PERFORMANCE INVESTIGATION OF A TWO-STAGE THERMOELECTRIC COOLER WITH INHOMOGENEOUS MATERIALS

by

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A novel model of two-stage thermoelectric cooler with inhomogeneous thermal conductivity in steady-state operating condition is established. The modification of the constant properties model allows controlling the distribution of Joule heat. Considering internal irreversibilities of the thermoelectric cooler, expressions for the cooling capacity, coefficient of performance and exergy efficiency are derived. By utilizing numerical methods, the temperature profile along the thermoelectric legs is presented. The optimal operating regions are explored. The coefficient of performance versus cooling capacity describing optimal operating regions in inhomogeneity materials are plotted. Meanwhile, the influence of the main parameters such as the variation of thermal conductivity distribution, cold-end temperature and the number of thermoelectric modules on the cooling performance is discussed in detail. Results indicate that the cooling capacity, coefficient of performance and exergy efficiency are improved compared to those of homogeneous two-stage thermoelectric coolers when an appropriate inhomogeneous property parameter is applied. The work can provide guidance on design of actual two stage thermoelectric coolers with inhomogeneous materials.

Key words: two-stage thermoelectric cooler, inhomogeneous thermal conductivity

1. Introduction

As the world strives to pay great attention to environment protection, the applications of thermoelectric materials in refrigeration are attracting significant attention [1, 2]. Thermoelectric material is a key enabler for sustainable development, which can realize the direct conversion of thermal energy and electric energy. Compared with traditional refrigeration devices, thermoelectric coolers have drawn attention for its absence of moving parts, silence in operation, small-scale applications, and continuous stability [3, 4]. Moreover, no refrigerant is required [5, 6]. Due to their advantages, thermoelectric coolers have great appeal in the fields of national defense, aerospace, medical, agriculture and microelectronic [7, 8]. For example, thermoelectric coolers provide a potential for temperature stabilization of semiconductor lasers [1]. However, its main problem is that the value of the dimensionless thermoelectric figure-of-merit (ZT) is relatively low [9-11]. Solving

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this problem of finding materials with high ZT has always been a challenge [2, 12]. Efforts have been made to investigate some approaches which can reinforce the cooling performance. Several strategies have been mainly focused on improving the geometrical structure of TE systems and the development of more efficient materials. Various forms of the thermoelectric devices and different types of the thermoelectric materials are investigated to offer better cooling performance. Sharma et al. [13] not only confirmed the most appropriate number of the thermoelectric elements for multi-stage TECs, but also analyzed the advantages compared with single thermoelectric coolers. Energy, exergy and exergoeconomic are considered as target functions of the performance evaluation for single- and two-stage TE device in different mode were compared by Nami et al. [10] On the other hand, concerns about inhomogeneous or graded thermoelectric materials are growing. Some studies have found that the asymmetry of Joule heat dissipation has a significant effect on TE devices. A new model, a single-stage thermoelectric cooling with inhomogeneous materials, was proposed by Lu et al. [14]. In the model, it was founded that optimizing the linear correlation coefficient of thermal conductivity play a significant role to improve the maximum cooling capacity and temperature difference. Bian et al. [15] offered the graded material and did numerical optimization. The results proved that it could achieve a 27% cooling enhancement compared to the best homogeneous material. The performance of TE coolers with the spatial- and temperature- dependent properties for a given the graded carrier concentration was investigated by Hu et al. [16]. They found that the asymmetric distribution of Joule heat is an effective way to enhance the performance of TE coolers. Huang et al.[17] studied a new type of parallel two-stage thermoelectric cooler by utilizing the space-dependent electrical conductivity of inhomogeneous materials. It was found that the spatial variation of conductivity would enhance the corresponding cooling capacity and coefficient of performance. Lam et al.[18] discussed the effect of pulse current on the properties of tapered non-uniform thermoelectric materials, and obtained the TE temperature distribution profiles.

However, no open literature has investigated a two-stage thermoelectric cooler (TTEC) constructed by inhomogeneous material with the spatial- dependent thermal conductivity. In order to break the symmetrical distribution of Joule heat and reinforce the cooling performance of thermoelectricity, a TTEC model with inhomogeneous thermal conductivity is theoretically established. This paper is organized as follows. In section 2, we derive analytical expressions of several important parameters of thermoelectric cooler based on the model. In section 3, the influence of the exponent of power function, cold-end temperature and distribution of the number of thermoelectric modules are analyzed. Several important conclusions are presented in section 4.

2. Model description

Schematic diagrams of the two-stage TECs considering inhomogeneous thermal conductivity in the electrically series configuration, which consists of p-type and n-type semiconductor elements, has been illustrated by Fig.1. Here, the colder and hotter stage with 'Nc' and 'Nh' numbers of TE elements are cascaded with \( N = N_c + N_h \) being the total number of thermocouples. \( r = N_h / N_c \) is the ratio of number of

Figure.1 Two stage series thermoelectric
modules between the two stages, respectively. $T_{cc}$ and $T_{ch}$ denote the temperatures of the cold-side and hot-side of colder end, and $T_{hc}$ and $T_{hh}$ are the temperatures of the cold-side and hot-side of hotter end, respectively.

Certain hypotheses were adopted in modeling of TTEC systems [16, 19-23]

1. The two-stage thermoelectric cooler is operated at a steady-state condition.
2. The Thomson effect in the thermoelectric devices is assumed negligible.
3. Heat losses generated by radiation and convection are not considered.
4. The Seebeck coefficient, thermal conductance and electrical resistance are independent of temperature.
5. Only a one-dimensional model that the heat flows through the element along the length direction of TE leg is considered.

2.1 Spatial-dependent thermal conductivity

In the steady-state condition, Domenicali’s equation [24] for the energy balance and the definition of heat flow $q(x)$ are formulated here in steady state,

$$\frac{d}{dx}[\lambda(x)\frac{dT(x)}{dx}] = -\frac{I^2\rho(x)}{A^2} + \frac{I}{A}T(x)\frac{d\alpha(x)}{dx}$$

$$q(x) = \alpha(x)IT(x) - \lambda(x)\frac{dT(x)}{dx}$$ (1)

Where $A$ is the cross-sectional area, $\alpha$ stands for the Seebeck coefficient, $x$ describes the distance from the cold end, $T(x)$ represents the temperature profile, $I$ is the electric current density.

The study utilizes explicit spatial dependent thermal conductivity with an aim to improve cooling performance. For simplicity, it is assumed that Seebeck coefficient and resistivity are independent of temperature and space, no thermal or electrical contact resistances, and no heat losses. A novel model is proposed that the inhomogeneous materials with the power function form $\lambda(x) = \lambda_0(x/L)^c$. Here, $c$ is a power exponent, and $\lambda_0$ is the initial thermal conductivity at hot end. Note that if $x = 0$ implies $\lambda(0) = 0$, there is no physical meaning, and we may exclude this possibility.

$$\frac{d^2T}{dx^2} + \frac{cx}{L} \frac{dT}{dx} = -\frac{I^2\rho}{\lambda_0(x/L)^c A^2}$$ (3)

Based on Dirichlet boundary condition, the cold ($x = 0$) and hot junction ($x = L$) of the TTEC temperatures are constant. The boundary conditions are expressed as:

$$x = 0 \quad T = T_c$$

$$x = L \quad T = T_h$$ (4)

Where $L$ is the length of the model. Considering the boundary conditions, Eq. 4 yields the expression for the differential of temperature with respect to $x$ with a finite $\lambda(x)$

$$\frac{dT}{dx} = -\frac{I^2\rho x}{\lambda_0(x/L)^c A^2} + (1-c)\frac{T_h-T_c}{x^c L^{1-c}} + \frac{I^2R(1-c)}{\lambda_0(x/L)^c A(2-c)}$$ (5)

The distribution of temperature is determined as

$$T(x) = -\frac{I^2\rho x^2}{(2-c)\lambda_0 A^2} + \left[\frac{T_h-T_c}{L^{1-c}} + \frac{I^2R}{L^{1-c}\lambda_0 A(2-c)}\right]x^{1-c}$$ (6)
Substituting Eq. (5) into Eq. (2), the absorption and released of heat are calculated as

\[ Q_c = \alpha I T_c - K \beta (T_c - T_e) - \omega R I^2 \]

\[ Q_h = \alpha I T_h - K \beta (T_h - T_f) + (1 - \omega) \dot{R} \]  

Where \( K = \lambda p A / L \) denotes the thermal conductivity, \( R = L \rho / A \) is the electrical resistance, \( \beta = 1 - c \) is defined as the normalized thermal conducted by presuming a homogeneous material with \( \lambda p \), \( \omega = (1 - c) / (2 - c) \) represents the distribution of the Joule heat, which flows to the cold end, and \( 1 - \omega \) is the partial Joule heat flowing to the hot end. In the generalized model, \( c \leq 1 \) is required. Note that for \( c = 0 \), this situation presents that the dumping of Joule heat into each end is symmetrical. For \( c = 1, \omega = 0 \) is obtained, which implies the whole of Joule heat is dumped in the hot end, and when \( c \to -\infty \), the Joule heat flowing to the hot end is zero. As shown by Fig. 2, the value of \( \omega \) in the model with inhomogeneous materials lies between 0 and 1. What is more, it is clear that \( \omega \) is a monotonically increasing function of \( c \). Whether \( c \) increases or decreases, it would increase the unevenness of Joule heat distribution.

### 2.2. Analysis of TTEC

For a TTEC considering inhomogeneous thermal conductivity, the heat absorbed at the cold side of the colder stage \( (Q_{cc}) \), the heat released at the hot side of the hotter stage \( (Q_{hh}) \), the heat released at the hot side of colder end \( (Q_{hc}) \) and the heat absorbed at the cold side of hotter end \( (Q_{hc}) \), based on the theory of non-equilibrium thermodynamics, are deduced as follows

\[ Q_{cc} = N_c [\alpha_c I_{cc} - K_c \beta_c (T_{cc} - T_e) - \omega_c R_c I_{cc}^2] \]

\[ Q_{cc} = N_c [\alpha_h I_{cc} - K_h \beta_h (T_{cc} - T_e) + (1 - \omega_h) R_c I_{cc}^2] \]

\[ Q_{hc} = N_h [\alpha_h I_{hc} - K_h \beta_h (T_{hc} - T_e) - \omega_h R_h I_{hc}^2] \]

\[ Q_{hh} = N_h [\alpha_h I_{hh} - K_h \beta_h (T_{hh} - T_h) + (1 - \omega_h) R_h I_{hh}^2] \]

Assume that the materials of both two stages possess the same properties,

\[ \alpha_c = \alpha_h = \alpha \quad R_c = R_h = R \quad K_c = K_h = K \quad c_c = c_h = c \]  

The following conditions are well observed in the electrically series configuration,

\[ U_m = U_c + U_h \quad I_c = I_h = I \]  

Without regard to the heat leakages, the thermal current balance equation is expressed as

\[ Q_{cc} = Q_{hc} \]  

One can presume that there is an intermediate junction temperature, \( T_m \), Hence, \( T_{cc} = T_{hc} = T_m \), the value of \( T_m \) comes out as

\[ T_m = \frac{K \beta_h (N_h T_{hh} + N_e T_{cc}) + (N_e (1 - \omega) + N_h \omega) R I^2}{\alpha I (N_h - N_e) + K (N_e T_{cc} + N_h T_{hh})} \]

The voltage and power output of the colder stage are calculated as

\[ V = V_{cc} = V_{ch} \]  

\[ P_m = \frac{K \beta_h (N_h T_{hh} + N_e T_{cc}) + (N_e (1 - \omega) + N_h \omega) R I^2}{\alpha I (N_h - N_e) + K (N_e T_{cc} + N_h T_{hh})} \]
\[ U_c = \frac{(Q_{hc} - Q_{cc})}{I} \]
\[ = \alpha N_c (T_m - T_{cc}) + N_c RI \]  
\[ P_c = (Q_{hc} - Q_{cc}) \]
\[ = \alpha IN_c (T_m - T_{cc}+) + N_c \hat{R}_i \]  

The voltage and power output of the hotter stage are written as
\[ U_h = \frac{(Q_{hh} - Q_{ha})}{I} \]
\[ = \alpha N_h (T_{hh} - T_m) + N_h RI \]  
\[ P_h = Q_{hh} - Q_{hc} \]
\[ = \alpha IN_h (T_{hh} - T_m) + N_h RI^2 \]  

The voltage \( V_{in} \) and power output \( P_{in} \) of the TTEC model are, respectively, expressed as
\[ U_{in} = \frac{(Q_{hh} - Q_{cc})}{I} \]
\[ = \alpha [N_c (T_m - T_{cc}) + N_h (T_{hh} - T_m)] + (N_h + N_c) RI \]  
\[ P_{in} = (Q_{hh} - Q_{cc}) \]
\[ = \alpha I N_c (T_m - T_{cc}+) + N_h (T_{hh} - T_m) + (N_h + N_c) \hat{R}_h \]  

Coefficient of performance (COP), exergy and exergy efficiency (\( \varepsilon \)) of the TTEC model are determined as follow, respectively [10, 25]:
\[ COP = \frac{Q_{cc}}{P_{in}} \]
\[ Ex_{Qcc} = Q_{cc} (T_o / T_{cc} - 1) \]
\[ \varepsilon_{TTEC} = \frac{Ex_{Qcc}}{P_{in}} \]  

Input parameters for numerical calculation are summarized in Table 1 [14].

<table>
<thead>
<tr>
<th>parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seebeck coefficient, ( \alpha ) [\mu V^{-1}K^{-1}]</td>
<td>200</td>
</tr>
<tr>
<td>electrical resistivity, ( R ) [\Omega m^{-1}]</td>
<td>( 10^{-5} )</td>
</tr>
<tr>
<td>thermal conductivity, ( \lambda ) [Wm^{-1}K^{-1}]</td>
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</tr>
<tr>
<td>length, ( L ) [m]</td>
<td>( 5 \cdot 10^{-3} )</td>
</tr>
<tr>
<td>area, ( A ) [m^2]</td>
<td>( 4 \cdot 10^{-6} )</td>
</tr>
<tr>
<td>cold end temperature, ( T_{cc} ) [K]</td>
<td>260</td>
</tr>
<tr>
<td>hot end temperature, ( T_{hh} ) [K]</td>
<td>300</td>
</tr>
<tr>
<td>ambient temperature, ( T_0 ) [K]</td>
<td>298.15</td>
</tr>
<tr>
<td>number of the colder stage thermocouples, ( N_c )</td>
<td>15</td>
</tr>
<tr>
<td>number of the hotter stage thermocouples, ( N_h )</td>
<td>15</td>
</tr>
</tbody>
</table>

2.3. Validation

The validation of the present model is executed by using the numerical and experimental results of Liu et al. [26] under identical operating conditions, where its hot end is fixed at 343.2K and cooling capacity is set at 5.5W. All the parameters and input data are identical with those reported by Liu et al. Fig. 3 illustrates a comparison of the present model results and the numerical and
experimental results of Liu et al. [26] for \( \text{COP} \) of the TTEC varying with current. A fairly good consistency between the model predictions and experimental results is observed, which means that the one-dimensional model adopted here can provide reliability in the calculations.

Figure. 3 Comparison between the \( \text{COP} \) of the present simulation results and the experimental data

3. Results and Discussions

3.1. Effect of power exponent \( c \)

The power exponent \( c \) plays a significant role in affecting the performance of TTEC. The \( I - Q_{\text{ev}} \), \( I - \text{COP} \) and \( I - \epsilon \) curves of the two-stage TECs with the power exponent \( c = -0.51, 0, 0.31 \) and 0.55 (in order to ensure the step of \( \omega \) is 0.1) are shown in Fig. 4. It is clearly seen from all curves that there exists a maximum cooling capacity and a corresponding current for a given power exponent \( c \). Increasing current improves cooling capacity and consumed electrical power. However, cooling capacity going up is dominant up to the optimal region of current, and then, the consumed power is overcoming, which causes a decrease in cooling capacity. What is more, for different power exponent \( c \), the maximum cooling capacity and corresponding current will be different. Obviously, the peak of the cooling capacity shifts towards right with power exponent \( c \) going up. Hence, for better power exponent \( c \), the maximum cooling capacity is obtained at higher current values. With an increase in power exponent \( c \), more Joule heat has poured at the hot end, and consequently, the cooling capacity increases.

Figure 4. Cooling capacity (a), \( \text{COP} \) (b) and the exergy efficiency (c) versus the electric current for different value \( c \)
In order to illustrate the effect of the Joule heat distribution parameter on the performance of the TTEC more clearly, the curves of $COP$ and exergy efficiency varying with current are plotted. As compared in Fig. 4(b) and Fig. 4(c), $COP$ and exergy efficiency first go up then decrease after attaining an optimum value. For example, the maximum values of $COP$ and exergy efficiency are 1.26 and 0.19 for $c = 0.55$, which is achieved at currents of 0.51A. The maximum of $COP$ and exergy efficiency decrease as the value of power exponent $c$ decreasing. It is obviously shown that $COP$ and exergy efficiency are highest for $c = 0.55$ and lowest for $c = -0.51$. What is more, the maximum $COP$ and exergy efficiency appear at relatively low current in the modeling of TTEC. Fact have been proved that exergy destruction keeps rising with increasement of current as the higher current results in more irreversibilities in the model.

As can be seen from Fig. 4, the maximum cooling capacity, coefficient of performance and exergy efficiency of an inhomogeneous TTEC are 38%, 57% and 52% higher than those of the homogeneous TTEC for inhomogeneous parameter $\omega = 0.4$. We conclude that the Joule heat flowing to the hot end is enhanced when the thermal conductivity in the vicinity of the hot end is larger than that in the vicinity of the cold end, which leads to a larger fraction of the Joule heat flowing towards the hot end, thus enhancing the cooling performance. The physical explanation for this phenomenon is that the existence of explicit spatial dependent thermal conductivity changes the space conversion syndrome.

3.2. The relation between the $COP$ and cooling capacity

Fig. 5 describes the curves of the cooling capacity $Q_{cc}$ as a function of the $COP$ for different values of power exponent $c$. The closed curves going through the origin are presented. There is a negative slope arc segment on which $Q_{cc}$ goes down as $COP$ rises. The optimal regions are situated in the parts of the $Q_{cc} \rightarrow COP$ curves with negative slope, and we can obtain a maximum cooling capacity $Q_{cc,\text{max}}$ whose corresponding coefficient of performance is $COP_m$ and a maximum coefficient of performance is $COP_{\text{max}}$ whose corresponding cooling capacity is $Q_{cc,m}$. To make the TTEC operate at the optimal state, the optimal regions of cooling capacity and coefficient of performance are determined by

$$Q_{cc,m} \leq Q_{cc} \leq Q_{cc,\text{max}}$$

$$COP_m \leq COP \leq COP_{\text{max}}$$

Figure. 5 Cooling capacity $Q_{cc}$ versus $COP$ for different value $\omega$

It is seen clearly that $Q_{cc,\text{max}}$ and $COP_m$ determine upper bounds of the cooling capacity and coefficient of performance, while $Q_{cc,m}$ and $COP_m$ offer the lower boundaries of the optimized the cooling capacity and coefficient of performance. What is more, it is worth noting that the enhancement of performance is more remarkable with the value of $\omega$ decreasing.
3.3. Effects of cold end temperature

Fig. 6 shows that the cooling capacity, \( \text{COP} \) and the exergy efficiency of the two different two-stage TECs designs with three cold end temperatures of 240K, 250K and 260K. For inhomogeneous materials, \( \omega = 0.3 \) is taken as an example, while \( \omega = 0.5 \) is homogeneous materials. Increment in the cold end temperature improves the cooling capacity, \( \text{COP} \) and exergy efficiency for both homogeneous and inhomogeneous TTECs. The reason is that the cooling capacity consumes more power by reducing the cold end temperature in a constant hot end temperature, which brings about lower values of \( \text{COP} \) and exergy efficiency. For all temperature differences, the cooling capacity, \( \text{COP} \) and exergy efficiency of inhomogeneous TTECs are higher than those of homogeneous TTECs. Moreover, based on the changing trend of the three parameters, the variation of cold end temperature has a greater impact on homogeneous TTECs.
3.4. Effects of thermocouples configuration on the model

The cooling capacity $Q_{cc}$ is plotted as a function of the ratio of number of modules in the cold stage to the hot stage $r = N_h / N_c$ for various values of power exponent $c$ in Fig. 7 (a). The trend of the cooling capacity curve is a parabola versus the ratio. There exists an optimum ratio, which contributes to the maximum cooling capacity. Moreover, comparing three curves, it is clearly seen that the cooling capacity are 0.34 for $\omega = 0.4$, 0.22 for $\omega = 0.5$ and 0.12 for $\omega = 0.6$. It is obvious that a lower dimensionless factor $\omega$ gives higher cooling capacity. The variation of COP and exergy efficiency are shown in figure 7 (b) and (c). These curves signify that two functions go up in the initial $r$ till optimal point and thereafter get reduced considerably. That is to say, the ratio of number of modules between the two stages $r$ has a significant effect on the design of TTEC.

4. Conclusions

The one-dimensional model of a two-stage thermoelectric cooler constructed by inhomogeneous material with the spatial- dependent thermal conductivity is developed. The expressions for temperature profile in the thermoelectric material is formulated. In light of a tradeoff between the cooling capacity and COP, the optimum operating regions are determined. The improvement of thermoelectric cooling confirms a fact that the inhomogeneous thermal conductivity conducted spatial dependence form contributes to the asymmetric dissipation of Joule heat when power exponent $c$ is located in the positive region. Thermal rectification can be realized through the modification of the constant properties model which can improve cooling capacity, coefficient of performance and exergy efficiency. In fact, the asymmetric flow of Joule heat in functionally graded thermoelectric materials can be achieved by varying carrier concentrations. The present analysis lays a foundation for the deeper investigation of practical multi-stage thermoelectric refrigeration devices made of inhomogeneous materials.

Nomenclature

- $\beta$ - the normalized thermal conducted, [-] $\beta = 1-c$
- $\omega$ - the distribution of the Joule heat, [-] $\omega = (1-c)/(2-c)$

Subscripts

- $c$ - colder stage
- $ch$ - hot side of colder stage
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