COMPARISON OF MATHEMATICAL MODELS TO ESTIMATE THE THERMAL CONDUCTIVITY OF TiO_2-WATER BASED NANOFLUID
A Review

by

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Heat transfer is a desirable phenomenon in many industries such as in refrigeration, transportation, power generation, cell preservation, incubator, metallurgy and material processing, health services, etc. Different types of fluids like water, oil, ethylene glycol etc. are being used as a heat transfer medium. Water is a commonly used as working fluid for transfer of heat. Nanofluids are developed by adding nanosized particle(s) in existing fluid to improve the heat transfer rate. Thermal conductivity of the nanofluid is an important parameter in estimation of heat transfer rate. Different types of mathematical models were developed by various investigators to predict the thermal conductivity of the nanofluids. In this review paper, the theoretical and mathematical model(s) have been compared to predict the thermal conductivity of nanofluids. The experimental data have been collected from literature and compared with Maxwell model, Hamilton and Crosser model, Maxwell-Garnetts model, Pak Cho model, Timofeeva et al. model, Li and Peterson model, Bhattacharya et al. model, respectively in detail. It has been observed that the prediction with the help of the mathematical models is good when the value of volume fraction was less than 0.01.

Key word: *nanofluid, thermal conductivity, mathematical model,* TiO_2*-water*

Introduction

Enhancement in the heat transfer rate is one of the most desirable properties in the current scenario. Nanofluid is one of the alternative method to enhance the heat transfer rate and mitigate the energy requirement. Nanofluid can be defined as the uniform dispersion of nanomaterial(s) (nanoparticles, nanofibers, nanotubes, nanowires, nanorods, nanosheets, or droplets) [1, 2] in the conventional fluid (water, ethylene-glycol or oil) [3]. Heat exchanger are widely used in order to remove heat from a system [4]. There are many methods to optimize heat exchanger efficiency like modification in design, material selection, twisted tape and working fluid replacement [2, 5]. This review paper focusses on the prediction of thermal conductivity for TiO_2-water base nanofluid using different models.

Miniaturization of devices is one of the most important requirement nowadays. Many studies are going on in order to reduce size of the thermal devices. Awais *et al*. [6] reviewed a paper on compact heat exchanger design, in which the main focus was on fin spacing, waffle height and colburn effect. Gulfam *et al*. [7] studied compact thermal management system with

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phase change material. Carneiro *et al*. [8] studied compact refrigeration system for thermal management. Most important way to achieve enhancement in heat transfer rate is by decreasing area to volume ratio of system [4]. Miniaturization of thermal system is important because of these parameters such as space limitation, material saving, cooling requirement of microelectronics system and easy to handle [4, 9, 10].

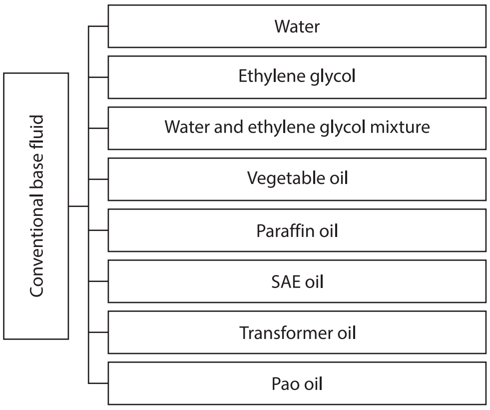

Figure 1. Base fluid used in different studies of nanofluid [12]

Nanofluid

Dispersion of nanosize material in base fluid known as nanofluid. The primary objective of Nanofluid is to enhance heat transfer rate at lowest possible volume concentration preferably less than 1% with uniform dispersion of nanoparticle size less than 10 nm [11]. However nanoparticle size up to 100 nm can used in preparation of nanofluid. Different types of base fluid used in different studies has been shown in fig. 1.

Nanofluid preparation methods

Synthesis of nanofluid can be done by two methods

One step method

In one step method (single step method), nanofluids are synthesised by direct evaporation of nanoparticle (by physical vapour deposition or chemical vapour deposition) and condensation of nanoparticle in base fluid to produce nanofluid [13-16]. The direct condensation and evaporation, laser ablation and submerged arc nanoparticle synthesis system methods are introduced in which nanoparticle are prepared with mechanical technology and dispersed in base fluid to get the desired stability and uniform dispersion of nanofluid [17]. One step method advantages are low agglomeration of nanoparticle and no drying and dispersion of nanoparticle while disadvantages are it cannot be produce in large quantity.

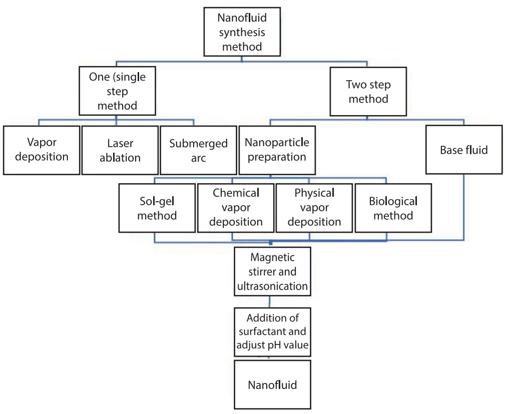

Figure 2. Method of nanofluid preparation

Two-step method

Synthesis of nanofluid on a large-scale is possible in this method. In this method, two step were used [18]. In first step nanoparticle is prepared with the mechanical or chemical process. Then these nanoparticles are mixed with the base fluid with the help of ball milling, high shear mixing, magnetic stirrer, ultrasonic vibration [19, 20]. Ultra sonication is one of the most preferred technique to increase stability of nanofluid. Nanofluid prepared by two step method with oxide particle are more stable than metallic nanoparticle [17]. Surfactant can improve stability time of nanofluid [21]. Gao *et al.* [22] studies six surfactant (APE10, CTAB, OP-10, SDBS, SDS, and TTAB) for CNT in which APE-10 base nanofluid stability was maximum.

Factors affecting heat transfer rate of nanofluid

Heat transfer rate of nanofluid was affected by many factors like thermal conductivity of base fluid, thermal conductivity of nanoparticle, volume fraction, particle size, particle shape, temperature, Brownian motion, interfacial layer, surface charge or pH value, dispersion technique, phonon-transport, the effect of clustering and agglomeration*, etc.*

Heat transfer rate is directly proportional to the thermal conductivity of the nanofluid. Thermal conductivity of nanofluid primarily depends upon the thermal conductivity of the base fluid and nanoparticles. If thermal conductivity of base fluid and nanoparticle is high then thermal conductivity of nanofluid will also be high [23].

Volume fraction is also directly proportional to the heat transfer rate up to a certain limit. Optimum limit is still unknown but maximum of 1% volume fraction is preferred [11]. Muthoka *et al.* [24] studied the effect of particle size on nanofluid and found that a maximum of 40% enhancement in viscosity and 37% decrement in viscosity is possible by altering the size of nanoparticle. Higher viscosity of nanofluid enhances pumping power which results in a decrease in efficiency of the system [25].

Shen *et al*. [26] studied different types of shapes (viz. spherical, hexahedron, tetrahedron, column, and lamina) of nanoparticles used in preparation of nanofluid. Spherical shaped nanoparticle-based nanofluid shows the best enhancement in heat conduction and gives minimum value of Nusselt number. Maheshwary *et al*. [27] studied TiO_2-water based nanofluid with particle shapes of different types viz. spherical, cubic, and rod. Cubic shape TiO_2-water-based nanofluid shows the maximum enhancement in thermal conductivity and viscosity. Synthesis of cubic shape particle is relatively difficult and costly. Spherical shape-based nanofluid result in minimum clogging and more stability. Reduction in particle size result in enhancement in thermal conductivity. Campos *et al*. [28] studied spherical and non-spherical based nanofluid for direct absorption solar collector. This paper shows that enhancement in thermal conductivity of non-spherical shape nanoparticle is relatively more than spherical nanoparticle.

Pavlović *et al*. [29] conducted study on TiO_2 based nanofluid and found that thermal conductivity increases with increasing temperature. Trend of increasing thermal conductivity of base fluid and nanofluid is approximately the same [30-34]. Duangthongsuk *et al*. [35] found decrement in the effective thermal conductivity ratio with increase in temperature. The nanofluid thermal conductivity increases with increasing temperature, but effective thermal conductivity ratio may increase, decrease or remain constant with increasing temperature.

In nanofluid(s) random motion of the particles is known as the Brownian motion [35]. Many studies claim that Brownian motion is the most important parameter in nanofluid. Lin *et al*. [36] studied the Brownian motion effect for Cu based nanofluid. A numerical simulation was done with Maxwel model, and it was compared with the traditional model. Both the model shows enhancement in heat transfer rate. Mittal *et al*. [37] studied the Brownian motion effect on the heat transfer rate for CuO-water based nanofluid. Decrement in Nusselt number was found with increment of Brownian motion. The values of Nusselt number, volume fraction, Prandtl and Reynolds numbers found to decrease with increase in Brownian motion. Reddy *et al*. [38] found that Brownian motion increases temperature of the nanofluid. Esfandiary *et al*. [39] studied Brownian motion effect on Al_2O_3-waterbased nanofluid with two-phase modelling. The predicted result was closer to the experimental results and he claimed that single-phase approach for modelling is not completely correct. It was also observed that Brownian motion decreases with increasing Rayleigh number and diameter of nanoparticle, and Brownian motion became negligible with high value of Rayleigh number and diameter of nanoparticle. Hayat *et al.* [40] studied the effect of Brownian motion on Carreau nanofluid, generalized nanofluid [41], and observed that Brownian motion and thermophoresis play a key role in the enhancement of heat transfer rate. Babu *et al*. [42] studied thermophoresis and Brownian motion effect for Cu and CuO-water based nanofluid. In this study Cu-water based nanofluid heat transfer rate was superior than CuO-water based nanofluid. Haddad *et al*. [43] studied thermophoresis and Brownian motion effect for heat transfer on nanofluid for natural-convection. Two cases were studied with and without thermophoresis and Brownian motion effect. It was observed that the enhancement in heat transfer rate with thermophoresis and Brownian motion is higher. It was also observed that Brownian motion and thermophoresis effect is relatively more when volume fraction is less.

Interfacial effect on the thermal conductivity has been studied in the last few decades, and this factor considers as the second most important parameter other than Brownian motion in nanofluid. Interfacial layer can be defined as a layer formed by fluid molecule around the solid particle. Khodayari *et al*. [44] studied interfacial layer resistance (kapitza resistance) effect and nanolayer. It has been reported that the effect of interfacial layer resistance is much higher than the nanolayer effect. It has been concluded in the study that for better prediction of the thermal conductivity of nanofluid, it is important to calculate accurate interfacial layer resistance and nanolayer. Wang *et al*. [45] studied thermal conductivity of the interfacial layer for Cu-liquid argon based nanofluid. Nanoparticle have diameter of 1 nm. It has been reported that the effect of the interfacial layer in the prediction of thermal conductivity can be neglected. Serebryakova *et al*. [46] studied influence of interfacial layer on the thermal conductivity. Hollow γ-Al_2O_3-water, ethylene glycol, isopropyl alcohol-based nanofluid has been considered with 2.2% volume fraction. It has been reported that the effect of interfacial layer in the prediction of thermal conductivity is remarkable. Most of the studies concluded interfacial layer as an important parameter in order to estimate thermal conductivity of nanofluid.

Lee *et al*. [47] studied surface charge effect on prediction on thermal conductivity of nanofluid. It was observed that surface charge is the most important parameter and it value must be high for enhancement of thermal conductivity. It has been also reported that if pH value goes far away from isoelectric point then stability of the nanofluid decreases. Gowda *et al*. [48] reported that the dispersion of the nanoparticle in base fluid could be improved by high surface charge or low pH value. Jung *et al*. [49] studied surface charge effect for nanofluid and observed that interparticle interaction generated by the surface charge is the responsible factor in prediction of thermal conductivity of nanofluid. Konakanchi *et al*. [50] studied pH effect for colloidal suspension of Al_2O_3, SiO_2, and ZnO nanoparticle with propylene and water mixture. It was also observed that the pH decreases with increase in temperature and pH increases with increase of volumetric concentration, larger particle size. Goudarzi *et al*. [51] studied pH variation effect on the thermal conductivity of nanofluid for solar collector application. It was observed that the 52% enhancement in efficiency when pH changes from 3-10.5 for CuO-water nanofluid and 64.5% enhancement in efficiency when pH changes from 9.2-10.5 for Al_2O_3.

Dispersion technique is one of the most important parameters which can affect other parameters. Siddiqui *et al.* [52] studied Cu-Al_2O_3-water based nanofluid. Stability of Cu based nanofluid was poor, but Al_2O_3 based nanofluid have good stability. Hybrid nanofluid of these two particle result in improved dispersion stability relative to the Cu-water based nanofluid. Gao *et al.* [23] studied dispersion mechanism with CNT-vegetable based nanofluid. Surfactant used in these studies for enhancement in stability are APE-10, OP-10, SDBS, SDS, CTAB, TTAB. Dispersion stability and viscosity with Surfactant APE-10 was better than other surfactants. Li *et al.* [53] studied the ultrasonication time effect for Cu-EG nanofluid. At the beginning, viscosity decreases rapidly but after some time viscosity increases, it means there is optimum ultrasonication time for nanofluid.

Thermal conductivity of nanofluid can also be affected by ballistic phonon-motion. Iacobazzi *et al.* [54] studied phonon-effect for Al_2O_3-water based nanofluids. Ballistic phonon-motion within the nanoparticle happens when the dimension of the particle is less than the mean free path in which phonon-travels. Phonon-effect can be neglected in nanofluid in which nanoparticle size greater than 35 nm [54, 55].

Daviran *et al.* [56] studied the effect of clustering and Brownian motion effect on the thermal conductivity of the nanofluid. It has been reported that Brownian motion does not play a significant role in thermal conductivity of nanofluid. Theoretical and experimental result shows that aggregation and cluster play a significant role in nanofluid heat transfer performance.

It can be concluded that every factor plays a significant role in prediction of thermal conductivity. Some studies reported that few factors have negligible effect, but most of them have the same view to consider all other parameter(s).

Material selection

For our study, we focused on TiO_2-water based nanofluid. Most of the studies are going on TiO_2-water based nanofluid because of its soluble nature in water, chemical stability, non-toxicity, sensational dispersitivity [30, 57, 58]. Most of the research reported a significant amount of enhancement in heat transfer rate with TiO_2-water based nanofluid. Properties of TiO_2 nanoparticle and water has been shown in tab. 1.

Table 1. Properties of TiO_2 nanoparticle and water [59, 60]

Properties [Unit]	Water	TiO_2
Thermal conductivity [$Wm^{-1}K^{-}1$]	0.645	11.7
Density [gcm^{-3}]	1	4.13

Comparison of thermal conductivity for TiO_2-water nanofluid with mathematical model(s) and experimental studies

In this study results of thermal conductivity of TiO_2-water based nanofluid having different volume fraction have been compared with different mathematical model(s) to get optimum model.

Ahmed [61] studied TiO_2 and water based nanofluid for improving car radiator performance. In this study volume fraction used was in the range of 0.1-0.3%, with Reynolds number ranging from 560-1650. Synthesis of nanofluid was done with two step method and nanoparticles were synthesized with 8000 ball milling with particle average diameter 44 nm. Surfactant used in this studies was Triton-X with a concentration of 0.5%. Enhancement of 47% effectiveness was observed with 2% volume fraction within the temperature range of 20-80 °C.

Fedele [62] studied viscosity and thermal conductivity for TiO_2-water based nanofluid. Mass percentage used in these studies is in the range of 1-35%. Temperature range used to measure other parameter is 283-343 K. Reading was taken within the temperature range of 293.7-352.4 K with a regular interval. 33.2% enhancement in thermal conductivity was observed at 352.4 K and 11.2% volume fraction (35 wt.%).

Ghadimi *et al.* [63] studied surfactant and ultrasonic processing effect on the stability, thermal conductivity and viscosity for TiO_2-water based nanofluid. Synthesis of nanofluid carried out with a concentration of 0.1 wt.% and (sodium dodecyl sulfate – SDS, surfactant. Nanoparticle diameter was an average size of 25 nm diameter. It has been reported that the variation in thermal conductivity, viscosity and stability time was dependent on surfactant and ultrasonic process.

Murshed *et al*. [64] studied thermal conductivity of TiO_2-water based nanofluid for rod shape and spherical shape nanoparticle, with volume fraction up to 5%. Ultrasonic processed 8-10 hours for better stability. The CTAB surfactant used with 0.01-0.02% volume fraction. Experimental data were compared with the Wasp model and H-C model and observed that the prediction of these model was much lower than the experimental result for both rod shape and spherical shape TiO_2 nanoparticle. For spherical shape maximum enhancement of 29.70% was observed and for rod shape maximum enhancement of 32.80% was observed.

Saleh *et al*. [65] studied TiO_2-water based nanofluid for heat transfer application. Volume fraction up to 1.0% has been studied within the temperature range of 10-60 °C. Nanofluid was prepared with two step method with ultrasonic process of two hours. Surfactant used are SDS, CTAB, and Span 80 in which enhancement in thermal conductivity with SDS was maximum. Comparison of experimental data with mathematical model was also done for relative thermal conductivity and it was observed that most of the model giving lower prediction result.

He *et al.* [66] studied TiO_2-water based nanofluid for heat transfer enhancement. Volume fraction used in preparation of nanofluid was in the range of 0.24-1.18%. Thermal conductivity in this study found to be directly proportional to the volume fraction and Reynolds number within the range, and it is also observed that heat transfer enhancement of experimental data was higher than theoretical value.

Yoo *et al.* [67] studied TiO_2-water, Al_2O_3-water, Fe-EG, and WO_3-EG. The VF in TiO_2-water based nanofluid was 0.1, 0.5, and 1. Measurement of thermal conductivity was done using transient hot-wire method. Enhancement in thermal conductivity was observed in all four cases.

Reddy *et al.* [59] studied TiO_2-water/EG based nanofluid for estimating thermal conductivity. Volume fraction used in the range of 0.2-1%. Thermal conductivity was measured in the range of 30-70 °C. Average diameter of nanoparticle was 21 nm. Experimental result was compared with H-C model, Bruggeman model, Yu model, Wasp model, and observed that prediction of all model was much lower than experimental result. A new correlation was also proposed in this study.

Duangthongsuk *et al.* [35] studied temperature-dependent thermal conductivity for TiO_2 and water based nanofluid. Volume fraction used in the range of 0.2-2%. Transient hot wire method used to measure thermal conductivity in the range of 15°-35°C. Experimental result was compared with the H-C model, Bruggeman model, Wasp model, Yu, and Cho model, EMT model and observed that mathematical models are giving lower prediction value. A new correlation was also proposed in this paper to estimate thermal conductivity and viscosity.

Variation in thermal conductivity in previous studies has been shown in fig. 3. Graph has been plotted between thermal conductivity vs volume fraction. Volume fraction used in the range of 0.0005-0.0554. Maximum thermal conductivity was achieved by Murshed *et al.* [64]. Duangthongsuk *et al.* [35] achieved 0.6 W/mK for 0.01 VF and Murshed [64] achieved 0.8127 W/mK thermal conductivity for 0.01 VF. Primary factor for 35.45% variation are temperature and shape. Volume fraction in most of the studies are under 0.01.

In 1873 a theoretical model was proposed by Maxwell to estimate thermal conductivity. Maxwell model [69] is the preferable model and it is applicable for millimetre to micrometre-sized particle [70]:

$$k_{\mathrm{nf}} = \frac{k_{\mathrm{p}} + 2k_{\mathrm{f}} + 2\Phi\left(k_{\mathrm{p}} - k_{\mathrm{f}}\right)}{k_{\mathrm{p}} + 2k_{\mathrm{f}} - \Phi\left(k_{\mathrm{p}} - k_{\mathrm{f}}\right)} k_{\mathrm{f}} \tag{1}$$

$$\Phi = \frac{1}{\left[\left(\dfrac{1-\Phi_{\mathrm{m}}}{\Phi_{\mathrm{m}}}\right)\left(\dfrac{\rho_{\mathrm{p}}}{\rho_{\mathrm{f}}}\right)\right] - 1} \tag{2}$$

In most of the study, it has been observed that Maxwell mode gives lower prediction result. In fig. 4, it can observed that Maxwell model prediction lies in the range of previous studies.

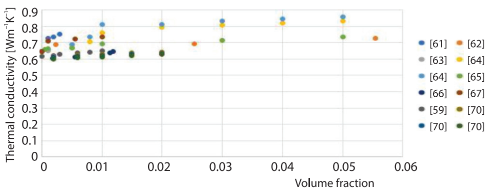

Figure 3. Variation in thermal conductivity *vs.* volume fraction for previous studies

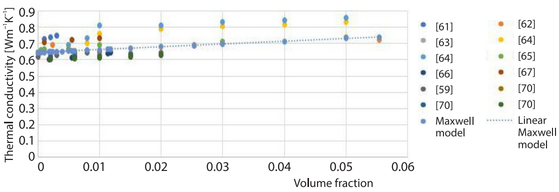

Figure 4. Variation in thermal conductivity *vs.* volume fraction for previous studies and Maxwell model

Gao *et al.* [71] studied graphene nanoplatelet with different base fluid DW, EG, and equal proportion of EG : DW. It was observed that Maxwell model prediction was lower than the experimental result. Islam [72] also observed that the prediction of thermal conductivity by the Maxwell model is lower than the experimental result.

Improvement in Maxwell model by considering shape factor is made in by Hamilton and crosser model [73, 74]:

$$\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = \frac{k_{\mathrm{p}} + (n-1)k_{\mathrm{f}} - (n-1)\Phi\left(k_{\mathrm{f}} - k_{\mathrm{p}}\right)}{k_{\mathrm{p}} + (n-1)k_{\mathrm{f}} + \Phi\left(k_{\mathrm{f}} - k_{\mathrm{p}}\right)} \tag{3}$$

Hayat *et al.* [75] compared Maxwell model and H-C model for biomedical engineering application. Different nanoparticle (including TiO_2)-water based nanofluid has been studied. It has been observed that trend line for Maxwell model and H-C model is similar but Maxwell model prediction superior compare to H-C model.

Maxwell-Garnetts model [69, 76] is based on the effective medium theory. In this model it is assumed that the particles are uniformly distributed and interaction between the particles is negligible. This model assumed that the volume fraction, thermal conductivity of particle and base fluid are the main reason for enhancement in thermal conductivity of nanofluid:

$$\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = \frac{k_{\mathrm{p}} + 2k_{\mathrm{f}} - 2\Phi(k_{\mathrm{f}} - k_{\mathrm{p}})}{k_{\mathrm{p}} + 2k_{\mathrm{f}} + \Phi(k_{\mathrm{f}} - k_{\mathrm{p}})} \tag{4}$$

In fig. 5 it can be observed that the prediction of thermal conductivity by Maxwell-Garnett's model is in the range of experimental result.

Pak *et al.* [77] and Kundan *et al.* [78] model assumes that only volume fraction is the responsible for the enhancement in thermal conductivity of the base fluid.

$$\frac{k_{\mathrm{eff}}}{k_{\mathrm{l}}} = 1 + 7.47\Phi_{\mathrm{p}} \tag{5}$$

In fig. 6, it can be observed that the prediction of thermal conductivity by Pak Cho model is linearly increasing and after 0.05 volume fraction prediction is more than experimental result. Since experiment studies above 0.05 volume fraction is less so we cannot judge.

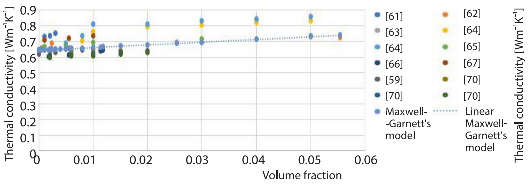

Figure 5. Variation in thermal conductivity *vs.* volume fraction for previous studies and Maxwell-Garnett's model

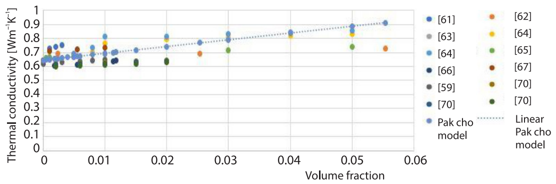

Figure 6. Variation in thermal conductivity *vs.* volume fraction for previous studies and Pak Cho model

Mehdi Bahiraei *et al.* Model [79] was derived on the basis of experimental studies, model has been derived for thermal conductivity using two-dimension regression:

$$k_{\mathrm{nf}} = 0.05099(\Phi + 1)^{0.07085}\, T^{0.4353} \tag{6}$$

Experimental data and prediction by Bahiraei *et al.* [79] model have very less deviation. But in fig. 7, it can be observed that the prediction of thermal conductivity is lower than experimental studies.

Timofeeva *et al.* model [80] based on EMT. This consider effect of agglomeration on nanofluids [78]:

$$k_{\mathrm{nf}} = k_{\mathrm{bf}}(1 + 3\Phi) \tag{7}$$

In fig. 8, it can be observed that the prediction of thermal conductivity by [80] model is within the range of experimental results.

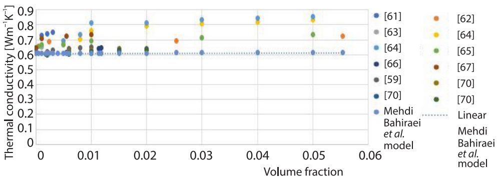

Figure 7. Variation in thermal conductivity *vs.* volume fraction for previous studies and [79] model

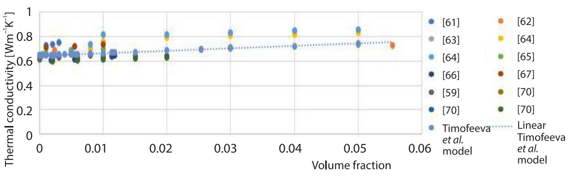

Figure 8. Variation in thermal conductivity *vs.* volume fraction for previous studies and [80] model

Li *et al*. model [81] assumed that the temperature and the volume fraction of the nanoparticle are the most important parameter for the enhancement in thermal conductivity of the nanofluid. Aggregation of nanoparticle in nanofluid is assumed to be negligible:

$$\frac{k_{\mathrm{eff}}}{k_{\mathrm{l}}} = 1 + 0.76448\Phi_{\mathrm{p}} + 0.01868867T - 0.46214175 \tag{8}$$

In fig. 9, it can be observed that the enhancement of thermal conductivity with respect to volume fraction is less as compared to experimental results. This model gives better result at lower volume fraction but when we increase volume fraction than deviation from experimental result is higher.

Bhattacharya *et al.* [82] model considers the effect of thermal conductivity of base fluid and nanoparticle along with the volume fraction. In fig. 10, it can be observed that the prediction of the thermal conductivity by [82] model is good up to 0.01 volume fraction. But at higher volume fraction prediction is higher than the experimental results.

$$\frac{k_{\text{eff}}}{k_\text{l}} = \varphi_\text{p} \frac{k_\text{p}}{k_\text{l}} + (1 - \varphi_\text{p}) \tag{9}$$

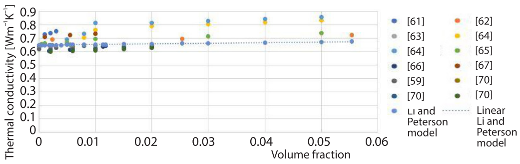

Figure 9. Variation in thermal conductivity *vs.* volume fraction previous for studies and [81] model

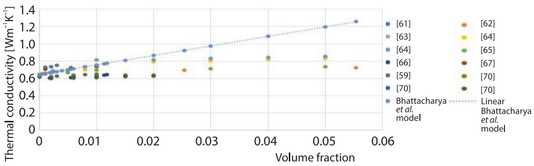

Figure 10. Variation in thermal conductivity *vs.* volume fraction for previous studies and [82] model

There are many other model available to estimate thermal conductivity of nanofluid, but we have been unable to use them because of limitation of available data and variation in environmental condition of different studies. Available mathematical models are Yu and Choi model, Xie model, Hui *et al.* model, Jeffery model, Xuan and Li model, Teng's *et al.* model, Lai *et al*. Model, Chu model, Koo and Klienstreuer, Das *et al*. model, Vajiha and Das model, Prashar *et al.* model, Yugao *et al.* model, Reddy *et al.* model, and Purohit *et al.* model. Some of these model are for specific nanofluid, some are for specific environmental condition like temperature range, volume fraction limit. Formula for determining thermal conductivity are different for different shape and size of the nanoparticle. Since experimental data used in this paper are from different environmental condition like particle shape, size, we can't be able to use these model to compare with experimental studies.

Conclusions

This review paper focusses on the thermal conductivity of the nanofluid. Preparation of nanofluid and factors influencing thermal conductivity of nanofluid has been discussed. Previous experimental studies for thermal conductivity has been compared for TiO_2-water nanofluid. Mathematical model are also compared with these experimental data. Due to unavailability of the data and different environmental condition we cannot be able to use all mathematical model. There are many other mathematical model available which can gives a satisfactory result. We can make the following conclusion with this paper as follows.

- Graph has been plotted for previous studies for TiO_2-water nanofluid. Volume fraction used in the range of 0.0005-0.0554.
- For 0.01 volume fraction thermal conductivity achieved was 0.6 W/mK and 0.8127 W/mK for TiO_2-water nanofluid in different studies. This shows that environment condition and other factor explained in this paper affect thermal conductivity of nanofluid.
- Maxwell model, Maxwell-Garnetts model, and Timofeeva model prediction of thermal conductivity was satisfactory with the experimental result. Pak Cho *et al*. model satisfactory up

to 0.03 volume fraction, Li and Peterson model result is satisfactory up to 0.02 volume fraction, Bhattacharya *et al*. Model result was satisfactory up to 0.01 volume fraction, above this limit prediction was lower are higher than experimental result limit. Bahiraei prediction was lower than experimental result. Since more studies required so, we cannot judge these model

- For better stability volume fraction should be less than 0.01. It can be observed that the prediction of all mathematical model are good below 0.01 volume fraction.

Nomenclature

k – thermal conductivity

Greek symbols

Φ – volume fraction
ρ – density

Acronyms

EG – ethylene glycol
SDS – sodium dodecyl sulfate
VF – volume fraction

Subscripts

f – fluid
m – mass
nf – nanofluid
p – nanoparticle

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