IMPROVED ISPH METHOD FOR NATURAL CONVECTION FROM HEATED T-OPEN PIPE OF AL$_2$O$_3$-WATER NANOFLUID IN A CAVITY: BUONGIORNO’S TWO-PHASE MODEL

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The unsteady natural convection of Al$_2$O$_3$–water nanofluid form heated open T-pipe inside a cavity has been investigated by ISPH method using non-homogenous two-phase Buongiorno’s model. Different lengths and heights of T-pipe shape are considered. The side walls of the cavity are kept at cool temperature $T_c$ and the horizontal walls are thermally insulated. The Lagrangian description of the controlling governing equations is discretized and solved using improved ISPH method. In this study, ISPH method is improved using kernel renormalization function for boundary treatment plus modification in the source term of pressure Poisson equation (PPE). The source term of PPE contains the velocity divergence plus density invariance multiply by relaxation coefficient. The calculations are performed for variable lengths of T-open pipe ($0.2 \leq L_b \leq 0.6$), variable widths of T-open pipe ($0.02 \leq W_b \leq 0.16$), ($0.02 \leq W_t \leq 0.16$) and variable concentration of nanoparticles volume fraction ($1\% \leq \varphi_{av} \leq 10\%$). The obtained results showed that the maximum values of the stream function are reduced by 80.8% when $\varphi_{av}$ is increased from 1% to 10%. Additionally, as lengths and widths of the T-pipe are raised, the average Nusselt numbers at the vertical walls are enhanced.

Key words: Buongiorno model; Improved ISPH; Nanofluid; Natural convection; Square cavity; T-open pipe.

Nomenclature

- $c_p$: Specific heat capacity ($J \ kg^{-1} \ K^{-1}$)
- $d_f, d_p$: Diameters of $H_2O$ and $Al_2O_3$ molecules,
- $D_B$: Brownian diffusion coefficient
- $D_T$: Thermophoresis coefficient
- $g$: Gravity acceleration ($m \ s^{-2}$)
- $k$: Thermal conductivity ($W \ m^{-1} \ K^{-1}$)
- $K_B$: Boltzmann’s constant ($J/k$)
- $L$: Size of the cavity ($m$)
- $Le$: Lewis number
- $Nu$: Local Nusselt number
- $\bar{Nu}$: Average Nusselt number
- $\tilde{N}_B$: Diffusivity ratio parameter
- $P$: Pressure ($N \ m^{-2}$)
- $Pr$: Prandtl number
- $Ra$: Rayleigh number
- $Sc$: Schmidt number
- $t$: Time ($s$)
There are several applications from studying natural convection in closed cavities. The applications are cooling of the electronically devices, humidification and heat exchangers. In addition, the nanofluid has many uses and contributions in the industry of the cooling systems of engines, fridges and electronics, nuclear plants, solar collectors [1, 2], solar energy [3-5], petroleum industries and lubricants and seals [6]. Mahian et al. [7, 8] introduced fundamental and applications review parts for the modeling and simulations of nanofluid flows. Rashidi et al. [7] summarized the impacts of the nanofluid on the performance of the condensing and evaporating systems. Rashidi et al. [9] introduced an additional review on the improvement of heat transfer using combinations between nanofluid and inserts. In the literature, there are two different models for simulation the nanofluid flows, namely single and two phase models. In the first model, namely single phase model, the fluid and nanoparticles are in thermal equilibrium [10-14]. In experimental studies [15], they reported that the enhancement on heat transfer by nanofluids was significant in the entrance region and the classical Shah equation cannot predict heat transfer of nanofluids. In the second model, namely two phase model, the influences of the Brownian motion, thermophoresis and other interactions between the base fluid and nanoparticles are considered [16, 17]. Buongiorno [18] introduced the non-homogenous model for nanofluids by considering the impacts of Brownian, diffusion and thermophoresis. After Buongiorno, there are numerous studies focused on the two phase model of the nanofluids [19-27]. Dogonchi et al. [28] studied the natural convection in a porous gap filled with nanofluid below magnetic field effects. Dogonchi et al. [29] used CVFEM to study the impacts of a magnetic field on natural convection of nanofluid in a wavy cavity. Rehman et al. [30] studied heat transfer from evenly
heated T-Shaped blade inside a trapezium enclosure. Sheikhzadeh et al. [31] compared between the homogeneous and non-homogenous models of the nanofluids. Moltlagh et al. [32] introduced non-homogenous two-phase Buongiorno’s model of Al₂O₃–water nanofluid inside an inclined cavity. Moltlagh et al. [33] used two-phase model for Fe₃O₄–water nanofluid in an inclined porous semi-annulus. In their studies, the impacts of the nanoparticles diffusion were considered according to the Buongiorno’s model. The smoothed particle hydrodynamics (SPH) method was originally introduced by [34, 35]. Later, SPH method and its incompressible version entitled ISPH have been received several attentions in the several fields [36-47].

Taking into account all the previous works, it is found that the authors focused on the homogeneous models of the nanofluid (one-phase model or two-phase model). Also, the authors considered regular fluid domains. However, the non-homogeneous nanofluid model within complex geometries is not presented. Therefore, the main objective of this study is to examine the flow and thermal fields within an enclosure that is heated from inside by an open T-shaped pipe and is filled by nanofluids. The non-homogenous two-phase Buongiorno’s model is applied to simulate the nanofluid case. The non-dimensionl governing equations are solved numerically using the smoothed particle hydrodynamic (ISPH) method. The main outcomes revealed that distributions of the nanofluid temperature, the average Nusselt number along the inner boundaries and the average Nusselt number at the left wall are supported as widths of the T-pipe channels is increased.

Fig. 1. Initial schematic and particles model of T-open pipe inside a cavity.

1. Mathematical formulations

Consider an unsteady, 2D (two-dimensional) and laminar flow of an incompressible (constant density) nanofluid inside a square enclosure as depicted in Figure 1. The mentioned figure consists of a T-shaped open pipe within a square enclosure. Length of the outer boundary is L while the length of the inner T-pipe channels is denoted by L_b. Channels of the T-shaped pipe have width W_b and W_t. The vertical walls of the enclosure have a relatively low temperature T_c while the horizontal walls are thermally insulated. The temperature differences inside the geometry are due to the uniform temperature condition T_h (T_h ≫ T_c) on the inner T-pipe. Uniformly shapes and equally sizes for the nanoparticles are assumed while the boundary conditions for the nanoparticles volume fractions at the vertical walls are zero mass fluxes. Variations of the density with the temperature are determined using the Boussinesq's approximation. The gravity acceleration is taken in the normal direction. The viscous dissipation and radiation impacts are neglected. The thermal equilibrium model between the base fluid
and the nanoparticles is considered and the thermophysical properties of the base fluid and the nanoparticles volume fraction at $310K$ are included in Table 1. Under all these assumptions, the partial controlling equations in Lagrangian form are described as, see Corcione et al. [19]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\rho_{nf} \frac{du}{dt} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\rho_{nf} \frac{du}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho_{nf} \frac{du}{\partial y}\right),$$

$$\rho_{nf} \frac{dv}{dt} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\rho_{nf} \frac{dv}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho_{nf} \frac{dv}{\partial y}\right) + (\rho \beta)_{nf} g(T - T_c),$$

$$\left(\rho c_p\right)_{nf} \frac{dT}{dt} = \nabla \cdot (k_{nf} \nabla T) + c_{p,p} \left[\rho_p D_B \nabla \varphi + \frac{\rho_p D_T}{T} \nabla T\right] \cdot \nabla T,$$

$$\rho_p \frac{d\varphi}{dt} = \nabla \cdot \left[\rho_p D_B \nabla \varphi + \frac{\rho_p D_T}{T} \nabla T\right].$$

### Table 1. Thermo-physical properties of water and nanoparticles at $310K$.

<table>
<thead>
<tr>
<th></th>
<th>$\left(\frac{kg}{m^3}\right)$</th>
<th>K (W/mK)</th>
<th>$C_p$ (J/kgK)</th>
<th>$\beta \times 10^{-5}$K$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al$_2$O$_3$</td>
<td>3970</td>
<td>40</td>
<td>765</td>
<td>0.85</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>993</td>
<td>0.628</td>
<td>4178</td>
<td>36.2</td>
</tr>
</tbody>
</table>

In the above system, $D_B$ is the Brownian diffusion coefficient and it is expressed as:

$$D_B = \frac{K_B T}{3 \pi \mu_f d_p}$$

where $K_B = 1.380648 \times 10^{-23}$J/k is the Boltzmann's constant, $\mu_f = 695 \times 10^{-6}$kg/ms is the dynamic viscosity of water and $d_p = 33$ nm is the diameter of the alumina molecule. In addition, $D_T$ refers to the thermophoresis coefficient that is given by:

$$D_T = 0.26 \frac{k_f}{2k_f + k_p \rho_f T} \varphi$$

Further, the thermophysical properties of the nanofluid are assumed to be functions in the nanoparticle volume fraction and the diameters of H$_2$O and Al$_2$O$_3$ molecules, $d_f$ and $d_p$ based on the experimental results of Corcione [48]; those are expressed as:

$$\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_p,$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$\left(\rho c_p\right)_{nf} = (1 - \varphi)\left(\rho c_p\right)_f + \varphi\left(\rho c_p\right)_p$$

$$\left(\rho \beta\right)_{nf} = (1 - \varphi)\left(\rho \beta\right)_f + \varphi\left(\rho \beta\right)_p$$

$$\mu_{nf} = \mu_f / \left(1 - 34.87 \left(d_p/d_f\right)^{-0.3} \varphi^{1.03}\right)$$

$$k_{nf} = k_f \left(1 + 4 A (Re_B)^{0.4} \Pr^{0.66} \left(\frac{T}{T_{ff}}\right)^{10} \left(\frac{k_p}{k_f}\right)^{0.03} \varphi^{0.66}\right)$$

$$Re_B = \rho_f u_B d_p / \mu_f$$

$$u_B = 2K_B T / (\pi \mu_f d_p^2)$$
where, $T_f$ is the freezing point of the base fluid, $u_B$ is the Brownian velocity. Here, the values of $d_f$ is assumed to be 0.385 nm.

The following dimensionless quantities are introduced:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \tau = \frac{t \alpha_f}{L^2}, \quad U = \frac{u L}{\alpha_f}, \quad \alpha_f = \frac{k_f}{(\rho C_p)_f},$$

$$V = \frac{v L}{\alpha_f}, \quad P = \frac{p L^2}{\rho \alpha_f}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \phi^* = \frac{\phi}{\phi_{avg}}, \quad D_B^* = \frac{D_B}{D_{Bo}^{*}}, \quad D_T^* = \frac{D_T}{D_{To}^{*}}, \quad \delta = \frac{T_c}{T_h - T_c}, \quad Ra = \frac{\beta_f (T_h - T_c) L^3}{\alpha_f v_f^2}$$

The following dimensionless quantities are introduced:

Then, the partial governing equations in Lagrangian and dimensionless forms are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

$$\frac{\rho_f}{\rho_f} \frac{dU}{dt} = \frac{\rho_f}{\rho_f} \frac{\partial P}{\partial X} + \frac{\partial}{\partial Y} \left( \frac{Pr \mu_{nf} \partial U}{\mu_f} \right) + \frac{\partial}{\partial Y} \left( \frac{Pr \mu_{nf} \partial U}{\mu_f} \right),$$

$$\frac{\rho_f}{\rho_f} \frac{dV}{dt} = \frac{\rho_f}{\rho_f} \frac{\partial P}{\partial Y} + \frac{\partial}{\partial X} \left( \frac{Pr \mu_{nf} \partial V}{\mu_f} \right) + \frac{\partial}{\partial Y} \left( \frac{Pr \mu_{nf} \partial V}{\mu_f} \right) + \frac{(\rho \beta)_{nf}}{\rho_f} Pr Ra \theta,$$

$$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \frac{d\theta}{dt} = \frac{\partial}{\partial X} \left( \frac{K_{nf}}{K_f} \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \frac{K_{nf}}{K_f} \frac{\partial \theta}{\partial Y} \right)$$

$$+ \frac{1}{Le} \left[ D_B^* \left( \frac{\partial \phi^*}{\partial X} \frac{\partial \theta}{\partial X} \right) + D_T^* \left( \frac{\partial \phi^*}{\partial Y} \frac{\partial \theta}{\partial Y} \right) \right] \frac{\partial \theta}{\partial Y}$$

$$d \phi^* = \frac{Pr}{Sc} \frac{\partial}{\partial X} \left( D_B^* \frac{\partial \phi^*}{\partial X} \right) + \frac{Pr}{Sc} \frac{\partial}{\partial Y} \left( D_B^* \frac{\partial \phi^*}{\partial Y} \right) + \frac{Pr}{Sc} \frac{\partial}{\partial X} \left( \frac{D_T^* \frac{\partial \theta}{\partial X}}{N_{BT} \left( 1 + \frac{\theta}{\delta} \right)} \right)$$

$$+ \frac{Pr}{Sc} \frac{\partial}{\partial Y} \left( \frac{D_T^* \frac{\partial \theta}{\partial Y}}{N_{BT} \left( 1 + \frac{\theta}{\delta} \right)} \right),$$

where, $Sc = \nu f / \mu f$ is the Schmidt number, $Le = \frac{K_f}{\rho C_p \phi_{avg} \phi_{Bo}}$ is Lewis number and $N_{BT} = \frac{\phi_{avg} D_{Bo} \phi_{Bo}}{D_{Bo (T_h - T_c)}}$ is diffusivity ratio parameter

On the side-walls,

$$U = 0; \quad V = 0; \quad \theta = 0; \quad \frac{\partial \phi^*}{\partial X} = 0; \quad \frac{\partial \phi^*}{\partial Y} = 0; \quad \frac{\partial \theta}{\partial Y} = 0,$$

On the horizontal-walls,

$$U = 0; \quad V = 0; \quad \frac{\partial \phi^*}{\partial Y} = 0; \quad \frac{\partial \theta}{\partial Y} = 0,$$

On the inner T-pipe shape,

$$\theta = 1, \quad \frac{\partial \phi^*}{\partial n} = - \frac{D_f^*}{D_B^*} \frac{1}{N_{BT} \left( 1 + \frac{\theta}{\delta} \right)} \frac{\partial \theta}{\partial n}$$

The local and average Nusselt numbers are defined as:

On the inner T-pipe:

$$\overline{N_u} = - \frac{K_{nf}}{k_f} \frac{1}{S} \int_{0}^{S} \frac{\partial \theta}{\partial n} \left| S \right| dS,$$

On left wall of a cavity:

$$\overline{N_u} = - \frac{K_{nf}}{k_f} \int_{0}^{L} \frac{\partial \theta}{\partial X} \left| X = 0 \right| dY$$

where $S$ is total interior shape length and $n$ is normal vector.
2. ISPH method

In this section, the solving technique of the partial controlling equations with formulation of ISPH method were discussed in details.

First: Projection scheme \cite{49} is used as a solving technique for the non-dimensional forms:

Initially, the intermediate velocities will be calculated as:

\[ U^* = U^n + \Delta t \rho_t \frac{\rho_t}{\rho_{nt}} \left( \frac{\partial}{\partial X} \left( \frac{\mu_{nt}}{\mu_f} \frac{\partial U}{\partial X} \right)^n + \frac{\partial}{\partial Y} \left( \frac{\mu_{nt}}{\mu_f} \frac{\partial U}{\partial Y} \right)^n \right), \]

\[ V^* = V^n + \Delta t \rho_t \frac{\rho_t}{\rho_{nt}} \left( \frac{\partial}{\partial X} \left( \frac{\mu_{nt}}{\mu_f} \frac{\partial V}{\partial X} \right)^n + \frac{\partial}{\partial Y} \left( \frac{\mu_{nt}}{\mu_f} \frac{\partial V}{\partial Y} \right)^n + \frac{(\rho_\beta)_n}{(\rho_\beta)_f} \Delta \theta^n \right). \]

The pressure Poisson equation (PPE):

\[ \nabla^2 p^{n+1} = \frac{1}{\rho_{nt}} \frac{1}{\Delta t} \left( \frac{\partial (\rho_{nt} U^*)}{\partial X} + \frac{\partial (\rho_{nt} V^*)}{\partial Y} \right) + \alpha \frac{\rho_f - \rho_{num}}{\Delta t^2} \]

where, \( \alpha \): \((0 \leq \alpha \leq 1)\) is a relaxation coefficient and \( \rho_{num} \) is the numerical density and it calculated from SPH approximation as:

\[ \rho_{num} = \sum_i m_j W_{ij} \]

Here, \( m_j \) refers to the mass of neighbor particle \( j \) and \( W_{ij} \) is the kernel function (Eq. 36) and \( i \) is the target particle.

Hence, the final velocity is calculated as:

\[ U^{n+1} = U^* - \Delta t \left( \frac{\partial P}{\partial X} \right)^{n+1} \]

\[ V^{n+1} = V^* - \Delta t \left( \frac{\partial P}{\partial Y} \right)^{n+1} \]

The temperature and nanoparticle volume fraction are calculated as:

\[ \theta^{n+1} = \theta^n + \Delta t \left( \frac{\rho C_p}{\rho C_p}_{nt} \right) \left( \frac{\partial}{\partial X} \left( \frac{K_{nt}}{K_f} \frac{\partial \theta}{\partial X} \right)^n + \frac{\partial}{\partial Y} \left( \frac{K_{nt}}{K_f} \frac{\partial \theta}{\partial Y} \right)^n \right) + \frac{1}{Le} \left( D_B \left[ \frac{\partial \phi^*}{\partial X} + \frac{\partial \phi^*}{\partial Y} \right]^n + \frac{D_B}{N_{BT}} \left[ \left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 \right] \right)^n \]

\[ \varphi^{n+1} = \varphi^n + \Delta t \left( \frac{Pr}{Sc} \frac{\partial}{\partial X} \left( \frac{D_B}{Sc} \frac{\partial \varphi^*}{\partial X} \right)^n \right) + \frac{Pr}{Sc} \frac{\partial}{\partial Y} \left( \frac{D_B}{Sc} \frac{\partial \varphi^*}{\partial Y} \right)^n + \frac{Pr}{Sc} \frac{\partial}{\partial X} \left( \frac{D_T}{N_{BT}} \frac{\partial \theta}{\partial Y} \right)^n \]

The positions of the fluid particles are updated at each time step as:

\[ X^{n+1} = X^n + \Delta \theta U^{n+1}, \]

\[ Y^{n+1} = Y^n + \Delta \theta V^{n+1} \]

Shifting technique \cite{50, 51} for preventing the particles disorders are:

\[ F_{i'} = F_i + \nabla F_i \cdot \Delta r_{ii'} + O(\Delta r_{ii'}^2), \]

\[ \Delta r_{ii'} = -D \nabla C_i. \]

where, \( F \) is any hydrodynamic function and \( \nabla C_i \) is gradient of particle concentration.
2.1. ISPH formulation

In the current simulations, a quantic kernel function \( W \) was used:

\[
W(q, h) = \frac{3}{16 \pi h^2} \begin{cases} 
(2 - q)^5 - 16(1 - q)^5 & 0 \leq q \leq 1 \\
(2 - q)^5 & 1 < q \leq 2, \\
0 & q > 2
\end{cases}
\]  

(36)

where, \( q = r_{ij}/h \). In this study, the main concept of SPH approximation using renormalization factor \( \gamma_i \) is expressed as:

\[
\langle f(r_i) \rangle = \frac{1}{\gamma_i} \sum_j m_j f(r_j) W(r_{ij}, h).
\]

(37)

The description of renormalization factor \( \gamma_i \) and its first derivative according to [52-53] are defined as follows:

\[
\gamma_i = \int_{\Omega_i} W(|r_{abl}|) d\Omega(r_j),
\]

(38)

\[
\nabla \gamma_i^e = - \int_{\Omega} n(r_j) W(r_{ij}) d\Gamma(r_j).
\]

(39)

The ISPH discretization of the solving technique (Eqs. (25) - (31)) has been discussed in details during our previous studies [52-54].

2.2. Treatment of boundary particles in ISPH method

In this study, the kernel renormalization function was adopted for the wall boundary treatment. This boundary treatment was introduced and validated for several applications by Nguyen et al. [48-50]. In renormalization wall boundary treatment, no need for extra dummy particles around the boundary wall as seen in figure 2. Another advantage of this treatment appears in applying Neumann boundary conditions without any special treatment as it appears directly in ISPH discretization. Fig. 2 (a) shows the schematic diagram of the renormalization wall boundary. Here, combined kernel renormalization function \( \gamma_i \) with the impact of boundary particles were used for treating renormalization wall boundary. In addition, Fig. 2 (b) shows the initial schematic diagram of dummy wall boundary particles. The dummy wall boundary considers an efficient way to treat the rigid wall boundary. The several layers of dummy boundary particles reduce the error of truncated kernel function.

Fig. 2 Schematic diagram of (a) Renormalization wall boundary and (b) Dummy wall boundary

3. Results and discussion

Fig. 3 shows the valuable comparisons for the implementation of the conditions of the nanoparticles volume fraction. The test consists of the streamlines, isotherms and nanoparticles volume fraction obtained in this study and those of Corcione et al. [19] at \( Ra = 3.37 \times 10^5 \) and \( \phi = 0.04 \). It is found that the results show excellent agreements with the previous published results.
In figure 4, impacts of the T-shaped pipe lengths $L_b = 0.2, 0.4, 0.5, 0.6$ on the isotherms contours are illustrated. There is an isothermal zone is noted around the inner shaped while the less-temperature zones occur near the vertical walls. In addition, the increase in lengths of the T-shaped pipe causes an extra heat generation inside the geometry and hence distributions of the temperature inside the flow domain are supported. In the same context, figure 5 shows the influences of the T-pipe length ($L_b = 0.2, 0.4, 0.5, 0.6$) on the streamlines contours at $Ra = 10^4$, $W_b = 0.08$, $W_t = 0.06$ and $\phi_{avg} = 0.03$. Here, it should be mentioned that the increase in lengths of the T-shaped pipe causes that the geometry be complicated and consequently the nanofluid flow is seen inactive, particularly, at the top part of the domain. Also, it is observed that the nanofluid flow is decreased when $L_b$ is growing from 0.2 to 0.4, however, more increase in $L_b$ causes that the nanofluid flow is concentrated at the lower part of the enclosure with an increase in the maximum values of the stream function.

Fig. 3. Comparison of (a) streamlines, (b) isotherms and (c) nanoparticle distribution at $Ra = 3.37 \times 10^5$ and $\phi = 0.04$.

Fig. 4. Impacts of T-pipe length ($L_b = 0.2, 0.4, 0.5, 0.6$) on the isothermal contours at $Ra = 10^4$, $W_b = 0.08$, $W_t = 0.06$ and $\phi_{avg} = 0.03$. 

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Fig. 5. Impacts of T-pipe length ($L_b = 0.2, 0.4, 0.5, 0.6$) on the streamlines contours at $Ra = 10^4$, $W_b = 0.08$, $W_t = 0.06$ and $\phi_{avg} = 0.03$.

Fig. 6 displays the streamlines contours for variations of the T-pipe width $W_b = W_t = 2, 6, 10, 16$ cm at $Ra = 10^4$. During these computations, value of $\phi_{avg}$ is fixed at 0.03. The figure disclosed that the thermal zone along the inner shape is enhanced as the width of the T-pipe channels are enhanced due to the increase in the temperature generation inside the flow domain. Additionally, distributions of the nanofluid temperature as well as the thermal boundary layers along the vertical walls are supported as $W_b$ and $W_t$ are varied. On the other hand, Figure 7 reveals that the low values of width of the T-pipe mean that the T-pipe channels are narrow therefore in this case the nanofluid flow is seen to be weak, particularly, at the top part of the enclosure. However, as width of the T-pipe channels are increased, the nanofluid inters inside the channels and hence there are streamlines are seen at the upper part of the domain. In the same context, the higher value of the stream function ($|\psi|_{max} = 0.356$) is noted at $W_b = W_t = 2$ cm while it’s low value is ($|\psi|_{max} = 0.325$) and it is noted at $W_b = W_t = 10$ cm.

Fig. 6. Impacts of T-pipe width ($W_b = W_t = 2, 6, 10, 16$) on the isothermal contours at $Ra = 10^4$, $L_b = 0.6$ and $\phi_{avg} = 0.03$.

Fig. 7. Impacts of T-pipe width ($W_b = W_t = 2, 6, 10, 16$) on the streamlines contours at $Ra = 10^4$, $L_b = 0.6$ and $\phi_{avg} = 0.03$. 

$|\psi|_{max} = 0.356$ $|\psi|_{max} = 0.337$ $|\psi|_{max} = 0.325$ $|\psi|_{max} = 0.331$
In Figure 8, impacts of the nanoparticles volume fraction on the isothermal contours at $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$ are checked. The results demonstrated that the increase in the average values of the nanoparticles volume fraction $\phi_{avg}$ enhanced the thermal conductivity of the liquid and thus the thermal boundary layers near the inner shape and along the vertical walls are increased. Fig. 9 displays effects of average of the nanoparticles volume fraction $\phi_{avg}$ on distributions of the normalized nanoparticle volume fractions $\phi^*$ at $Ra = 10^4$, $W_b = 0.08$ and $W_t = 0.06$. Based on the inverse relation between $\phi_{avg}$ and $\phi^*$, it is clear that the increase in $\phi_{avg}$ causes a reduction in distributions of $\phi^*$. Also, it is seen that there are contours of $\phi^*$ inside the horizontal channel of the T-pipe and this is due to loading the nanoparticles by the fluid during the flow. However, the viscosity of the mixture is enhanced as $\phi_{avg}$ is varied from 0.01 to 0.1 and consequently there is a deactivation in the nanofluid flow is observed as $\phi_{avg}$ increases. This behavior is presented, clearly, in Figure 10. In this figure the maximum values of the stream function is reduced by 80.8% when $\phi_{avg}$ is varied from 1% to 10%.

Fig. 8. Impacts of the average value of the nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on the isothermal contours at $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$.

Fig. 9. Impacts of average value of nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on nanoparticles volume fraction contours $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$.

Fig. 10. Impacts of the average value of the nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on nanoparticles volume fraction contours $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$. 

Fig. 10. Impacts of the average value of the nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on nanoparticles volume fraction contours $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$. 

Fig. 10. Impacts of the average value of the nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on nanoparticles volume fraction contours $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$. 

Fig. 10. Impacts of the average value of the nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on nanoparticles volume fraction contours $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$. 

Fig. 10. Impacts of the average value of the nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on nanoparticles volume fraction contours $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$. 

Fig. 10. Impacts of the average value of the nanoparticles volume fractions ($\phi_{avg} = 0.01$, $0.03$, $0.05$, $0.1$) on nanoparticles volume fraction contours $Ra = 10^4$, $L_b = 0.6$, $W_b = 0.08$ and $W_t = 0.06$.
on the streamlines contours \( Ra = 10^4, L_b = 0.6, W_b = 0.08 \) and \( W_t = 0.06 \).

In this study, the average Nusselt numbers are calculated along the inner T-Pipe and at the left wall of the enclosure. Behaviors of this coefficient under effects of the controlling parameters, namely, lengths of the T-pipe channels \( L_b \), width of the T-pipe channels \( W_b \) and \( W_t \) and the average values of the nanoparticles volume fraction are presented in figures 11-13. Fig. 11 shows that the average Nusselt number along the inner shape boundaries is reduced as \( L_b \) is increased while the increase in \( L_b \) causes that the temperature gradients near the vertical walls are enhanced and thus the average Nusselt number is increased due to the increase in the temperature differences. Also, figure 12 reveals an enhancement in the average Nusselt number along the inner shape and at the left wall as width of the T-pipe is increased due to the support in the thermal boundary layers along the domain boundaries. In the same context, figure 13 shows that the increase in the average values of the nanoparticles volume fraction results in an enhancement in the thermal conductivity of the liquid and consequently the average Nusselt number at either the inner boundaries or the outer boundaries is increased as \( \phi_{avg} \) is growing.

Fig. 11. Average Nusselt number on T-pipe and left cavity wall under the impacts of T-pipe length \( (L_b = 0.2, 0.4, 0.5, 0.6) \) at \( Ra = 10^4, W_b = 0.08, W_t = 0.06 \) and \( \phi_{avg} = 0.03 \).

Fig. 12. Average Nusselt number on T-pipe and left cavity wall under the impacts of T-pipe width \( (W_b = W_t = 0.2, 0.4, 0.5, 0.6) \) at \( Ra = 10^4 \), and \( \phi_{avg} = 0.03 \).
Fig. 13. Average Nusselt number on T-pipe and left cavity wall under the impacts of T-pipe width \((W_b = W_t = 0.2, 0.4, 0.5, 0.6)\) at \(Ra = 10^4\), and \(\phi_{avg} = 0.03\).

4. Conclusions

Numerical study based on the incompressible smoothed particles hydrodynamic (ISPH) method has been conducted for the nanofluid and thermal fields within enclosures containing open T-shaped pipes and are filled by \(\text{Al}_2\text{O}_3-\text{H}_2\text{O}\) nano-mixture. The non-homogenous Buongiorno’s two-phase model is applied to simulate the nanofluid transport. Non-fixed lengths and widths for the inner T-pipe were considered while the thermophysical properties of the nanofluid were assumed to be functions in diameters of the base fluid and nanoparticles molecules. During the investigation of this physical situation, the following main results are summarized:

- An increase in lengths of the T-pipe channels enhances the temperature distributions as well as the average Nusselt number at the left wall.
- Distributions of the nanofluid temperature, the average Nusselt number along the inner boundaries and the average Nusselt number at the left wall are supported as widths of the T-pipe channels is increased.
- The increase in the average values of the nanoparticles volume fraction causes an enhancement in the thermal boundary layers while a deactivation in the nanofluid flow is noted.
- Both the average Nusselt number along the inner boundaries and at the left wall are boosted when the average values of the nanoparticles volume fraction is growing.

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References


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