k+n}\quad\left(k,\ n=0,\ 1,\ 2,...;\ k\vee n\neq 0\right) \tag{14}$$

– magnetic:

$$g_{k,\,n}=U^{k-1}\frac{\partial^{k-1+n}N}{\partial x^{k-1}\partial t^n}z^{k+n}\quad\left(k,\ n=0,\ 1,\ 2,...;\ k\neq 0\right) \tag{15}$$

– and the constant parameter:

$$g=\frac{\partial z}{\partial t}=const. \tag{16}$$

where the constant in the equation (16) may take different values. Sets of these independent parameters reflect the nature of free stream velocity change, alteration characteristic of variable N, and a part from that, in the integral form (by means of z and $\partial z/\partial t$) pre-history of flow in boundary layer.

The first parameters of the sets are:

$$f_{1,0} = z\frac{\partial U}{\partial x}\ ;\quad f_{0,1} = \frac{z}{U}\frac{\partial U}{\partial t}\ ;\quad g_{1,0} = Nz \tag{17}$$

The introduced parameters allow further transformation of the differential equation (11) into a universal form. It is a universal form in the sense that neither the equation, nor the corresponding boundary conditions will not explicitly depend on the free stream velocity and outer magnetic field.

In order to obtain universal equation the following differential operator is used:

$$\frac{\partial}{\partial s} = \sum_{\substack{k,\,n=0\\ k\vee n\neq 0}}^{\infty} \frac{\partial f_{k,n}}{\partial s}\frac{\partial}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty} \frac{\partial g_{k,n}}{\partial s}\frac{\partial}{\partial g_{k,n}}\ ;\quad s = x,t \tag{18}$$

where the derivations of parameters with respect to x and t are determined by immediate differentiation of the expressions (14) and (15) and they have the forms:

$$\frac{\partial f_{k,n}}{\partial x} = \frac{D_{k,n}}{Uz}\ ;\quad \frac{\partial f_{k,n}}{\partial t} = \frac{\partial E_{k,n}}{z}\ ;\quad \frac{\partial g_{k,n}}{\partial x} = \frac{K_{k,n}}{Uz}\ :\quad \frac{\partial g_{k,n}}{\partial t} = \frac{L_{k,n}}{z} \tag{19}$$

where:

$$D_{k,n} = (k+n) f_{k,n}F + A_{k,n};\quad A_{k,n} = (k-1) f_{1,0} f_{k,n} + f_{k+1,n};\quad F = U\frac{\partial z}{\partial x} \tag{20}$$

$$K_{k,n} = (k+n) g_{k,n}F + B_{k,n};\quad B_{k,n} = (k-1) f_{1,0} g_{k,n} + g_{k+1,n} \tag{21}$$

$$E_{k,n} = \left[(k-1) f_{0,1} + (k+n) g\right] f_{k,n} + f_{k,n+1} \tag{22}$$

$$L_{k,n} = \left[(k-1) f_{0,1} + (k+n) g\right] g_{k,n} + g_{k,n+1} \tag{23}$$

Using the parameters (14)-(16), differential operator (18) and expressions (19)-(23), the equation (11) is transformed to the form:

$$\begin{aligned} &D^2\frac{\partial^3\varphi}{\partial\eta^3} + \left(f_{1,0} + \frac{1}{2}F\right)\ \varphi\frac{\partial^2\varphi}{\partial\eta^2} + f_{1,0}\left[1-\left(\frac{\partial\varphi}{\partial\eta}\right)^2\right] + f_{0,1}\left(1-\frac{\partial\varphi}{\partial\eta}\right) + \frac{1}{2}\eta g\frac{\partial^2\varphi}{\partial\eta^2} + \\ &+\left\{\begin{matrix} g_{1,0}\left(1-\dfrac{\partial\varphi}{\partial\eta}\right) \quad \text{for}\ \ n=0 \\ -g_{1,0}\dfrac{\partial\varphi}{\partial\eta}\left(1-\dfrac{\partial\varphi}{\partial\eta}\right)^n \quad \text{for}\ \ n\in\mathbb{N} \end{matrix}\right\} = \sum_{\substack{k,\,n=0\\ k\vee n\neq 0}}^{\infty}\left[E_{k,n}\frac{\partial^2\varphi}{\partial\eta\partial f_{k,n}} + D_{k,n}X\left(\eta;\ \ f_{k,n}\right)\right] + \\ &+\sum_{\substack{k=1\\ n=0}}^{\infty}\left[L_{k,n}\frac{\partial^2\varphi}{\partial\eta\partial g_{k,n}} + K_{k,n}X\left(\eta; g_{k,n}\right)\right] \end{aligned} \tag{24}$$

From the equation (24) can be seen, that the characteristics of external flow dominate by the means of function F . In order the equation (24) to be independent of the outer flow characteristics *i. e.* to be universal, it is necessary to show the existence of the equality:

$$F = F\left(f_{k,n}, g_{k,n}\right) \tag{25}$$

In order to show that we start from the impulse equation of the discussed problem:

$$\frac{\partial}{\partial t}\left(U\delta^{*}\right) + \frac{\partial}{\partial x}\left(U^{2}\delta^{**}\right) + U\delta^{*}\frac{\partial U}{\partial x} + NU\tilde{\delta}^{**} - \frac{\tau_{w}}{\rho} = 0 \tag{26}$$

where:

$$\delta^{*}\left(x,\ t\right) = \int_{0}^{\infty}\left(1 - \frac{u}{U}\right)\ dy \ \text{– displacement thickness} \tag{27}$$

$$\delta^{**}\left(x,\ t\right) = \int_{0}^{\infty}\frac{u}{U}\left(1 - \frac{u}{U}\right)\ dy \ \text{– momentum thickness} \tag{28}$$

$$\tau_{w}\left(x,\ t\right) = \nu\rho\left.\frac{\partial u}{\partial y}\right|_{y=0} \ \text{– shear stress on the body;} \tag{29}$$

$$\tilde{\delta}^{**}\left(x,\ t\right) = \left\{\begin{matrix} \delta^{*} \ \text{ for } \ n = 0 \\ -\int_{0}^{\infty}\frac{u}{U}\left(1 - \frac{u}{U}\right)^{n} dy \ \text{ for } \ n \in \mathbb{N} \end{matrix}\right\} \tag{30}$$

Introducing the parameters given by equations (14)-(16) into the equation (26), and using the following values:

$$H^{*}\left(x,\ t\right) = \frac{\delta^{*}}{h} = \frac{1}{D}\int_{0}^{\infty}\left(1 - \frac{\partial\varphi}{\partial\eta}\right)d\eta \tag{31}$$

$$H^{**}\left(x,\ t\right) = \frac{\delta^{**}}{h} = \frac{1}{D}\int_{0}^{\infty}\frac{\partial\varphi}{\partial\eta}\left(1 - \frac{\partial\varphi}{\partial\eta}\right)d\eta \tag{32}$$

$$\tilde{H}^{**}\left(x,\ t\right) = \frac{\tilde{\delta}^{**}}{h} = \left\{\begin{matrix} H^{*}\left(x,\ t\right) \ \text{ for } \ n = 0 \\ -\frac{1}{D}\int_{0}^{\infty}\frac{\partial\varphi}{\partial\eta}\left(1 - \frac{\partial\varphi}{\partial\eta}\right)^{n} d\eta \ \text{ for } \ n \in \mathbb{N} \end{matrix}\right\} \tag{33}$$

$$\zeta\left(x,\ t\right) = \frac{\tau_{w}h}{\mu U} = D\left.\frac{\partial^{2}\varphi}{\partial\eta^{2}}\right|_{\eta=0} \tag{34}$$

equation (26) is transformed into a new form out of which the function F can be expressed as:

$$F=\frac{\zeta-\left(f_{1,0}+f_{0,1}+\frac{1}{2}g\right)H^{*}-2f_{1,0}H^{**}-g_{1,0}\tilde{H}^{**}-S}{\frac{1}{2}H^{**}+\sum\limits_{\substack{k,n=0\\k\vee n\neq 0}}^{\infty}(k+n)f_{k,n}\frac{\partial H^{**}}{\partial f_{k,n}}+\sum\limits_{\substack{k=1\\n=0}}^{\infty}(k+n)g_{k,n}\frac{\partial H^{**}}{\partial g_{k,n}}} \tag{35}$$

where:

$$S=\sum_{\substack{k,n=0\\k\vee n\neq 0}}^{\infty}\left(A_{k,n}\frac{\partial H^{**}}{\partial f_{k,n}}+E_{k,n}\frac{\partial H^{*}}{\partial f_{k,n}}\right)+\sum_{\substack{k=1\\n=0}}^{\infty}\left(B_{k,n}\frac{\partial H^{**}}{\partial g_{k,n}}+L_{k,n}\frac{\partial H^{*}}{\partial g_{k,n}}\right) \tag{36}$$

It can be noted from the expression (35) that the function F depends only on the introduced parameters (14)-(16). The equation (24) does not contain the free stream velocity distribution and external magnetic field, which defines each particular case of the considered flow and for a selected change in electrical conductivity – power n, this mathematical model is considered to be universal. The corresponding boundary conditions, also universal, have the following form:

$$\varphi=0\ ,\ \frac{\partial\varphi}{\partial\eta}=0\ \text{ for }\ \eta=0;\ \frac{\partial\varphi}{\partial\eta}\to 1\ \text{ for }\ \eta\to\infty \tag{37}$$

$$\varphi=\varphi_0(\eta)\ \text{ for }\ \left\{\begin{array}{c} f_{k,n}=0,\ (k,\ n=0,\ 1,\ 2,...,\ k\vee n\neq 0)\\ g_{k,n}=0,\ (k,\ n=0,\ 1,\ 2,...,\ k\neq 0)\\ g=0\end{array}\right\} \tag{38}$$

where $\varphi_0(\eta)$ – is Blasius solution for laminar flat-plate boundary layer.

The boundary-layer equation is generalized such that the equation and the boundary conditions are independent of the particular conditions of the problem, *i. e.* obtained equations and corresponding boundary conditions are the same for all possible values of functions U and N.

Numerical integration of the equation (24) with boundary conditions (37) and (38) is carried out once for all, taking on the right side of the equation (24) a finite number of terms. Thus obtained "universal" results are used to draw general conclusions on the development of the boundary layer, as well as to calculate particular cases.

Before integration of the universal equation, a characteristic value should be selected for scale $h(x,t)$ of transversal coordinate in boundary layer. In this case it is convenient to select the value $h=\delta^{**}$, and according to the equations (31) and (32), $H^{*}=\delta^{*}/\delta^{**}=H$ and $H^{**}=1$. The equation (35) is now reduced to the following form:

$$F = 2\left[\zeta - \left(f_{1,0} + f_{0,1} + \frac{1}{2}g\right)H - 2f_{1,0} - g_{1,0}\tilde{H}^{**}\right] - 2\left[+\sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty} E_{k,n}\frac{\partial H}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty} L_{k,n}\frac{\partial H}{\partial g_{k,n}}\right] \quad (39)$$

Further in this paper, adequate approximation of equation (24) is given in which influence of the parameters $f_{1,0}$; $f_{0,1}$; $g_{1,0}$ and g are detained, and the influence of all other parameters and their derivates are disregarded. The equation (24) in the so-called four-parameter twice localized approximation has the form of:

$$\Im = g\left(f_{1,0}\frac{\partial^2\varphi}{\partial\eta\partial f_{1,0}} + g_{1,0}\frac{\partial^2\varphi}{\partial\eta\partial g_{1,0}}\right) + F\left[f_{1,0}X\left(\eta; f_{1,0}\right) + g_{1,0}X\left(\eta; g_{1,0}\right)\right] \quad (40)$$

where $\Im$ is the left-hand side of the equation (24). The function F, in the same approximation, is obtained from the equation (39) and has the form of:

$$F = 2\left[\zeta - \left(f_{1,0} + f_{0,1} + \frac{1}{2}g\right)H - 2f_{1,0} - g_{1,0}\tilde{H}^{**} - g\left(f_{1,0}\frac{\partial H}{\partial f_{1,0}} + g_{1,0}\frac{\partial H}{\partial g_{1,0}}\right)\right] \quad (41)$$

The corresponding boundary conditions obtained from the conditions (37) and (28) are given with the following equations:

$$\varphi = 0\,,\ \frac{\partial\varphi}{\partial\eta} = 0 \ \text{ for } \ \eta = 0; \ \frac{\partial\varphi}{\partial\eta} \to 1 \ \text{ for } \ \eta \to \infty \quad (42)$$

$$\varphi = \varphi_0\left(\eta\right) \ \text{ for } \ f_{1,0} = 0; \ f_{0,1} = 0; \ g_{1,0} = 0; \ g = 0 \quad (43)$$

The value of the constant D is $D = 0.47$, and it is determined by matching the equation (40) with equation of the stationary boundary layer on a flat plate [1].

Results

The equation (40) with the boundary conditions (42) and (43) is solved using three-diagonal method, known in the East literature as the "progonka" method, for $n = 0$ and $n = 1$. A part of the obtained results is presented in the figures 1,2,3,4,5 and 6. Figures 1,2 3 and 4 show the results for $n = 0$, *i. e.* the case of constant electrical conductivity, and figures 5 and 6 for $n = 1$, *i. e.* the case of electrical conductivity change in the form of Rossow [18].

Figures 1 and 2 shows the variations of the variables ζ , F and H in function of the parameter $f_{1,0}$, for several values of the magnetic parameter $g_{1,0}$, and for values of the unsteadiness parameter $f_{0,1} = 0.10$ and the constant parameter $g = 0.05$. It can be noticed that the magnetic parameter $g_{1,0}$ (magnetic field) influences considerably to the position of the boundary layer separation point. The increase in the magnetic parameter move the point of boundary layer separation downstream and in that sense its influence can be considered positive. The functions F and H decrease with the increase of magnetic parameter value. These conclusions are drawn for the case of free stream acceleration ($f_{0,1} > 0$), as well as for free stream deceleration ($f_{0,1} < 0$).

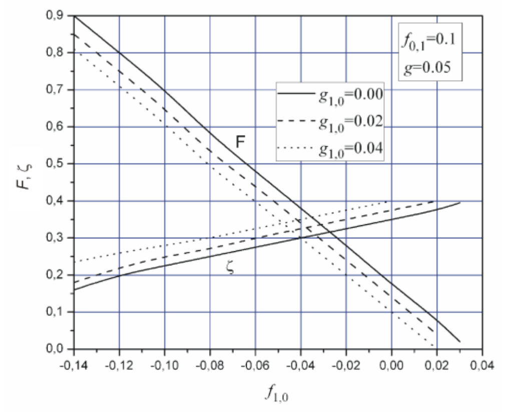

Figure 1. Variations of variables F, ζ in function of dynamic parameter $f_{1,0}$ for different values of magnetic parameter $g_{1,0}$

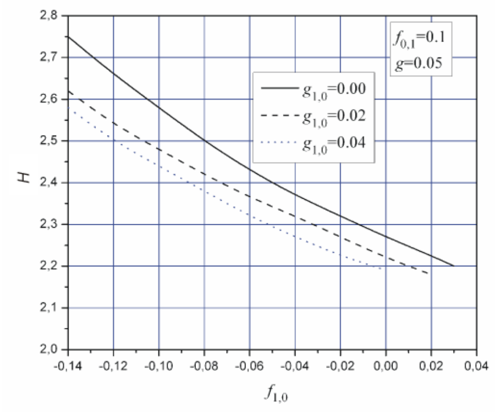

Figure 2. Variations of variable H in function of dynamic parameter $f_{1,0}$ for different values of magnetic parameter $g_{1,0}$

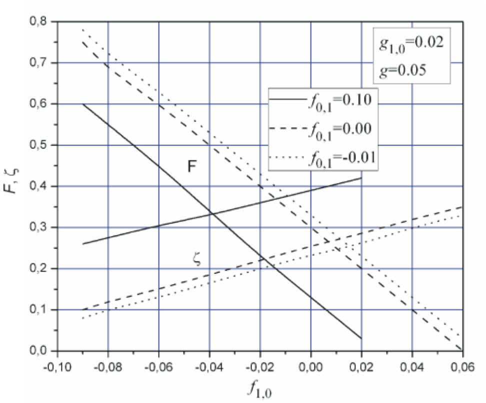

Figure 3. Variations of variables F, ζ in function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$

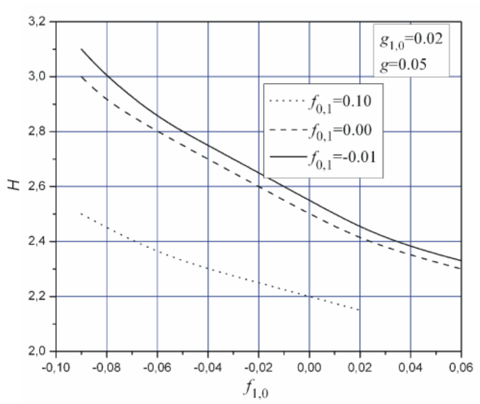

Figure 4. Variations of variables H in function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$

It can be noted from figure 5 that for the case of electrical conductivity change in the form of Rossow ($n=1$), the influence of magnetic field is negative, because the increase of magnetic parameter move the point of boundary layer separation upstream – towards the front stagnation point. The function F increases with the increase in parameter $g_{1,\,0}$. This conclusion is valid both for free stream acceleration ($f_{0,\,1} > 0$), as well as for free stream deceleration ($f_{0,\,1} < 0$).

Figures 3, 4 and 6 shows the graphs of characteristic functions for different values of the parameter $f_{0,\,1}$. On the figures 3 and 4 parameters values are $g_{1,\,0} = 0.02$; $g = 0.05$, and on the figure 6, $g_{1,\,0} = 0.04$; $g = 0.01$. The results obtained for the case of constant electrical conductivity are shown in the figures 3 and 4, while the figure 6 give results for the case of variable electrical conductivity in the form of Rossow $(n = 1)$.

It can be noticed from these figures that free stream acceleration causes a delay in boundary layer separation, *i. e.* it moves the boundary layer separation point downstream, while free stream deceleration moves the boundary layer separation point upstream. Thus the influence of free stream acceleration is positive and the influence of free stream deceleration is negative. These conclusions are valid also for other values of the magnetic parameter and parameter g which have not been given in the figures.

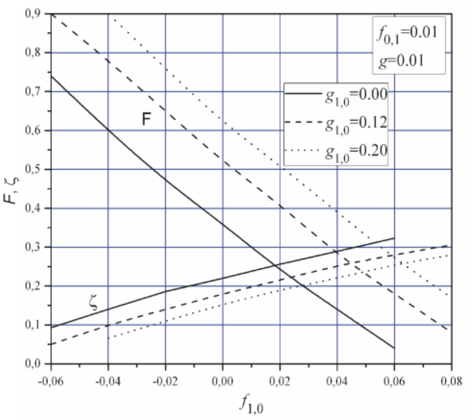

Figure 5. Variations of variables F,ζ in function of dynamic parameter $f_{1,0}$ for different values of magnetic parameter $g_{1,0}$

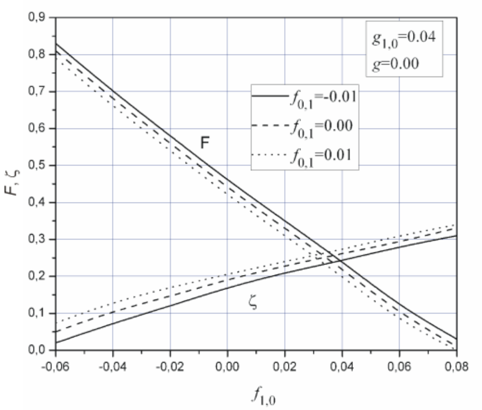

Figure 6. Variations of variables F,ζ in function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$

Conclusions

In this paper unsteady plane laminar MHD boundary layer flow of incompressible and variable electrical conductivity fluid is considered. This problem can be analyzed for each particular case, *i. e.* for given free stream velocity. Here is used quite different approach in order to use benefits of generalized similarity method and universal equation of observed problem is derived. This equation is solved numerically in some approximation and integration results are given in the form of diagrams and conclusions. The obtained results can be used to draw general conclusions on the boundary-layer development, as well as to calculate particular problems.

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Nomenclature

B – magnetic field induction [T]
D – standardization constant [–]
F – characteristic function [–]
$f_{k,n}$ – dynamical parameters [–]
g – constant parameter [–]
$g_{k,n}$ – magnetic parameters [–]
H – characteristic function [–]
H^* – characteristic function [–]
H^{**} – characteristic function [–]
N – characteristic function [s^{-1}]
t – time [s]
u,v – longitudinal and transversal velocity in boundary layer, respectively, [ms^{-1}]
U – free stream velocity [ms^{-1}]
x,y – longitudinal and transversal coordinate, respectively, [m]
z – characteristic function [s]

Greek letters

δ^* – displacement thickness [m]
δ^{**} – momentum thickness [m]
φ – dimensionless stream function [–]
η – dimensionless transversal coordinate [–]
μ – viscosity [Pa s]
ν – kinematic viscosity [m^2s^{-1}]
ρ – fluid density [kgm^{-3}]
σ – electrical conductivity [Sm^{-1}]
τ – shear stress [Pa]
Ψ – stream function [m^2s^{-1}]
ζ – characteristic function [–]

Subscripts and Superscripts

0 – initial time moment
1 – known boundary layer cross-section

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