COMPREHENSIVE PERFORMANCE INVESTIGATION AND OPTIMIZATION OF A PLATE FIN HEAT EXCHANGER WITH WAVY FINS

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As the pressure drop and pump power increase with the enhancement of heat transfer, it is of great value to investigate the comprehensive performance of the heat exchanger based on common accurate correlations of heat transfer and flow friction. This paper adopts a generalized air-side thermal-hydraulic correlation to study the comprehensive performances of the plate fin heat exchanger with wavy fins. To better understand the fin characteristics, performance indexes under the same flow rate, pressure drop and pump power are employed to estimate the comprehensive flow and heat transfer performances. The nonlinear optimization problem is established in consideration of the multiple independent variables with the maximum effectiveness or the minimum modified entropy generation number as the optimization objective function, which is solved by the genetic algorithm. Comparative analysis is conducted for results obtained from the parametric analysis and heat exchanger optimization, indicating that the objective function of the modified entropy generation number is effective for the design optimization of the comprehensive performance.

Key words: plate-fin heat exchanger; wavy fin; comprehensive performance; performance index; optimization

1. Introduction

As it is widely used in fields of aerospace, refrigeration, air conditioning and oil refining, the heat transfer enhancement of the compact heat exchanger has been of industrial and academic interests for decades [1, 2]. The extended surface [3] is one of the effective ways to enhance heat transfer by interrupting the boundary layer, restarting it, creating secondary flows, and/or generating flow unsteadiness [4] with typical channels of wavy fins [5], offset strip fins [6-8] and louvered fins [9]. However, as the pressure drop and pump power increase with the enhancement of heat transfer, it is important to evaluate the comprehensive performance [7, 10, 11] of the heat exchanger.

The performance indexes [4, 12-14] have been adopted for the estimation of the comprehensive heat transfer and resistance characteristics by investigating the performances of individual extended
surfaces. Criteria for evaluating the enhanced heat transfer [13, 15] are mainly focus on the first law of thermodynamics [16-18] and the second law of thermodynamics [19-21], based on which the compact heat exchanger is optimized under single/multiple objective functions and various constraints of conflicting to each other. In this regard, the study based on the heat transfer and flow friction correlations [22-24] is target-shooting for the improvement of the comprehensive performance of heat exchanger. Other interesting topics include optimizing the entire system [25, 26] or adopting the total annual cost [27] as the objective function. However, as from our previous research [28], the single optimization with the objective function of the modified entropy generation number coordinates the performance of the heat exchanger in both economic and thermodynamic aspects.

Based on different correlations of heat transfer and flow friction, such as the Manglik and Bergles correlation [27, 29] and the Joshi and Webb correlation [30], various optimization algorithms have been employed for the optimization of plate-fin heat exchangers with offset strip fins, including the fast and elitist non-dominated sorting genetic-algorithm (NSGA-II) [31], the particle swarm optimization (PSO) algorithm [32], the imperialist competitive algorithm (ICA) [33], and the harmony search algorithm [34]. In addition to the development of more efficient algorithms, it is of great importance to adopt commonly accurate formulas [6] for heat exchanger optimization [28], for which the experimentally validated computational fluid dynamic simulation approach [25, 26, 35, 36] has been widely used for the improvement of heat transfer and friction correlations in applications of different geometries and operating conditions.

Benefit from the effective flow interruptions and improved heat transfer by the waveform, the plate fin heat exchanger with wavy fins is attractive for industrial application with advantages of efficient thermal performance, proper compactness, low weight, robust structure, low cost and uncut surfaces in the flow direction [37, 38]. Since there are no cuts in the surface, wavy fins can be employed in applications where an interrupted fin might be subject to a potential fouling or clogging problem due to particulates, freezing moisture, bridging louvers due to condensate, and so on [37]. Thus, the comprehensive performance optimization of the plate-fin heat exchanger with wavy fins is of significant profits for both academia and industry. Various correlations [36, 37, 39-41] of heat transfer and flow friction characteristics for wavy fins have been proposed for limited application ranges. Recently, Naef A.A. Qasem and Syed M. Zubair [5] proposed and validated a more accurate and generalized air-side thermal-hydraulic correlation for wavy-fin compact heat exchangers from experimental and numerical data, which is adopted for the performance evaluation and optimization for this work.

In this paper, we employ a generalized air-side thermal-hydraulic correlation to study the comprehensive performances of the plate fin heat exchanger with wavy fins by both parametric analysis and heat exchanger optimization. The comprehensive heat transfer and resistance characteristics are estimated by the performance indexes under the same flow rate, pressure drop and pump power. Single optimization is performed by GA with the objective functions of the maximum effectiveness and the minimum modified entropy generation number for the heat exchanger. Comparative analysis between the obtained results from the performance index calculation and GA optimization is investigated.

2. Thermal analysis

The thermal analysis of a typical cross-flow gas/gas plate fin heat exchanger with offset strip
fins [28] is applied to wavy fins in this work.

2.1. Wavy fins

The differences between the offset strip fins and the wavy fins lie in the correlation of flow and heat transfer characteristics, the fluid flow area, and the heat transfer surface area. For a plate fin heat exchanger with wavy fins (Fig. 1), a generalized air-side thermal-hydraulic correlation for Reynolds number between 400 and 11500 is expressed in Eq. (1)-(3), which has been validated by both experimental and numerical data in ref. [5]. The available geometrical parameter ranges include fin pitch ($F_p$) of 1.4-6.5mm, fin height ($F_h$) of 4-10.5mm, amplitude ($A$) of 0.15-3mm, flow length ($L_d$) of 20-65mm, wavelength ($L$) of 5-15mm, and fin thickness ($t$) of 0.05-0.45 mm.

\[
j = 0.656927Re^{-0.3338}\left(\frac{F_h}{D_h}\right)^{0.571367}\left(\frac{F_p}{D_h}\right)^{0.283142}\left(\frac{2A}{D_h}\right)^{0.056066}\left(\frac{L_d}{D_h}\right)^{-1.0107}\left(\frac{L}{D_h}\right)^{-0.229603}\left(\frac{t}{D_h}\right)^{-0.0476}
\]

\[
f = 3.86916Re^{-0.3569}\left(\frac{F_h}{D_h}\right)^{-0.51098}\left(\frac{F_p}{D_h}\right)^{0.05031}\left(\frac{2A}{D_h}\right)^{0.78254}\left(\frac{L_d}{D_h}\right)^{0.08671}\left(\frac{L}{D_h}\right)^{-0.4177}\left(\frac{t}{D_h}\right)^{0.05775}
\]

\[
D_h = \frac{2(F_h-t)(F_p-t)}{(F_h+F_p-2t)}
\]

Although the original work [5] for obtaining eq. (1)-(3) simulated the whole fluid flow domain with the inlet and outlet zone, the inlet zone was set at a uniform frontal velocity as a distributor, the outlet zone is used to avoid the back flow, and no entrance and outlet back-mixing effect were taken into consideration for the inlet and outlet zone, respectively. So several periodic fin configurations with the same Reynolds number are assumed to satisfy the requirements of heat transfer as the flow and heat transfer correlations are applicable for a limited flow length ($L_d$) from 20 to 65mm. Since the correlations are only related to the fin parameters and the Reynolds number, the heat transfer and flow friction coefficients for each fluid side remain the same.

The fluid flow area $A_c$ is expressed in eq. (4)-(5),

\[
A_{ch} = N_{layer,h}\left(F_{p,h} - t_h\right)\left(F_{h,h} - t_h\right)\frac{L_d\delta_p L_{1c}}{F_{p,h}}
\]
\[ A_{c,c} = N_{layer,c} \left( F_{p,c} - t_c \right) \left( F_{h,c} - t_c \right) \frac{L_{slh} P_{th}}{F_{p,c}} \]  \hspace{1cm} (5)

The heat transfer surface area \( A_i \) is shown in eq. (6)-(7),

\[ A_{h} = 2N_{L,h}N_{C,h}N_{layer,h}\lambda_h \left[ \left( F_{p,h} - t_h \right) + \left( F_{h,h} - t_h \right) \right] \]  \hspace{1cm} (6)

\[ A_{c,c} = 2N_{L,c}N_{C,c}N_{layer,c}\lambda_c \left[ \left( F_{p,c} - t_c \right) + \left( F_{h,c} - t_c \right) \right] \]  \hspace{1cm} (7)

where, \( N_i \) is the number of wavelengths per flow channel, shown in eq. (8); \( N_C \) is the number of flow channel per plate, shown in eq. (9); \( \lambda \) is the length of a periodic wave, mm, shown in eq. (10).

\[ N_L = \frac{L_P}{L} \]  \hspace{1cm} (8)

\[ N_{C,h} = \frac{L_{slh} P_{th}}{F_{p,h}}, N_{C,c} = \frac{L_{slc} P_{th}}{F_{p,c}} \]  \hspace{1cm} (9)

\[ \lambda = \int_0^L \sqrt{1 + \left( \frac{2\pi A}{L} \right)^2 \cos^2 \left( \frac{2\pi x}{L} \right)} dx \]  \hspace{1cm} (10)

### 2.2. Plain fins

For comparison, the heat transfer and flow friction characteristics of the full developed rectangular plain fins are referred from [42, 43], shown in eq. (11)-(16). The laminar flow is assumed under the Reynolds number of 2200, while the Gnielinski correlation is adopted for calculating the turbulent flow [43].

The laminar flow \( (Re \leq 2200) \),

\[ Nu = 5.331 \]  \hspace{1cm} (11)

\[ j = \frac{Nu}{RePr^{\frac{3}{2}}} \]  \hspace{1cm} (12)

\[ f = \frac{18.233}{Re} \]  \hspace{1cm} (13)

The turbulent flow \( (Re > 2200) \),

\[ Nu = \frac{(f / 8)(Re - 1000)Pr}{1 + 12.7(f / 8)^{0.3}(Pr^{0.3} - 1)} \]  \hspace{1cm} (14)

\[ j = \frac{Nu}{RePr^{\frac{3}{2}}} \]  \hspace{1cm} (15)

\[ f = (0.79\ln(Re) - 1.64)^{-2} \]  \hspace{1cm} (16)

### 3. Parametric investigation

Three performance indexes [4, 12-14] are employed for the evaluation of the comprehensive heat transfer and resistance characteristics of wavy fins under the same flow rate (eq. 17), pressure drop (eq. 18) and pump power (eq. 19) in this paper. Typical wavy fins with various groups of
configuration parameters are collected from literature [5, 39] shown in Tab. 1.

\[ \eta_1 = \frac{j}{f} \]  \hspace{1cm} (17)
\[ \eta_2 = \frac{j}{f^\frac{1}{2}} \]  \hspace{1cm} (18)
\[ \eta_3 = \frac{j}{f^\frac{3}{4}} \]  \hspace{1cm} (19)

### Table 1. Typical wavy fin configuration parameters

<table>
<thead>
<tr>
<th>Fin No.</th>
<th>( F_h ) [mm]</th>
<th>( F_p ) [mm]</th>
<th>2A [mm]</th>
<th>( L_{d1} ) [mm]</th>
<th>( L ) [mm]</th>
<th>( t ) [mm]</th>
<th>( D_h ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GJ1</td>
<td>8</td>
<td>2</td>
<td>1.5</td>
<td>65</td>
<td>10.8</td>
<td>0.2</td>
<td>2.925</td>
</tr>
<tr>
<td>GJ2</td>
<td>8</td>
<td>2.25</td>
<td>1.5</td>
<td>65</td>
<td>10.8</td>
<td>0.2</td>
<td>3.247</td>
</tr>
<tr>
<td>GJ3</td>
<td>8</td>
<td>2.5</td>
<td>1.5</td>
<td>65</td>
<td>10.8</td>
<td>0.2</td>
<td>3.552</td>
</tr>
<tr>
<td>GJ4</td>
<td>8</td>
<td>2</td>
<td>1.5</td>
<td>53</td>
<td>10.8</td>
<td>0.2</td>
<td>2.925</td>
</tr>
<tr>
<td>GJ5</td>
<td>8</td>
<td>2.25</td>
<td>1.5</td>
<td>53</td>
<td>10.8</td>
<td>0.2</td>
<td>3.247</td>
</tr>
<tr>
<td>GJ6</td>
<td>8</td>
<td>2.5</td>
<td>1.5</td>
<td>53</td>
<td>10.8</td>
<td>0.2</td>
<td>3.552</td>
</tr>
<tr>
<td>GJ7</td>
<td>7</td>
<td>2</td>
<td>1.5</td>
<td>43</td>
<td>10.8</td>
<td>0.2</td>
<td>2.847</td>
</tr>
<tr>
<td>GJ8</td>
<td>7</td>
<td>2.25</td>
<td>1.5</td>
<td>43</td>
<td>10.8</td>
<td>0.2</td>
<td>3.150</td>
</tr>
<tr>
<td>GJ9</td>
<td>7</td>
<td>2.5</td>
<td>1.5</td>
<td>43</td>
<td>10.8</td>
<td>0.2</td>
<td>3.437</td>
</tr>
<tr>
<td>GJ10</td>
<td>8</td>
<td>2</td>
<td>1.5</td>
<td>43</td>
<td>10.8</td>
<td>0.2</td>
<td>2.925</td>
</tr>
<tr>
<td>GJ11</td>
<td>10</td>
<td>2</td>
<td>1.5</td>
<td>43</td>
<td>10.8</td>
<td>0.2</td>
<td>3.041</td>
</tr>
<tr>
<td>GN1</td>
<td>5</td>
<td>5</td>
<td>0.3</td>
<td>30</td>
<td>15</td>
<td>0.2</td>
<td>4.800</td>
</tr>
<tr>
<td>GN2</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>30</td>
<td>15</td>
<td>0.2</td>
<td>4.800</td>
</tr>
<tr>
<td>GN3</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>30</td>
<td>15</td>
<td>0.2</td>
<td>4.800</td>
</tr>
<tr>
<td>GN10</td>
<td>4</td>
<td>5.5</td>
<td>0.6</td>
<td>30</td>
<td>15</td>
<td>0.2</td>
<td>4.426</td>
</tr>
<tr>
<td>GN15</td>
<td>4</td>
<td>5.5</td>
<td>0.6</td>
<td>30</td>
<td>15</td>
<td>0.45</td>
<td>4.169</td>
</tr>
<tr>
<td>GN7</td>
<td>4</td>
<td>5.5</td>
<td>0.6</td>
<td>30</td>
<td>15</td>
<td>0.05</td>
<td>4.580</td>
</tr>
<tr>
<td>GN19</td>
<td>4</td>
<td>5.5</td>
<td>0.6</td>
<td>20</td>
<td>5</td>
<td>0.05</td>
<td>4.580</td>
</tr>
<tr>
<td>GN20</td>
<td>4</td>
<td>6</td>
<td>0.6</td>
<td>20</td>
<td>5</td>
<td>0.2</td>
<td>4.592</td>
</tr>
<tr>
<td>GN8</td>
<td>4</td>
<td>6</td>
<td>0.6</td>
<td>30</td>
<td>15</td>
<td>0.2</td>
<td>4.592</td>
</tr>
</tbody>
</table>

Note: (GJ1, GJ2 and GJ3), (GJ4, GJ5 and GJ6) and (GJ7, GJ8 and GJ9) is a group of similar wavy fins with different \( F_p \), respectively; (GJ1, GJ4 and GJ7), (GJ2, GJ5 and GJ8) and (GJ3, GJ6 and GJ9) is a group of similar wavy fins with different \( L_{d1} \), respectively; (GJ7, GJ10 and GJ11) is a group of similar wavy fins with different \( F_h \); (GN1, GN2 and GN31) is a group of similar wavy fins with different 2A; (GN10, GN15 and GN7) is a group of similar wavy fins with different \( t \); both (GN7, GN19) and (GN20, GN8) is a group of similar wavy fins with different \( L \) and \( L_{d1} \).

As for the generalized thermal-hydraulic correlation of wavy fins in eq. (1)-(3), the effective Reynolds numbers range from 400 to 11500. The comprehensive indexes of flow and heat transfer within certain Reynolds numbers are illustrated in Fig. 2. For comparison, the comprehensive performances with the similar heights (\( F_p \)-\( t \)) and widths (\( F_h \)-\( t \)) of plain fins (Tab. 2) under different Reynolds numbers are also listed in Fig. 2. As the correlations in eq. (11)-(16) are only related to Reynolds number, the \( j \) and \( f \) factor keep the same for the various plain fin configurations under the
Figure 2. Comprehensive performances of wavy fins with different fin configurations along with the Reynolds numbers: (a) is for the same flow rate; (b) is for the same pressure drop; (c) is for the same pump power.

Table 2. Plain fin configuration parameters

<table>
<thead>
<tr>
<th>Fin No.</th>
<th>$F_h$ [mm]</th>
<th>$F_p$ [mm]</th>
<th>$t$ [mm]</th>
<th>For comparison with</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN1</td>
<td>8</td>
<td>2</td>
<td>0.2</td>
<td>GJ1, GJ4, GJ10</td>
</tr>
<tr>
<td>PN2</td>
<td>8</td>
<td>2.25</td>
<td>0.2</td>
<td>GJ2, GJ5</td>
</tr>
<tr>
<td>PN3</td>
<td>8</td>
<td>2.5</td>
<td>0.2</td>
<td>GJ3, GJ6</td>
</tr>
<tr>
<td>PN4</td>
<td>7</td>
<td>2</td>
<td>0.2</td>
<td>GJ7</td>
</tr>
<tr>
<td>PN5</td>
<td>7</td>
<td>2.25</td>
<td>0.2</td>
<td>GJ8</td>
</tr>
<tr>
<td>PN6</td>
<td>7</td>
<td>2.5</td>
<td>0.2</td>
<td>GJ9</td>
</tr>
<tr>
<td>PN7</td>
<td>10</td>
<td>2</td>
<td>0.2</td>
<td>GJ11</td>
</tr>
<tr>
<td>PN8</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
<td>GN1, GN2, GN3</td>
</tr>
<tr>
<td>PN9</td>
<td>4</td>
<td>5.5</td>
<td>0.2</td>
<td>GN10</td>
</tr>
<tr>
<td>PN10</td>
<td>4</td>
<td>5.5</td>
<td>0.45</td>
<td>GN15</td>
</tr>
<tr>
<td>PN11</td>
<td>4</td>
<td>5.5</td>
<td>0.05</td>
<td>GN7, GN19</td>
</tr>
<tr>
<td>PN12</td>
<td>4</td>
<td>6</td>
<td>0.2</td>
<td>GN20, GN8</td>
</tr>
</tbody>
</table>

As it is shown in Fig. 2, the comprehensive evaluation index of wavy fins under the same flow rate changes little with the Reynolds number, which is different from that of the plain fins as a result of the separately calculating methods for laminar and turbulence flows. While the evaluation indexes under the same pressure drop and the same pump power demonstrate better performances under low
Reynolds numbers, and fall sharply as the Reynolds number increases, among which the plain fins almost display the worst comprehensive performances, showing that good compromises between heat transfer and flow friction can be achieved by the wavy fins through adjusting the fin geometries. In this way, the comprehensive performances of wavy fins are significantly improved. However, the latter two indexes are more significant and meaningful for the energy conservation and efficiency improvement in actual industrial applications. Thus, the low Reynolds number is preferred to obtain the better comprehensive performances regarding both heat transfer and flow friction.

Among all the selected fins, GN1 is observed to achieve the highest comprehensive performances, with the lowest wavy length amplitude and highest hydraulic diameter. According to the expression of the comprehensive indexes, the performances are negatively related the ratio of the wavy length amplitude to the hydraulic diameter. On one hand, the wavy fins promote the heat transfer compared with the plain fins; on the other hand, the smallest wavy amplitude leads to the lowest fluctuation, causing the minimum friction loss. GN3 obtains the worst performance for the comprehensive evaluation index under the same flow rate with the largest wavy length amplitude. Although the heat transfer is increased, the friction loss caused is higher. However, this fin configuration does not lead to the lowest performance for the indexes under the same pressure drop and the same pump power, so the proper amplitude selection is important for the comprehensive performance.

With the smallest fin thickness and the largest wavelength, GN7 shows competitive advantages for the comprehensive evaluation indexes under the same flow rate and the same pressure drop among the other fins except GN1. It is obvious that the performances are also negatively related the ratio of the fin thickness to the hydraulic diameter, but not as much as the ratio of the wavy length amplitude to the hydraulic diameter. Thus, attentions should be paid on the fin thickness and wavelength to achieve relatively high performances.

With the longest flow length, GJ1, GJ2 and GJ3 achieve the worst comprehensive performances even to the level of plain fins under the same pressure drop and the same pump power since the flow friction increases with the flow length. Obviously, the evaluation indexes are in negatively relationship with the ratio of the flow length to the hydraulic diameter. However, it is not wise to sacrifice the flow length for the improvement of comprehensive performances as the proper flow length is essential to fulfill the function of heat transfer. As for the influence of the fin pitch on the comprehensive indexes, it is apparent that the comprehensive indexes are in exponential relationships with the ratio of the fin pitch to the hydraulic diameter. So it is with the fin height.

However, as the correlations are influenced by multiple parameters, the single parameter analysis for the impact of the fin configurations is not suitable for industrial application. The comprehensive index analysis is used for rough guide though, and it depends on the optimization for practical applications under certain circumstances.

4. Optimization

4.1. Optimization problem

The target of the maximum effectiveness and the minimum modified entropy generation number [28] are selected for the optimization of the plate fin heat exchanger with wavy fins.

The optimization problem is expressed by eq. (20) with the independent variables, presented in
eq. (21) and the constraints in eq. (22).

\[
\min f(X)
\]
\[
s.t. \ h_i(X) \leq 0 \quad (v = 1, 2, L, p)
\]

\[
X = \begin{bmatrix}
F_{h\text{h}}, F_{h\text{c}}, F_{p\text{h}}, F_{p\text{c}}, A_n, A_c, L_{d\text{h}}, L_{d\text{c}}, L_n, L_c, t_h, t_c, P_{f\text{h}}, P_{f\text{c}}, N_{\text{layer,h}}
\end{bmatrix}
\] (21)

Besides the allowable pressure drop \((dp^0)\), the heat transfer and flow friction correlation is constrained by the available application scope of Reynolds number: \(400 \leq Re \leq 11500\).

In this way, the constraints can be transferred into the penalty function by eq. (22), where the matrix is employed to show whether the constraints are satisfied, including the pressure drop and the Reynolds number scope. The function \(cons1(X)\) summarizes the deviation from the constraints, where \(con2(X)\) compares the maximum deviation with 0 to assess whether it is within the limitation, otherwise a penalty factor \((r)\) is applied in eq. (23).

\[
deltap(X) = \Delta p(i) \quad (i = h, c)
\]
\[
R = Re(i) \quad (i = h, c)
\]
\[
cons1(X) = \{\{deltap - dp^0\}, \{400 - R\}, \{R - 11500\}\}
\]
\[
cons2(X) = \max(0, \max(cons1(X)))
\]

In this way, the fitness function of the optimization is obtained by eq. (23),

\[
\min p(X)
\]
\[
p(X) = f(X) + r * cons2(X)
\] (23)

4.2. Objective functions and performance indicators

The GA optimization approach, the assumptions, the input parameters of the cross-flow gas/gas plate fin heat exchanger, basic settings of the algorithm and necessary parameters used for the calculation can be found in ref. [28]. The geometrical parameter ranges are set between the lower and upper bounds of the independent variables, shown in Tab. 3. The GA toolbox in MATLAB is employed for the optimization. Two significant digits are set for the fin configurations as the fin height, fin pitch, wavy length amplitude, flow length, wavelength of the wavy fin and the fin thickness. The \textit{round} function is adopted for the roundness of the periodic flow lengths and layers.

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|}
\hline
Parameter & Lower bound & Upper bound \\
\hline
\(F_p\) [mm] & 1.4 & 6.5 \\
\hline
\(F_h\) [mm] & 4 & 10.5 \\
\hline
\(A\) [mm] & 0.15 & 3 \\
\hline
\(L_{d\text{h}}\) [mm] & 20 & 65 \\
\hline
\(L\) [mm] & 5 & 15 \\
\hline
\(t\) [mm] & 0.05 & 0.45 \\
\hline
\(P_f\) & 1 & 150 \\
\hline
\(N_{\text{layer,h}}\) & 1 & 300 \\
\hline
\end{tabular}
\end{table}
The objectiveness functions of the effectiveness $\varepsilon$ and the modified entropy generation number $N_{s1}$ are shown in eq. (24) and eq. (25)-(27), respectively. Performance indicators as TAC and the pump power are expressed in eq. (28)-(30) and eq. (31), respectively.

$$\varepsilon = 1 - \exp \left\{ \frac{1}{C^*} NTU^{0.22} \left[ \exp \left( -C^* NTU^{0.78} \right) - 1 \right] \right\}$$

$$N_{s1} = N_{s1,p} + N_{s1,T}$$

$$N_{s1,p} = \frac{m_h R_{g,h} \ln \frac{P_{m,h}}{P_{out,h}} + m_c R_{g,c} \ln \frac{P_{m,c}}{P_{out,c}}}{Q} T_{in,c}$$

$$N_{s1,T} = \frac{m_h C_{p,h} \ln \frac{T_{out,h}}{T_{in,h}} + m_c C_{p,c} \ln \frac{T_{out,c}}{T_{in,c}}}{Q}$$

$$TAC = CC + OC$$

$$CC = A_x C A^n$$

$$OC = \frac{\zeta \tau}{1000} \left[ \left( \Delta p_m \frac{\eta \rho_b}{\eta \rho} \right)_h + \left( \Delta p_m \frac{\eta \rho_b}{\eta \rho} \right)_c \right]$$

$$W = \frac{1}{\eta} \left( \frac{m_h}{\rho_b} \Delta p_h + \frac{m_c}{\rho_c} \Delta p_c \right)$$

4.2.1. Maximum effectiveness

For the optimization with the objective function of maximum effectiveness, the fitness function is the minimum of the reciprocal of the effectiveness, with $f(X)$ in eq. (23) as $\varepsilon \ (1/\text{eff})$ shown in Fig. 3 along with the generation. The corresponding thermodynamic and economic indicators are shown in Fig. 5. As it is can be seen from Fig. 3, the fitness value of the minimum reciprocal of effectiveness falls quickly in the first generation, and approaches to be steady from the 10th generation, which illustrates the efficiency of the optimization method.
4.2.2. Minimum modified entropy generation number

For the optimization with the objective function of minimum modified entropy generation number, \( N_{s1} \), the fitness function is the minimum modified entropy generation number (ent) for eq. (23). The curve of the fitness function of varying with generation is shown in Fig. 4, with the thermodynamic and economic indicators presented in Fig. 5. With the objective function of the minimum modified entropy generation number, the fitness value decreases rapidly and turns to be stable from the 20\(^{th}\) generation, which achieves the stopping criteria in the 132\(^{nd}\) generation, indicating the effectiveness of GA in optimizing the plate fin heat exchanger with wavy fins.

![Fitness vs Generation](image)

**Figure 4.** The fitness value of the minimum modified entropy generation number along with the generation

5. Comparison and analysis

According to the parametric analysis of flow and heat transfer by evaluating \( jff, jff^{A2}, jff^{A3} \), GN1 shows the best performance, the fin configuration of which is selected for the comparison of the optimization results. GN1 with the same total flow channel length \( (L_d \times P_f) \) and the same flow layer as the optimized results, are named GN1-1, GN1-2, GN1-3. Fig. 5 presents the results from single objective optimization and fin configurations obtained by comparative parametric investigations with various performance indicators. For simplification, O1, O2 represents the optimization results with the objective function of the maximum effectiveness (1/eff) and the minimum modified entropy generation number (ent), respectively; P1, P2, P3 represents the comparative parametric investigations of GN1-1, GN1-2, GN1-3, respectively. While both \( P_1 \) of GN1-1 and GN1-2 is beyond the bound of the set independent variable in Tab. 3, the upper bound of 150 is adopted for GN1-3. RMB Yuan is the Chinese currency, the ratio of which to US dollar is about 100/15.46.

In comparison of the performance indicators (Fig. 5) of the maximum effectiveness (1/eff) and the minimum modified entropy generation number (ent), the pressure drop of the cold side \( (\Delta p_c) \) significantly decreases for about 93\% with smaller wavy length amplitude for the optimization results of O2, leading to the reduction of the compressor power consumption and the operating cost (\(~ 92\%)\. The lower total heat transfer area contributes to the decrease of the capitalized cost (\(~ 27\%)\. Both of them results in less total annual cost (\(~ 86\%)\), while the thermodynamic performance is approximately the same for effectiveness (0.9091 for O1 and 0.8968 for O2).
Although P3 is set with less number of periodic flow lengths ($P_f$), the pressure drop of the cold side is higher than that of P1 and P2, which is caused by the same inlet parameters of fluid. That is to say, a more compact heat exchanger (smaller fluid flow area) with shorter flow length leads to higher mass flow velocity, resulting in the stronger fluid disturbance and higher pressure drop. Thus, the pressure drop of the compact heat exchanger should be adjusted to be in coordination with the enhanced heat transfer, indicating the necessity for the comprehensive analysis and optimization.

GN1 shows the best performance in the comprehensive analysis of wavy fins, so the thermodynamic performances of heat exchangers with that configuration (P1, P2, P3) seem to be good in terms of effectiveness (from 0.8625 to 0.9171). While as a complex system, the performance of heat exchanger is not simply determined by the fin configuration, so the fact that the total annual costs (TAC) of parametric analysis is about 23% to 54% higher than that of the optimization result of the minimum modified entropy generation number ($ent$) prove the superiority of the optimization.

Thus, it is recommended that the objective function of the modified entropy generation number should be employed preferentially in the design optimization of the plate fin heat exchanger. The calculated performance indicators in Fig. 5 show the superiority of this optimization methodology.

As the flow and heat transfer correlations are applicable for a limited flow length ($L_d$), the periodic fin configurations ($P_f$) have been set to satisfy the requirements of heat transfer, which definitely affects the optimization results. The flow and heat transfer correlations for larger range of flow length are expected in future research to include more fin configurations correctly.

6. Conclusions and perspectives

(1) The performance index analysis provides rough guide to better understand the fin characteristics. The low Reynolds number is preferred to obtain the better comprehensive performance in compromise with heat transfer and flow friction. The proper wavy amplitude on the one hand promotes the heat transfer, and significantly affects the fluid fluctuation and friction loss on the other hand. The amplitude selection is important for the comprehensive performance. Attention should be paid on the fin thickness and wavelength to achieve relatively high performances.

(2) GA is proved to be effective and efficient for the optimization of the plate fin heat exchanger
with wavy fins with fast convergence and reliable results.

(3) It is recommended that the objective function of the modified entropy generation number should be employed preferentially in the design optimization of the plate fin heat exchanger, which improves the comprehensive performance of the heat exchanger.

(4) A more compact heat exchanger with shorter flow length leads to higher mass flow velocity, resulting in the stronger fluid disturbance and higher pressure drop, which should be adjusted to be in coordination with the enhanced heat transfer, indicating the necessity for the comprehensive analysis and optimization.

(5) Regarding the GA optimization, the validation is focus on performance calculation, whereas a further validation with the experiments is required.

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Nomenclature

\[
\begin{align*}
A & \quad \text{wavy length amplitude [mm];} \\
A_c & \quad \text{fluid flow area, mm}^2; \\
A_{sf} & \quad \text{annual co-efficient factor;} \\
C_A & \quad \text{capitalized cost unit heat transfer area, yuan/m}^2; \\
CC & \quad \text{capitalized cost;} \\
C_p & \quad \text{specific heat at constant pressure, J/kg·K;} \\
C' & \quad \text{ratio of heat capacity;} \\
D_h & \quad \text{hydraulic diameter, mm;} \\
f & \quad \text{friction factor;} \\
F_h & \quad \text{fin height, mm;} \\
F_p & \quad \text{fin pitch, mm;} \\
j & \quad \text{heat transfer factor;} \\
L & \quad \text{wavelength of the wavy fin, mm;} \\
L_d & \quad \text{flow length, mm;} \\
m & \quad \text{mass flow of fluid, kg/s;} \\
n_1 & \quad \text{exponent of non-linear increase with area increase;} \\
N_C & \quad \text{number of flow channel per plate;} \\
N_L & \quad \text{number of wavelengths per flow channel;} \\
N_{layer} & \quad \text{number of fin layers;} \\
N_{s1} & \quad \text{modified entropy generation number;} \\
N_{s1,p} & \quad \text{modified entropy generation number caused by friction resistance;} \\
N_{s1,T} & \quad \text{modified entropy generation number caused by heat transfer;} \\
Nu & \quad \text{Nusselt number;} \\
NTU & \quad \text{number of transfer units;} \\
OC & \quad \text{operating cost, yuan;} \\
p & \quad \text{pressure, Pa;} \\
P_f & \quad \text{number of the periodic flow lengths;} \\
Pr & \quad \text{Prandtl number;} \\
r & \quad \text{penalty factor;} \\
Re & \quad \text{Reynolds number;} \\
R_g & \quad \text{universal gas constant, J/kg·K;} \\
t & \quad \text{fin thickness, mm;} \\
T & \quad \text{temperature, K;} \\
TAC & \quad \text{total annual cost, yuan;} \\
W & \quad \text{compressor power consumption caused by pressure drop, W;}
\end{align*}
\]

Symbols

\[
\begin{align*}
\varepsilon & \quad \text{effectiveness of the heat exchanger;} \\
\lambda & \quad \text{length of a periodic wave, mm;} \\
\zeta & \quad \text{electricity price, yuan/kW·h;} \\
\tau & \quad \text{annual operating time, h;} \\
\eta & \quad \text{efficiency of compressor;} \\
\eta_1 & \quad \text{comprehensive flow and heat transfer evaluation index under the same flow rate;} \\
\eta_2 & \quad \text{comprehensive flow and heat transfer evaluation index under the same pressure drop;}
\end{align*}
\]

References

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