COMMENTS ON "SIGNIFICANCE OF IMPROVED FOURIER-FICK LAWS IN NON-LINEAR CONVECTIVE MICROPOLAR MATERIAL STRATIFIED FLOW WITH VARIABLE PROPERTIES"

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Abstract: The θ(η) and ϕ(η) equations are invalid.

Waqas et al. [1] studied the effectiveness of temperature-dependent thermal conductivity and improved Fourier-Fick fluxes on the 2-D, steady incompressible micropolar material flow past a stretchable surface. The researchers considered non-linear mixed convection, heat generation and double stratification aspects. Waqas et al. [1] presented the energy and concentration equations (Eqs. (4) and (5) in Ref. [1]) as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_1\begin{pmatrix} u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x}+ \\ +2uv\frac{\partial^2 T}{\partial x\partial y}+u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}- \\ -\frac{Q}{\rho c_p}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)\end{pmatrix}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left[K(T)\frac{\partial T}{\partial y}\right]+\frac{Q}{\rho c_p}(T-T_\infty) \quad (1)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\lambda_2\begin{pmatrix} u\frac{\partial u}{\partial x}\frac{\partial C}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial C}{\partial x}+ \\ 2uv\frac{\partial^2 C}{\partial x\partial y}+u^2\frac{\partial^2 C}{\partial x^2}+v^2\frac{\partial^2 C}{\partial y^2}\end{pmatrix}=D\frac{\partial^2 C}{\partial y^2} \quad (2)$$

From the above, concentration (C) and temperature (T) depend on x, y.

Waqas et al. [1] introduced these variables to convert PDEs into ODEs (Eq. (8) in Ref. [1]):

$$\eta = y\sqrt{\frac{c}{\nu}} \tag{3}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_0} \tag{4}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_0} \tag{5}$$

From Eq. (3), the similarity variable (η) depends on y only. From Eq. (4), the temperature θ(η) depends on y only whilst RHS $\frac{T(x,y) - T_\infty}{T_w - T_0}$ depends on x, y. Hence, there is a disagreement between LHS & RHS so that Eq. (4) is invalid. From Eq. (5), the concentration ϕ(η) depends on y only whilst RHS $\frac{C(x,y) - C_\infty}{C_w - C_0}$ depends on x, y. Hence, there is a disagreement between LHS & RHS so that Eq. (5) is invalid.

The same errors were revealed by Pantokratoras [2-5]. As indicated by Pantokratoras [5], the similarity variable (η) was defined by Minkowycz and Sparrow [6] as:

$$\eta = \left[\frac{g\beta(T_w - T_\infty)}{4\nu^2}\right]^{1/4} \frac{y}{x^{1/4}} \tag{6}$$

to agree with their energy equation.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{7}$$

Awad [7] revealed recently the same errors.

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